DYNAMICS OF CUTTING VISCOELASTIC MATERIALS

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ABSTRACT

Mechanical cutting of visco-elastic polymers is experimentally investigated using sharp knives. The knife is aligned orthogonally to the substrate's surface, and is forced into the substrate. Two cutting scenarios are investigated i) plunge cutting, where the knife edge moves directly into the substrate, and ii) slicing, where the knife shears tangentially as it moves into the substrate. For slicing fracture lines emanating from the cut are observed, and the threshold forces for cutting are considerably smaller, indicating that the two cutting processes are very different.

1 INTRODUCTION

One of the major aspects of many manufacturing processes involves cutting of material. Modern techniques include i) lasers for vaporizing the material, ii) strong water jets for blasting away the material, iii) electrical discharge machining (EDM) for electrically heating and removing the material as well as iv) wet chemistry for etching away material. More traditional methods are simple mechanical cutting, which can roughly be divided into two categories: a) orthogonal cutting (sawing, grinding, drilling etc.) and b) tangential cutting (slicing, guillotining). Orthogonal cutting works well for harder materials, and examples are milling steel and sawing wood [1, 2, 3, 4]. Tangential cutting often is better suited for softer, viscoelastic materials [5, 6], and examples are slicing food or sectioning biological tissues into micron-thick slices [7]. Although there is a significant body of research on the mechanics of orthogonal cutting, motivated in part by the desire to shape hard materials such as metals, surprisingly little is known about the mechanics of tangential cutting, even though this process combines aspects of materials engineering with tribology (friction), wear and fracture (crack formation).

2 EXPERIMENTAL PROCEDURE

Two experiments were developed to investigate the cutting of soft, viscoelastic materials. We adapted an advanced rheometer by replacing the standard rheological geometries with custom-built knives for investigating slicing, and we constructed a setup for investigating simple plunge cutting. Our model substrate for cutting was PDMS, which is silicone oil, combined with a cross-linker to create an elastic material. The parameter space for exploration consists of varying the cutting knives, the cutting procedures (i.e. force, displacement etc.) and the material properties of the PDMS substrate.

The rheometer is able to rotate the circular knife at a controlled torque, and move the knife in the axial direction. With feedback it then is possible to control the angular velocity and the normal force on the knife edge. Figure 1a) shows one such knife, which is similar in appearance to a cookie-cutter. In the figure the rotation is along the s-direction, as indicated by the arrow, and the axial (downwards) direction is chosen along the z-axis. Figure 1b) shows a schematic fur such a knife that is cutting a PDMS sample. The vertical force and torque are distributed among frictional forces along the knife's walls, and forces on the actual knife edge, which is responsible for cutting.

The custom-built apparatus for plunge cutting consists of a laboratory razor blade mounted to a translation stage, and a precision balance. The substrate rests on the balance, and measures the downwards force transmitted by the moving razor blade.

The model viscoelastic material we have chosen for the cutting experiments is the cross-linked silicone polymer, PDMS [8]. The monomer units are $\mathrm{Si}\,(\mathrm{CH}_3)_2\,\mathrm{O}$, and the polymer back-bone is composed of $\mathrm{Si}-\mathrm{O}$ bonds, that are extremely strong ($\sim$ 5 eV). A silicone cross-linker is added and the sample is cured at moderate temperatures (~ 100 °C) to create the viscoelastic solid. By controlling the curing times and amount of cross-linker, the elastic modulus can be varied over a decade from $0.5\mathrm{N/mm}^2 \lesssim E \lesssim 5\mathrm{N/mm}^2$. The top surface of the PDMS, is exposed to air during the curing process, resulting in a very smooth surface. This is the surface that the knife encounters.

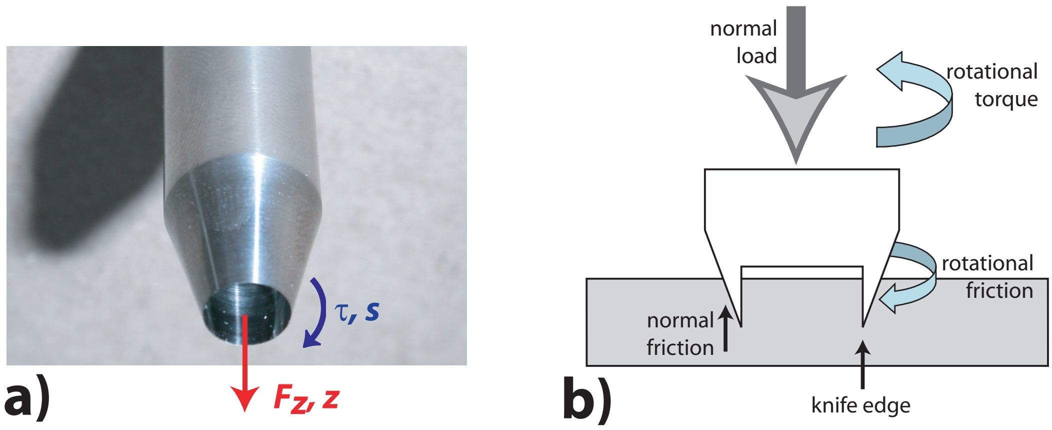

Figure 1: Circular cutters, with diameter of about 1 cm. a) shows the orientation of the cutter. b) Force diagram for tangential cutting. The applied load and rotational force are distributed among the side walls and the sharp knife edge.

3 RESULTS

Plunge cutting can be performed either by controlling the normal force on the knife or the displacement of the knife in the substrate. The material will yield viscoelastically or by breaking bonds (i.e. being cut). In figure 2a the forces on the knife blade are shown during a step-wise cutting of the PDMS substrate. The nominal depth of the knife edge relative to the top surface of the unstrained PDMS sample increases from $z = 0.3, 0.6, \ldots, 2.4$ mm. Between displacements the force on the knife relaxes in a characteristic fashion, plotted in 2b. The log-log plot indicates that there is a long-term relaxation process which can be fairly well fitted by the functional form

$$F_{\text{relax}} \propto t_{\text{relax}}^{\gamma} , \tag{1}$$

where the exponent γ decreases from $\gamma \approx -0.09$ to $\gamma \approx -0.12$ with increasing depth, see figure 2c. The sample thickness is 3.2 mm, and the experiments were conducted at room temperature.

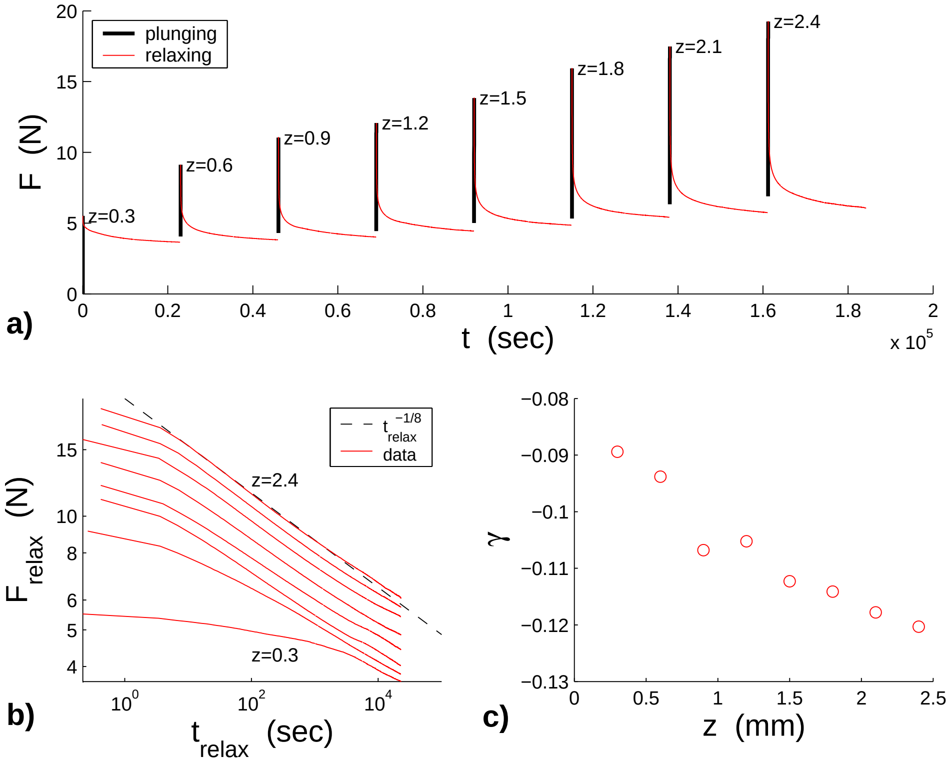

Figure 2: Direct plunge cutting of PDMS with a razor blade. a) The knife edge's position was successively increased from $z = 0.3, 0.6, \ldots 2.4$ mm, and the force was allowed to relax for about six hours between steps. b) Log-log plot of the force relation for all six cuts. The dashed line showing $t_{\text{relax}}^{-1/8}$ is included for comparison. c) Shows the relaxation exponents, γ, as a function of the depth within the sample.

The slow dynamical process, seen in figure 2b where the forces drop to half their initial values over hours, can be attributed to stress relaxation of the bulk modulus, $G(t)$, rather than a slow cutting process. Separate experiments show that the cutting depth does not substantially change over time, for $t_{\text{relax}} > 10$ sec. Thus actual cutting must occur on shorter time scales and/or during the plunging process (which takes about 20 seconds for each step) where the forces on the knife are rapidly increasing.

Rotating the cutter introduces new dynamics and can significantly increase the cutting rates as well as decrease the threshold force required for cutting as compared with plunge cutting. Figure 3 shows the depth of the circular knife inside the substrate, along with the torque required to maintain a normal force $F_z = 1$ N, and angular velocity $\omega = 0.02$ rad/sec. The top plot shows the entire experiment, which took about 22 hours, and the lower plot focuses on the startup regime.

Most of the cutting occurs within the first $s = 50$ cm of shearing, after which the cutting depth has reached a plateau. Initially the torque increases linearly with the azimuthal distance, indicating that the material is elastically being twisted by the rotating knife. At $s \approx 1.5$ mm, the torque behaves differently, indicating that the knife is sliding across the surface, and wearing down the surface [9, 10]. Somewhat later, at $s \approx 3.5$ mm, indicated in the figure, the knife is starting to penetrate the substrate. Much later, after shearing over a distance of $s = 5$ m, the knife is actually being forced out of the substrate [11].

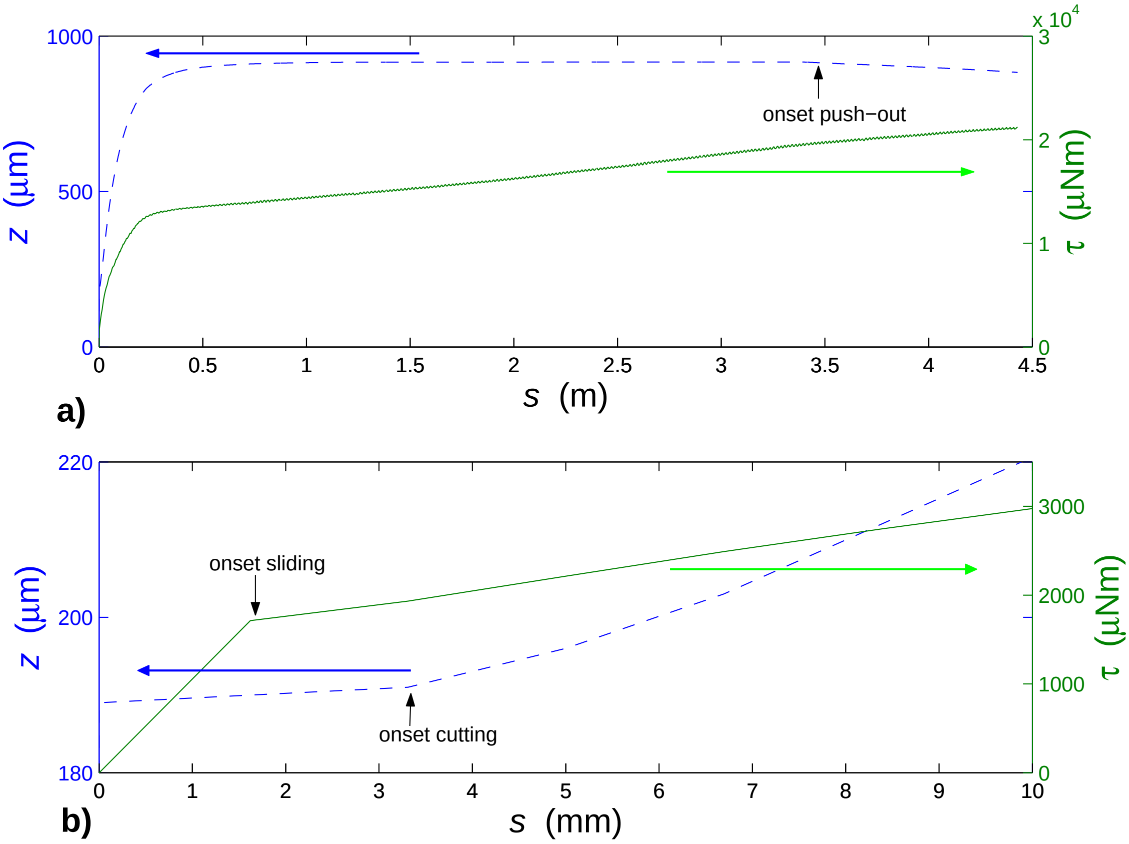

Figure 3: Slicing with a circular cutter at a normal force $F_z = 1$ N, and velocity $\omega = 0.02$ rad/s. The plots show the dependence of the torque and cutter depth on the distance the knife has rotated. a) Long-term dynamics, where the cutting depth monotonically increases until $s \approx 3.5$ m, beyond which the cutter is "pushed out" of the substrate. b) Shows the initial startup, where the torque linearly increases $s \leq 1.5$ mm, and the onset of cutting occurs later at $s \approx 3.5$ mm.

Performing experiments similar to the one shown in figure 3 at lower normal forces increases the shearing distance at which cutting occurs. Before this onset of cutting, the knife edge appears to be benignly sliding over the smooth PDMS surface.

Provided that the angular velocities are fairly slow, $\omega \lesssim 10$ rad/sec, the cutting rate scales with the angular velocity in a simple fashion for a given normal force:

$$\frac{dz}{dt} \sim \left(\frac{\partial s}{\partial t}\right)\left(\frac{\partial z}{\partial s}\right) , \tag{2}$$

where the last term depends on the normal force. Thus by doubling the angular velocity, the cutting rate dz/dt roughly doubles.

Some of the cut samples show unusual crack formation when viewed under the microscope, see figure 4. In order to produce such radial cracks, the cutter must perform many rotations. The cracks are fairly regularly spaced around the outside of the cutter at a characteristic angle close to 45°, pointing inwards in the direction of the rotation (which is counter-clockwise here). Larger cracks emanate from the cut and have a characteristic length of 300 μm, however smaller cracks exist that are hundreds of microns away from the cut.

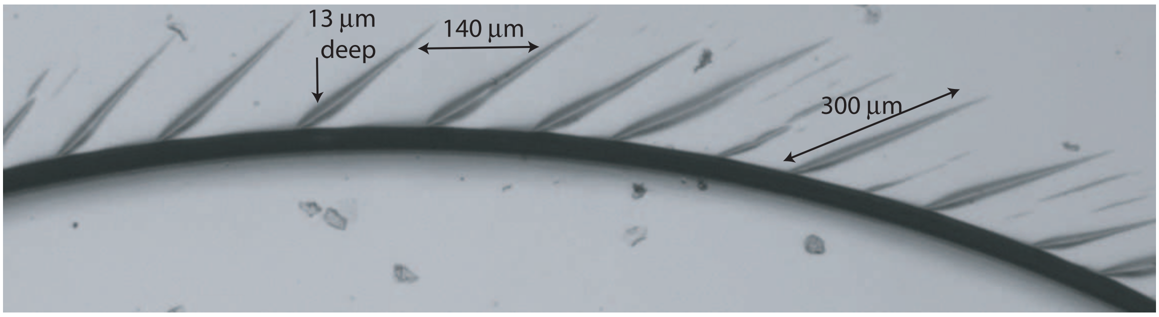

Figure 4: Cracks formed by rotational cutting shown in figure 3. The main cut is ≈ 0.8 mm deep, and the side cuts are ≈ 10 μm deep.

CONCLUSIONS

The cutting process involves several mechanisms, other than simply breaking bonds. The large localized stresses exerted by the knife lead to large stress gradients, which in turn cause viscoelastic deformation. Depending on how quickly the stresses are applied the material the polymers will re-arrange via reptation, pullout of the polymer chains or simply break [12, 13]. Comparing estimates for the work required to break the bonds and actual work performed by the knife shows that the cutting efficiency is very small, indicating that energy is distributed among many mechanisms other than simple cutting.

ACKNOWLEDGEMENTS:

The author thanks Glen Hill for his help in performing many of the experiments.

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[7] Du Pont manufactures an ultra-microtome that can cut micron-thick sections (Sorvall MT-5000 Ultamicrotome).

[8] Made by Down Corning under the name of Sylgard 184.

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[11] A possible explanation is that the crack collects debris, which provides a normal force on the knife and forces it out.

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