PROCEDURE OF STATISTICAL SIZE EFFECT PREDICTION FOR CRACK INITIATION PROBLEMS

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ABSTRACT

An improved generalized law for combined energetic-probabilistic size effect on the nominal strength for structures failing by crack initiation from a smooth surface is used for practical purposes – the paper proposes a procedure to capture both deterministic and statistical size effects on the nominal strength of quasibrittle structures failing at crack initiation. The advantage of the proposed approach is that the necessity of time consuming statistical simulation is avoided, only deterministic nonlinear fracture mechanics FEM calculation must be performed. Results of deterministic nonlinear FEM calculation should follow deterministic-energetic formula, a superimposition with the Weibull size effect, which dominates for large sizes using the energetic-statistical formula, is possible. As the procedure does not require a numerical simulation of Monte Carlo type and uses only the results obtained by deterministic computation using any commercial FEM code (which can capture satisfactorily deterministic size effect), it can be a simple practical engineering tool.

1 INTRODUCTION AND SIZE EFFECT FORMULAE

Practical and simple approach to incorporate the statistical size effect into the design or the assessment of very large unreinforced concrete structures (such as arch dams, foundations and earth retaining structures, where the statistical size effect plays a significant role) is important. Failure load prediction can be done without simulation of Monte Carlo type utilizing the energetic-statistical size effect formula in mean sense together with deterministic results of FEM nonlinear fracture mechanics codes.

This work is based on the latest achievements of Bažant, Vořechovský and Novák [1] who proposes a new improved law with two scaling lengths (deterministic and statistical) for combined energetic-probabilistic size effect on the nominal strength for structures failing by crack initiation from smooth surface. The role of these two lengths in the transition from energetic to statistical size effect of Weibull type is clarified. Relations to the recently developed deterministic-energetic and energetic-statistical formulas are presented. The paper by Bažant, Vořechovský and Novák [1] also clarifies the role and interplay of two material lengths: deterministic and statistical.

The deterministic energetic size effect formula for crack initiation from smooth surface reads (e.g. Bažant, Bažant and Planas, Bažant and Novák) [2,3,4]:

$$\sigma_N(D) = f_r^\infty \left[1 + \frac{rD_b}{D + l_p} \right]^{1/r} \tag{1}$$

where σ_N is the nominal strength depending on the structural size D. Parameters f_r^∞, D_b and r are positive constants representing the unknown empirical parameters to be determined. Parameter f_r^∞ represents solution of the elastic-brittle strength which is reached as a nominal strength for very large structural sizes. The exponent r (a constant) controls the curvature and the slope of the law. The exponent offers a degree of freedom while having no effect on the expansion in derivation of

the law (Bažant, Bažant and Planas) [2,3]. Parameter D_b has the meaning of the thickness of cracked layer. Variation of the parameter D_b moves the whole curve left or right; it represents the deterministic scaling parameter and is in principle related to grain size and drives the transition from elastic brittle (D_b=0) to quasibrittle (D_b >0) behavior.

By considering the fact that extremely small structures (smaller than D_b) must exhibit the plastic limit, a parameter l_p is introduced to control this convergence. The formula (1) represents the full size range transition from perfectly plastic behavior (when $D \to 0; D \ll l_p$) to elastic brittle behavior ($D \to \infty; D \gg D_b$) through quasibrittle behavior. Parameter l_p governs the transition to plasticity for small sizes D (crack band models or averaging in nonlocal models leads to horizontal asymptote). The case of $l_p \neq 0$ shows the plastic limit for vanishing size D and the cohesive crack and perfectly plastic material in the crack both predicts equivalent plastic behavior. For large sizes the influence of l_p decays fast and therefore the cases of $l_p \neq 0$ are asymptotically equivalent to case of $l_p = 0$ for large D.

The large-size asymptote of the deterministic energetic size effect formula (1) is horizontal: $\sigma_N(D)/f_r^\infty$=1, see fig. 1a). But this is not in agreement with the results of nonlocal Weibull theory as applied to modulus of rupture (Bažant and Novák [5]), in which the large-size asymptote in the logarithmic plot has the slope $-n/m$ corresponding to the power law of the classical Weibull statistical theory (Weibull [6]). In view of this theoretical evidence, there is a need to superimpose the energetic and statistical theories. Such superimposition is important, for example, for analyzing the size effect in vertical bending fracture of arch dams, foundation plinths or retaining walls.

The statistical part of size effect and the existence of statistical length scale have been investigated in detail (Vořechovský and Chudoba [7]) for particular case of glass fibers. The work shows, briefly, that the statistical part of size effect in structures with stationary strength random field has a large-size asymptote in the classical Weibull form (straight line in double-log plot $-n/m$) while the left (small size) asymptote is horizontal. The value of the horizontal asymptote for $D \to 0$ is the mean strength of the random field, and in Weibull understanding it is the mean strength measured for the reference length being equal to the autocorrelation length l_s. So by introduction of the random strength field we introduce the length scale (l_s).

By incorporating this result (statistical part) into the formula (1) we get a final law (Bažant, Vořechovský and Novák [1]):

$$\sigma_N = f_r^\infty \left[\left(\frac{L_0}{L_0 + D} \right)^{rn/m} + \frac{rD_b}{l_p + D} \right]^{1/r} \tag{2}$$

This formula (which is very close to a general law derived by Bažant [8]) exhibits the following features:

- Small size left asymptote is correct (deterministic), parameter l_p drives to fully plastic transition for small sizes.
- Large size asymptote is the Weibull power law (statistical size effect, a straight line with the slope $-n/m$ in the double-logarithmic plot of size versus nominal strength)
- The formula introduces two scaling lengths: deterministic (D_b) and statistical (L_0). The mean size effect is partitioned into deterministic and statistical parts. Each have its own length scale, the interplay of both embodies behavior expected and justified by previous research. Parameter D_b drives the transition from elastic-brittle to quasibrittle and L_0 drives the transitional zone from constant property to local Weibull via strength random field. Note that the autocor-

relation length l_s has direct connection to our statistical length L_0. This correspondence is explained in papers by Vořechovský and Chudoba [7] or Bažant, Vořechovský and Novák [1]. Having the summation in the denominators limit both the statistical and deterministic parts from growing to infinity for small D. So it remedies the problem that the previous energetic-statistical formulas (Bažant and Novák [4,5]) intersect the deterministic law at the size $D=D_b$ and therefore gives higher mean nominal strength prediction for small structures compared to the deterministic case.

Note that for $m\rightarrow\infty$ it degenerates to deterministic formula (1). The same applies if $L_0\rightarrow\infty$. The interplay of two scaling lengths using the ratio L_0/D_b is demonstrated by Bažant, Vořechovský and Novák [1]. The question arises what is in reality the ratio L_0/D_b? Since both scaling lengths are in concrete probably driven mainly by grain sizes, we expect $L_0\approx D_b$, so the simpler law with $L_0=D_b$ should be an excellent performer in practical cases.

2 SUPERIMPOSITION OF FEM DETERMINISTIC-ENERGETIC AND STATISTICAL SIZE EFFECTS

As was already mentioned deterministic modeling with NLFEM can capture only deterministic size effect. A procedure of superimposition with statistical part should be established. Such procedure of the improvement of the failure load (nominal stress at failure, deterministic size effect prediction) obtained by a nonlinear fracture mechanics computer code can be as follows:

1) Suppose that the modeled structure has characteristic dimension D_t. The natural first step is to create FEM computational model for this real size. At this level the computational model should be tuned and calibrated as much as possible (meshing, boundary conditions, material etc.). Note that we obtain a prediction of nominal strength of the structure (using failure load corresponding to the peak load of load-deflection diagram) for size D_t, but it reflects only deterministic-energetic features of fracture. Simply, the strength is usually overestimated at this (first) step; the overestimation is more significant as real structure is larger. Result of this step is a point in the size effect plot presented by a filled circle in figure 1a).

2) Scale down and up geometry of our computational model in order to obtain the set of similar structures with characteristic sizes D_i, $i=1,\ldots, N$. Based on numerical experience a reasonable number is around 10 sizes and depends how the sizes cover transition phases. Therefore, sizes D_i should span over large region from very small to very large sizes. Then calculate nominal strength for each size σ_i, $i=1,\ldots, N$. Note that for two very large sizes nominal strengths should be almost identical as this calculation follows energetic size effect with horizontal asymptote. If not, failure mechanism is not just only crack initiation, other phenomena (stress redistribution) plays more significant role and the procedure suggested herein cannot be applied. The computational model has to be mesh-objective in order to obtain objective results (e.g. crack band model, nonlocal damage continuum) for all sizes. In order to ensure that phenomenon of stress redistribution (causing the deterministic size effect for the range of sizes) is correctly captured, well tested models are recommended for strength prediction. A special attention should be paid to the selection of constitutive law and localization limiter. The result of this step is a set of points (circles) in the size effect plot as shown in figure 1a).

3) The next step is to obtain the optimum fit of the deterministic-energetic formula (1) using the set of N pairs $\{D_i, \sigma_i\}$ $i=1,\ldots, t, \ldots, N$. The result of this step is the set of values of four parameters: f_r^∞, D_b, r and l_p. The parameter l_p can be excluded from the fit based on the plastic analysis (this is fully described by Bažant, Vořechovský and Novák [1]). Fit of the parameter f_r^∞ can also be avoided because this limit can be estimated from nonlinear FEM analysis as the value to which the nominal strength converges with increasing size. So we can be prescribe (for very large sizes), $\sigma_N/f_r^\infty=1$ as asymptotic limit. The result of this step is illustrated by a fitted curve to the set of points in figure 1a).

4) There are three remaining parameters which should be substituted into statistical-energetic formula (2): n, m and L_0. Parameter n is the number of spatial dimensions ($n=1,2$ or 3). Parameter m represents the Weibull modulus of FPZ with Weibull distribution of random strength. Recent study due to Bažant and Novák [4] reveals that, for concrete and mortar, the asymptotic value of Weibull modulus $m\approx24$ rather than 12, the value widely accepted so far. The ratio n/m therefore represents the slope of MSEC in the size effect plot for $D\rightarrow\infty$. This means that for extreme sizes the nominal strength decreases, for two-dimensional (2D) similarity ($n=2$), as the -1/12 power of the structure size. Note, that for different material the asymptotic value of Weibull modulus is different, e.g. for laminates much higher than 24. Result of these 4 steps are shown for illustration in fig 1a).

Parameter L_0 is now only remaining parameter to be determined. As it represents statistical length scale it seems to be that we will need to utilize statistical software incorporated into your NLFM code. But there is much simpler alternative based on simple calculation of local Weibull integral.

Figure 1: Illustration of the superimposition steps. a) Steps 1-4 resulting in deterministic fit; b) step 5 – determination of parameter L_0; c) final formula and nominal strength prediction for the real structural size

A choice of statistical length scale l_s is a primary task and must be made (a good judge may be probably $l_s\approx D_b$. Since the choice about a scatter of FPZ strength must be made (Weibull modulus
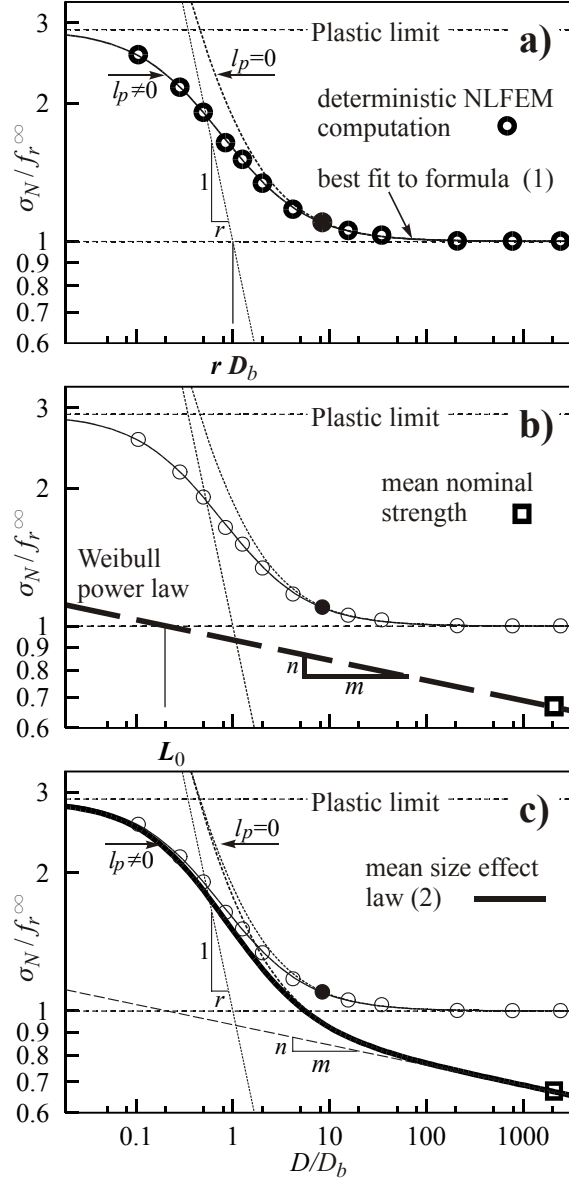
m driving the power of size effect for large sizes), one can compute large size structure having the Weibull strength of each FPZ. Once the mean strength of such large structure is known (a point in the size effect plot with coordinates D_{stat}, σ_{stat}, one can pass a straight line of slope n/m through the point (Weibull asymptote). Graphically, the intersection of the statistical (Weibull) asymptote with deterministic strength for infinite structure size (horizontal asymptote) f_r^∞ gives the statistical scaling length on *D*-axis, see figure 1b). The numerical solution to L_0 can be written as:

$$L_0 = D_{stat} \left(\frac{\sigma_{stat}}{f_r^\infty} \right)^{n/m} \tag{3}$$

and therefore this parameter does not need to be fitted, analytical expression can be used. Note that the large size strength (mean strength σ_{stat}) can be computed by Weibull integral, where the choice of reference volume V_0 and Weibull modulus (scatter) must be made (this is described in detail e.g. by Bažant and Planas [3]):

$$P_f = 1 - \exp\left(-\int_V \left\langle \frac{\sigma(x)}{s_0} \right\rangle^m \frac{dV(x)}{V_0} \right) \tag{4}$$

where *V* is the volume (area, length) of the structure depending on dimension (*n*), s_0 is the Weibull scaling parameter and V_0 is an elementary volume of the material for which the Weibull distribution has parameters *m* and s_0. The function $\sigma(x)$ is the maximum principal stress at a point of coordinate vector *x*. One can avoid the computation of nonlocal integral (and determination of load leading to P_f corresponding to the mean load) by means of numerical simulation of Monte Carlo type. In such case we recommend to use the stability postulate of extreme values for discretization of random blocks and their association with scaled PDF. This approach has been used in the numerical example and is described in detail by Novák, Bažant and Vořechovský [9,1].

5) As all parameters of statistical-energetic formula are determined, nominal strength can be calculated for any size. Using real size of the structure D_t the prediction of the corresponding nominal strength $\sigma_{N,t}$ can be done using (2). This prediction will be generally different (lower) from initial deterministic prediction, figure 1c). The larger structure the larger difference is. The formula will provide us the strength prediction for the mean strength. Additionally, a scatter of strength can be determined just using the fundamental assumption of Weibull distribution. For the distribution we know two parameters, shape parameter *m* is prescribed initially, and scale parameters *s* can be calculated easily from predicted mean and Weibull modulus.

3 SUMMARY AND CONCLUSIONS

The paper presents an analytical formula for the nominal mean strength prediction of crack initiation problems. The paper suggests a practical procedure of superimposition of deterministic and statistical size effect at crack initiation. It requires only a few FEM analysis using scaled sizes and simple linear stochastic simulation of a large size structure. The prediction can be done without complicated and time consuming Monte Carlo simulation, which is usually used to deal with influence of uncertainties on structural strength.

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