MODES OF DYNAMIC RUPTURE ON INTERFACES WITH NONLINEAR RATE AND STATE FRICTION LAWS

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ABSTRACT

Dynamic ruptures on frictional interfaces propagate as dynamic frictional shear cracks with well-defined rupture fronts and rupture velocities comparable to the wave speeds of the surrounding elastic media. On a frictional interface, the faces of the shear crack behind the rupture front are not traction-free and, in general, slide past each other as prescribed by the imposed friction law. In this study, laboratory-derived nonlinear rate and state friction laws are considered in which the frictional strength (or resistance) of the interface depends on the local sliding rate (or relative particle velocity) V and a state variable. The state variable incorporates dependence on the history of slip and describes the evolving properties of the microscopic contacts on the interface.

Zheng and Rice [1] showed that if an interface between two identical elastic half-spaces is described by these laws of steady-state rate-weakening type, the dynamic rupture can proceed in two ways: as the so-called crack-like mode or pulse-like mode. In the crack-like mode, the slip duration at the points on the interface is comparable to the duration of the dynamic rupture itself. In the pulse-like mode, the points slip for a much shorter time and then dynamically heal, so that the resulting mode resembles a wrinkle (in a shear sense) propagating along the interface. The rupture mode is determined by two factors: How high the interface shear stress is before the dynamic rupture and how strong the rate weakening is (Zheng and Rice [1]).

We find that there is another possibility, the multi-pulse mode, in which there are several pulses of slip propagating along the interface (Lapusta and Rice [2]). These multiple pulses occur in our simulations of dynamic ruptures in a two-dimensional anti-plane problem. The rupture starts in the crack-like mode but then the sliding behind the rupture front destabilizes with a wave-like pattern which grows into separated pulses of slip. The multiple pulses and their nucleation are well-resolved numerically, both in time and in space, and are reproducible in simulations with higher resolution. They are present only when the rate and state formulation incorporates sufficiently strong rate weakening so that the steady-state frictional resistance varies approximately as $1/V$ at high sliding rates V. For the logarithmic rate dependence of the traditional Dieterich-Ruina laws, only the crack-like mode results. The new multi-pulse mode is consistent with the classification of Zheng and Rice [1] and fits in the transitional regime between the crack-like and pulse-like modes.

The origin of these multiple pulses and their spacing can be explained by the linearized stability analysis of the steady elastodynamic sliding to Fourier mode perturbation. The analysis shows that there is the critical wavelength such that perturbations with shorter wavelengths decay and perturbations with longer wavelengths grow (Rice et al. [3]). We study the behavior of the growing wavelengths and find that there is a wavelength with the maximum growth rate (i.e., the perturbation with that wavelength grows the fastest). This is exactly the wavelength that we see during the nucleation of the multiple pulses behind the rupture front. The growth rate of that wavelength strongly depends on the amount of rate weakening in the friction law. The multiple pulses appear during the life-time of the dynamic rupture only if this growth rate is high enough. That is why we see multiple pulses only in the cases with strong rate weakening.

1 INTRODUCTION

Dynamic behavior of frictional interfaces is an important and challenging problem in both engineering and geophysical applications. Parts of frictionally held structural interfaces (in ship hulls, fiber-reinforced composites etc.) can fail under dynamic loading and the details of the resulting dynamic rupture may determine whether the structure or its components will fail.

Earthquakes propagate in the Earth's crust as dynamic frictional shear cracks and the resulting ground motion depends on the details of the dynamic rupture and, in particular, on its mode. Dynamic frictional sliding is also important in other problems such as machine cutting.

This study considers propagation of dynamic ruptures on a planar frictional interface between two identical elastic half-spaces. The interface is governed by the laboratory-derived rate and state friction laws which are presented in section 2. An interesting and important problem is how the frictional sliding behind the rupture front proceeds. Points on the interface may continue sliding as long as the sliding region keeps expanding. Such a mode of sliding is called crack-like. In this case, the duration of slip at each point is comparable to the duration of the dynamic rupture itself. Recently, observations showed that earthquake ruptures may propagate in such a way that the duration of sliding at each point along the fault is much shorter than the overall duration of the earthquake (Heaton [4]). This mode is called pulse-like.

Zheng and Rice [1] showed that both crack-like and pulse-like modes can be obtained on homogeneous rate and state interfaces with rate-weakening steady-state frictional properties and established what determines the rupture mode. In simplified terms, the rupture is pulse-like when the interface is under low pre-stress and crack-like when the interface is under high pre-stress where what is low or high in stress is quantified by the amount of rate weakening of the friction law. In the pulse-like mode, the sliding is dynamically arrested; this process is called self-healing. Note that while pulse-like ruptures can be generated by self-healing on rate-weakening interfaces with appropriate properties, this is not the only mechanism for producing slip pulses. Other candidate mechanisms include dynamic normal stress variations on interfaces between materials of different elastic properties and arrest waves from heterogeneities on the interfaces.

In our simulations, we observe another, muti-pulse, mode of dynamic rupture propagation as discussed in section 4. Section 3 discusses the linearized stability analysis of steady sliding that allows us to explain the initiation and spacing of the multiple pulses. We conclude in section 5 by discussing the implications of this new multi-pulse mode of dynamic rupture propagation and where it fits in the crack-like vs. pulse-like picture.

2 NONLINEAR RATE AND STATE FRICTION LAWS

The interfaces considered in this study are governed by nonlinear rate and state friction laws that have been developed in the last 25 years to explain various experimental results and geophysical observations (Lapusta et al. [5], Rice et al. [3], and references therein). The laws incorporate the dependence of frictional resistance on the instantaneous sliding rate V (given by the relative particle velocity across the interface) and state variables. The state variables describe the evolving properties of the microscopic asperity contacts on the interface.

For the case of constant in time normal stress σ and one state variable θ, the general formulation of these laws is given by

$$\tau = \sigma f(V,\theta), \quad d\theta / dt = \varphi(V,\theta), \tag{1}$$

where τ is the frictional strength (resistance) of the interface which is equal to the shear stress during sliding, $\sigma > 0$ is the compressive normal stress acting across the interface, and f is the friction coefficient. V and θ, and hence f and τ, depend on space variables and time. Note that σ in this formulation can vary in space but not with time. This is appropriate for the present study that considers sliding on a planar interface between identical elastic half-spaces because such sliding does not alter normal stress. If elastic properties were different on the two sides of the interface or the interface were non-planar, the sliding would dynamically alter normal stress. To describe properly the effects of normal stress that varies in time, eqns (1) would have to incorporate additional dependencies as discussed in Rice et al. [3] and references therein.

For laws (1) to reflect laboratory experiments, nonlinear functions $f(V,\theta)$ and $\varphi(V,\theta)$ should have certain properties. In sliding at a constant sliding rate V, the frictional resistance τ evolves toward the corresponding steady-state value $\tau_{ss}(V)$. This is described by the evolution of θ toward the constant value $\theta_{ss}(V)$ that satisfies $\varphi(V,\theta_{ss}) = d\theta/dt = 0$. In the vicinity of the steady state corresponding to an arbitrary sliding velocity V^*, eqns (1) linearize to

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = \frac{a^*\sigma}{V^*}\frac{\mathrm{d}V}{\mathrm{d}t} - \frac{V^*}{L^*}\left[\tau - \tau_{ss}(V)\right], \quad \tau_{ss}(V) = \tau_{ss}(V^*) + \frac{(a^*-b^*)\sigma}{V^*}(V - V^*), \tag{2}$$

where a^*, b^*, and L^* are obtained by evaluating the appropriate combinations of the partial derivatives of functions $f(V,\theta)$ and $\varphi(V,\theta)$ at the steady state values $V = V^*$ and $\theta = \theta_{ss}(V^*)$. Hence a^*, b^*, and L^* are numbers, in general dependent on V^*, that describe frictional properties of the interface for sliding velocities V in the vicinity of V^*. The term with a^* reflects the experimentally observed direct effect of the change in rate V on the change in τ. That effect is always positive in experiments and hence we assume $a^* > 0$ for any V^*. The next term incorporates the observation that during sliding with constant V the frictional stress τ evolves toward its steady-state value $\tau_{ss}(V)$ through characteristic slip of order L^*. The dependence of the steady-state value of stress on V in the vicinity of V^*, given by the second equation of (2), depends on the parameter $(a^* - b^*)$. The friction has steady-state rate-strengthening properties if $(a^* - b^*) > 0$ and rate-weakening properties if $(a^* - b^*) < 0$.

3 LINEARIZED STABILITY ANALYSIS OF STEADY FRICTIONAL SLIDING

To understand properties of the nonlinear laws (1), we pursue 2D linearized stability analysis of the steady sliding on a planar interface governed by laws (1) (Rice et al. [3] and references therein). Consider two elastic half-spaces sliding past each other steadily on the interface $y = 0$ with the uniform sliding rate V^* and shear stress $\tau^* = \tau_{ss}(V^*)$. For simplicity and comparison with the simulations of section 4, we consider anti-plane perturbations here; the in-plane problem can be analyzed in a similar manner and has more involved, but conceptually analogous results. To investigate the stability of such sliding to Fourier mode perturbations, we look for the anti-plane displacement solution $u_z(x,y,t)$ in the perturbed form

$$u_z(x,y,t) = V^*t \ \operatorname{sign}(y)/2 + \operatorname{Re}[U(y)\exp(\mathrm{i}kx + pt)]. \tag{3}$$

Hence we impose a perturbation of wavenumber k (and wavelength $2\pi/|k|$) and look for its behavior in time characterized by the (complex) parameter p. The equation for p can be found by finding $U(y)$ from the elastodynamics equations and then making the shear stress that corresponds to the solution (3) on the interface y = 0 equal to the linearized frictional resistance (2). The resulting equation for p is

$$\frac{1}{2}\mu|k|\sqrt{1 + \frac{p^2}{k^2c_s^2}} + \frac{p(pa^*\sigma/V^* + (a^*-b^*)\sigma/L^*)}{p + V^*/L^*} = 0, \tag{4}$$

where μ is the shear modulus, c_s is the shear wave speed, and the square-root term has branch cuts from $\mathrm{i}|k|c_s$ to $\mathrm{i}\infty$ and from $-\mathrm{i}\infty$ to $-\mathrm{i}|k|c_s$ in the complex p plane so that the real part of the square root is non-negative. The details of deriving (4) can be found in Rice et al. [3] where eqn (4) are analyzed to determine whether the perturbations are stable or unstable. Rice et al. [3] show that, for interfaces with steady-state rate weakening, there is a critical wavelength such that the perturbations with shorter wavelengths decay and the perturbations with longer wavelengths grow. Sliding on steady-state rate-strengthening interfaces is stable to perturbations of any wavelengths.

Here we study the behavior of the growing wavelengths in the case of steady-state rate weakening (Lapusta and Rice [2]). The nondimensional parameters of the problem are

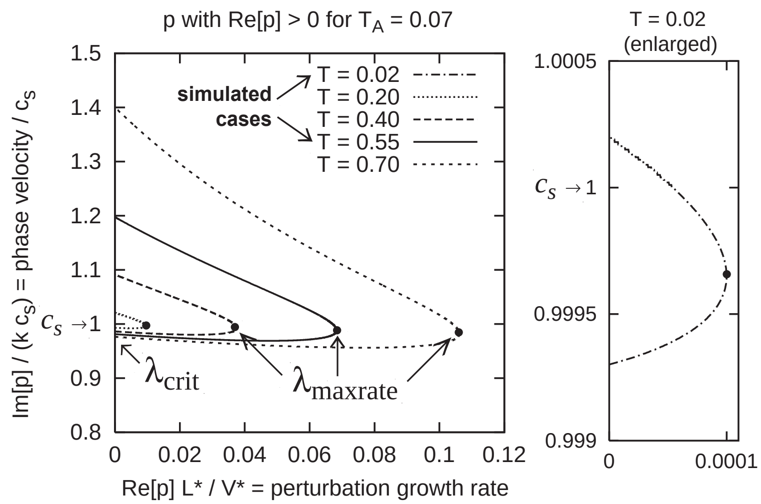

Figure 1: Behavior of the growing perturbations.

$$\lambda = \frac{V^*}{|k| c_s L^*} > 0, \quad T = \frac{(b^* - a^*)\sigma}{\mu V^* /(2c_s)} > 0, \quad T_A = \frac{a^* \sigma}{\mu V^* /(2c_s)} > 0. \tag{5}$$

In eqns (5), λ is the nondimensional wavelength of the perturbation, T describes the steady-state rate weakening, and T_A describes the direct rate effect. In terms of parameters T and T_A, the critical wavelength is given by $\lambda_{\text{crit}} = 1/(T^2 + T/T_A)^{1/2}$. We find that for each combination of parameters $T > 0$ and $T_A > 0$ and each wavelength $\lambda > \lambda_{\text{crit}}$, there is a pair of roots p with Re(p) > 0. For $0 < T < 1$, these roots are complex conjugate. For $T > 1$, they are complex conjugate for λ not too different from λ_{crit} but for larger λ they first coincide as a double positive root and then, as $\lambda \to \propto$, one of these roots goes to 0 while the other goes to $(T+1)/(T_A - 1)$.

We find the roots by solving the equation (4) numerically. For every combination of T and T_A we considered there is a wavelength of maximum growth rate. Figure 1 shows solutions p with Im(p) > 0 for a fixed value of T_A and several values of T. The values of T and T_A were picked to enable comparison with simulations discussed in section 4. Each curve is parameterized by the wavelength λ: Points at the Im(p) axis correspond to $\lambda = \lambda_{\text{crit}}$ and λ increases as the curves move out into the half-plane Re(p) > 0. Black dots mark solutions p that correspond to the wavelength λ_{maxrate} of the maximum growth rate. We see that the maximum growth rate depends significantly on the value of T, varying from ~0.0001 for T = 0.02 to ~0.1 for T = 0.7. Another interesting feature is the supersonic phase velocity of some larger wavelengths.

4 MULTI-PULSE MODE OF RUPTURE PROPAGATION

We simulate dynamic rupture propagation along the interface governed by the rate and state friction laws of the form

$$\tau = \sigma \frac{f_o + a \ln(V/V_o)}{1 - b/f_o \ln(V_o\theta/L) + L/(V_{weak}\theta)}, \quad \frac{\mathrm{d}\theta}{\mathrm{d}t} = 1 - \frac{V\theta}{L}, \quad \tau_{\text{ss}} = \sigma \frac{f_o + a\ln(V/V_o)}{1 + b/f_o \ln(V/V_o) + V/V_{weak}}, \tag{6}$$

where a, b, f_o, V_o, L, and V_{weak} are frictional parameters. The laws (6) are based on the standard Dieterich-Ruina formulation (Lapusta et al. [5] and references therein) but allow for much stronger

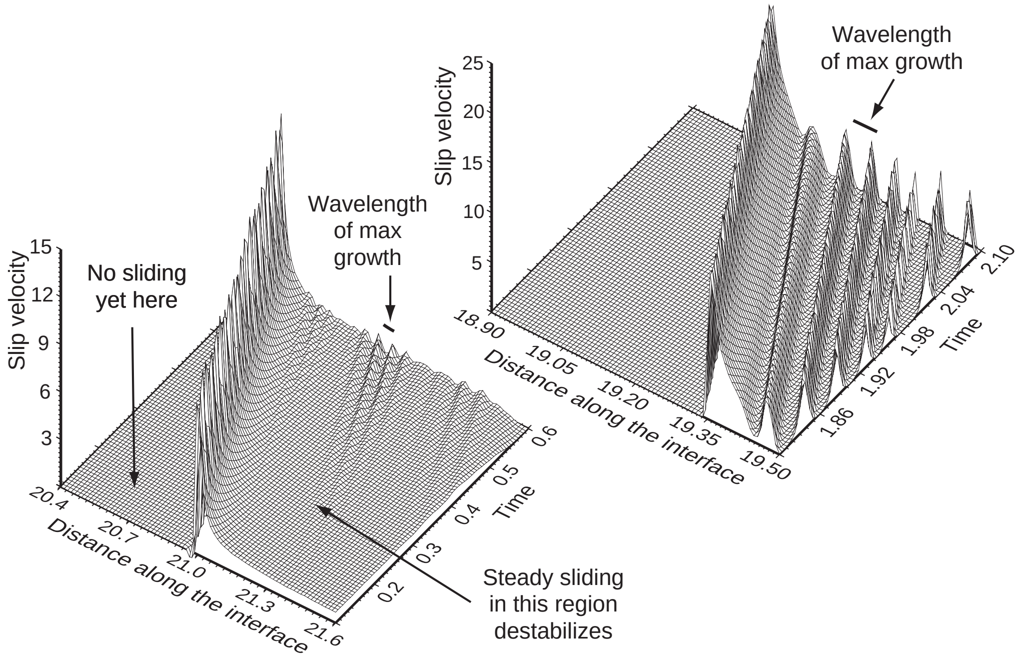

Figure 2: Initiation of multiple pulses (left). Well-developed pulses later in time and further along the interface (right). The axes show nondimensional values. Note the difference in scale between the two plots.

steady-state rate weakening at sliding rates V comparable to or larger than parameter V_{weak}. Such stronger weakening might occur at high sliding rates due to thermal processes such as flash heating (Rice [6]) or other effects (e.g., Di Toro et al. [7]). If $V_{weak} = \infty$ then eqns (6) resemble the standard formulation that allows only for logarithmic rate weakening.

The simulations are done in the 2D anti-plane elastic context; the methodology is similar to Lapusta et al. [5]. The dynamic ruptures nucleate at the rheological transitions between steadily sliding regions with steady-state rate-strengthening properties and locked regions with steady-state rate-weakening properties. The ruptures then propagate into the rate-weakening regions.

The example of the rupture progression in space and time is shown in Figure 2. The parameters of the law (6) used in this simulation are $a = 0.01$, $b = 0.014$, $f_o = 0.6$, $V_o = 10^{-6}$ m/s, and $V_{\text{weak}} = 20$ m/s. In Figure 2 (left), the points with high sliding rates correspond to the rupture front. Behind the front, the sliding is originally in the crack-like mode (e.g., at 0.2 in nondimensional time) but later the sliding destabilizes into a wavy pattern. Analyzing the stability of the nearly steady sliding behind the rupture front as in section 3, we find that the wavelength of maximum growth rate λ_{maxrate} predicted by the analysis is consistent with the wavelength of the observed instability as marked in Figure 2. The relevant (for this case) curve in Figure 1 is the solid curve that corresponds to $T = 0.55$. During subsequent rupture propagation, the instability grows into well-separated pulses. Figure 2 (right) shows rupture propagation at later times and further along the interface. The maximum-growth wavelength λ_{maxrate} is again marked and corresponds very well to the distance between pulses; note that the scale is different on all axes compared to Figure 2 (left). The pulses are well-resolved numerically both in time and in space and are reproducible in simulations with higher numerical resolution.

In an analogous simulation but with V_{weak} essentially equal to infinity (the change that results in much smaller rate weakening), no instability is observed behind the rupture front and the rupture remains crack-like (results not shown due to space restrictions). The relevant curve in Figure 1 is now the one enlarged on the right that corresponds to $T = 0.02$. The value of T_A is approximately equal in both cases. The first case (shown in Figure 2) has much higher effective rate weakening behind the rupture front and hence much higher growth rate of the wavelength $\lambda_{\mathrm{maxrate}}$ (the growth is exponential and the exponent is about 70 times larger in the first case). That growth rate is high enough to initiate multiple pulses sufficiently quickly, before the friction properties at the spatial region of interest change significantly. In the second case, the growth rates are too small to manifest themselves in the changing frictional environment behind the rupture front.

5 DISCUSSION

Multiple pulses can propagate along interfaces governed by rate and state friction laws of steady-state rate-weakening type when the steady-state rate weakening is strong enough. In terms of the classification given by Zheng and Rice [1], this new multi-pulse mode of dynamic rupture propagation fits in the transitional regime between the crack-like and pulse-like modes. The multiple pulses nucleate when the approximately steady-state sliding behind the rupture front destabilizes. The wavelength of this process can be predicted from the linearized analysis of steady sliding and corresponds to the wavelength of the maximum growth rate. To produce multiple pulses, the steady-state rate weakening should be strong enough at high sliding rates to result in sufficiently large instability growth rates. The classical logarithmic Dieterich-Ruine laws do not provide adequate weakening but they have been formulated based on slow experiments with sliding velocities in the range of 1 μm to 1 mm. When the formulation is expanded to include the possibility of enhanced rate weakening at high slip rates, multiple pulses start to occur. Such enhanced rate weakening may take place due to thermal or other effects (e.g., Rice [6], Di Toro et al. [7]). We also see single pulses as the simulation parameters are changed appropriately, e.g., in the direction of even stronger weakening, as predicted by Zheng and Rice [1]. The observed multi-pulse mode can accumulate slip comparable to the crack-like mode while having local properties of the pulse-like mode. This new mode can have significant implications for the frequency content of ground motions generated by earthquakes and it would be important to study whether this multi-pulse mode occurs during earthquakes.

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