NUMERICAL MODELING OF SHEAR LOCALIZATION IN ELASTOPLASTIC MATERIALS

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ABSTRUCT

The work addresses the problem of describing shear localization at mesoscale level of deformation. In the planar problem the shear localization lines are represented as cuts. The stress vector is continuous at the banks of the cuts. The discontinuities of the tangential displacements are allowed by imposing boundary conditions at the cuts that allow gliding along the lines. In particular, the condition of the constancy of the shearing stress along the line is used. It is assumed that plasticity is localized at the shear lines, while outside the lines the material can be considered to be linearly elastic.

The developed numerical algorithm realizes the method of finite elements on the problem-oriented meshes with double nodes, and allows studying the initiation and propagation of an arbitrary number of curvilinear cuts with arbitrary orientations. The lower boundary for the inter-cut distances is determined solely by the size of the finite elements. Thus both the number of the shear lines and the distances between the lines in the system can be arbitrary.

The algorithm was applied for numerical analysis of the deformation mode in the vicinity of a circular hole. Shear localization at the system of cuts in the shape of the logarithmic spiral leads to overall plastic properties of the bulk material. It is demonstrated that for sufficiently close positioning of shear localization lines the obtained solutions are close to the classical solutions in the framework of continuous models. The suggested approach can be used to advantage to describe the intermediate state of material, between the classical elasticity with no shear lines, and continual plasticity when the shear lines are infinitely close to each other.

1 INTRODUCTION

Shear localization in physical mesomechanics is treated in a multilevel approach (Panin [1]). Within this framework the classical models of deformation of solids occupy a certain well-defined place and correspond to various limiting cases. Thus, it is well known that no combination with the dimension of length can be built from parameters describing a linearly elastic body, the Young modulus and the Poisson coefficient. This means that classical elasticity corresponds to medium without characteristic linear scales. In other words, elasticity behaves similarly at all scale levels. However, the equations of plasticity belong to the class of hyperbolic equations and have well-defined orientation of their characteristic lines (shear lines) in every point. It can thus be said that ideal plasticity describes an “anisotropic” structure with infinitely close shear lines. Shear lines in real processes are certainly discrete, but it is reasonable to expect that if the lines are sufficiently close to each other on the characteristic linear scale of the problem, the discreteness will not be important. However, when the inter-line distances approach the characteristic scale of the problem the continuos model becomes insufficient, and the discreteness of the system of shear lines should be explicitly considered. Numerous experiments clearly demonstrate that such a localization scale is important in practical applications, including pressure processing of metals, plastic deformation in the vicinity of holes, in various processes involving deformation of geomaterials, etc. (Nadai [2], Sokolovsky [3], Revuzhenko [4]).

2 GENERAL FORMULATION OF PROBLEMS WITH DISCRETE SYSTEMS OF SHEAR LINES

Let us consider a region of material in the conditions of planar deformation. Generation of the shear lines, their shape and position are inherently related to properties of the material, its state equations, conditions of deformation, and criteria for the onset of localization of plastic deformations. This process should either be treated explicitly by the method of successive loading, or rely on a priori data from independent experimental and theoretical studies.

Let us assume that the shapes and positions of the shear lines have already been determined at the current load parameter step, the distances between the lines are comparable to the characteristic scale of the problem, and their shape can be curvilinear. The widths of the shear lines are usually so small that they can be adequately modeled by cuts. Shear lines are technically the surfaces of strong discontinuity – discontinuity of tangential displacement. Let us pose the problem of finding the fields of displacement increments and stress increments that satisfy conditions of equilibrium in the given region at every loading step.

The developed algorithm accepts any model that can adequately describe deformation of material outside the shear lines. To isolate the effects related only to localization of shears we chose the simplest model of linearly elastic body for the bulk material outside the lines.

Let us cast the state equations for material in the zone of shear localization in the form of boundary conditions describing interaction between the banks of the cut. The normal component of the vector of displacement increments u_n and the stress vector $\boldsymbol{p^n}$ are continuous on the area of the tangential discontinuity of displacements:

$$u_n^+ = u_n^-, \tag{1}$$

$$(\boldsymbol{p^n})^+ = (\boldsymbol{p^n})^-. \tag{2}$$

Here, the subscripts plus and minus refer to different sides of the discontinuity line. Direction cosines of the normal and the tangent here are equal at both sides of the shear line. Modeling shear localization lines with cuts provides the possibility for relative translation of initially coinciding points along the lines. Whether such translations actually take place or no depends on the criterion of shear localization and on the conditions imposed at the banks of the cut. If criterion of localization for this particular loading step is not satisfied at certain parts of the cut, there is no tangential discontinuity of displacement. When the localization criterion is met, generation of tangential displacement at certain point discontinuity becomes possible if one of the following conditions is fulfilled at different parts of the cuts:

$$p^n{}_t = f_1(p^n{}_n, x, y), \tag{3}$$

$$p^n{}_t = f_2(u_t^+ - u_t^-, x, y). \tag{4}$$

Here $p^n{}_n$ and $p^n{}_t$ are the normal and tangential components of the vector of stress increments, u_t is the tangential component of the vector of displacement increments, x and y are Cartesian coordinates of the point, and f_1 and f_2 are given functions.

An important advantage of the suggested formulation is the possibility of imposing the relations between stresses and displacements at the boundaries instead of specifying their particular values. The range of conditions that can be realized at the banks of the cuts within the developed numerical algorithm is rather wide. Furthermore, besides describing shear lines in the conditions of continuous normal displacement, the algorithm accepts discontinuities of the normal displacement increments determined by other parameters of the problem.

3 FINITE ELEMENT MODELING OF THE POSSIBILITY OF STRONG DISPLACEMENT DISCONTINUITIES

For numerical modeling of the lines of localization of plastic deformations in a planar region we chose the method of finite elements. We also assume here that part of the lines connecting the nodes of the finite element mesh pass along the surfaces of localization of plastic deformations – the shear lines. To have varying and possibly rather large number of shear lines in the modeled region we let each node of the region belong to two banks of the cut. In this case the region can include curvilinear cuts with lower limit on the distances between the cuts determined solely by the sizes of the chosen finite elements.

A particular feature of the initial mesh of finite elements is the double nature of each node which have two numbers in global numeration. This allows to separate each modeling point into two distinct nodes with different values of displacement induced by deformation. Node numeration of the finite elements is optimized to reduce data storage requirements of the program package taking into account the positions of the cuts. The geometry of the mesh can be adjusted in the course of computation.

The relevant numerical algorithm, its realization as a universal program package and solution of several typical problems are developed (Bushmanova [5], [6]).

4 SHEAR LOCALIZATION AT DISCRETE SYSTEM OF LINES IN THE VICINITY OF A CIRCULAR HOLE

The suggested algorithm can be applied to describe inelastic properties of the medium localized at discrete system of shear lines. Apparently the correct solution of this problem should provide transition to the known results, which can be used as a test for the suggested formulation of the problem and the chosen method to solve it.

Let us consider the problem of shear localization lines on the example of a system for which the geometry of the lines is known from experimental and analytical studies. This will help us demonstrate the potential of the numerical algorithm for realization of strong discontinuity of displacement along the system of a large number of shear lines. Consider the known classical solutions. Let the region under study be a ring in the conditions of planar symmetric deformation. The problem is solved in dimensionless variables, with elasticity modulus E taken as the characteristic stress value, and hole radius r taken as the characteristic linear scale. External radius of the ring is denoted R. Let us set normal radial stress at the internal boundary contour and radial displacement at the external contour with load parameter u. Tangential stress is equal to zero at both internal and external boundaries. In this case it leads to zero tangential stress in the entire region. This problem has a known solution within the framework of linear theory of elasticity (Timoshenko [7]).

Let us assume that maximum tangential stress τ in the modeled region does not exceed shear yield point τ^* for the material. Increasing the load parameter leads to formation of a plastic region around the hole in which the condition of ideal plasticity holds true. In this case we obtain the classical elastoplastic solution of the problem (Nadai [2]). Shear lines in the plastic region have the shape of logarithmic spirals intersecting polar radii at angles $\pm\pi/4$. Tangential stress remains constant all shear lines. Formally the continual plastic solution of the problem corresponds to an infinite set of shear lines. The radius of the plastic zone is determined by the load parameter and the yield point of the material.

Let us consider an intermediate state between elastic and elastoplastic for the ring described above. Let the load parameter have such a value that a plastic region is formed around the hole in the elastoplastic solution. In the numerical solution of the problem we assume that a finite number of shear lines is generated and the condition of the constancy of the shearing stress along the line is met. Thus we consider n cuts along logarithmic spirals in the ring of radius c. We choose the mesh of finite elements determined by the families of logarithmic spirals and concentric circles. Along certain lines from one of the families of logarithmic spirals we place the cuts – shear lines that begin at the internal contour of the boundary and end inside the region. The material is assumed to deform elastically outside the cuts. Discontinuity of tangential displacement is allowed along the cuts. If we treat the two banks of the cut as two distinct boundary fragments, then for two points having identical coordinates but belonging to the opposite banks conditions (1)-(3) hold true.

Consider the solution of this problem under the following parameters: Poisson coefficient $\nu = 0.3$, $\tau^* = 0.03 \cdot E$, $R = 3.248 \cdot r$, $u = 0.016 \cdot r$, $c = 1.988 \cdot r$, $n = 1, 2, 4, 8, 16, 32, 48, 96$.

The maximum tangential stress τ grows larger as the distances between the cuts are increased. Maximum values τ are attained at the hole boundary between the cuts. Figure 1 and figure 2 show surface $\tau(x,y)$ and isolines $\tau(x,y)$ for $n = 8$, respectively.

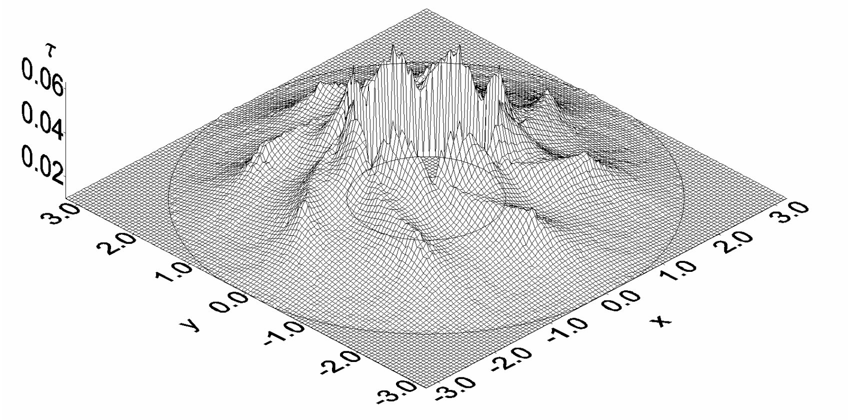

Figure 1: Surface of maximum tangential stress $\tau(x,y)$.

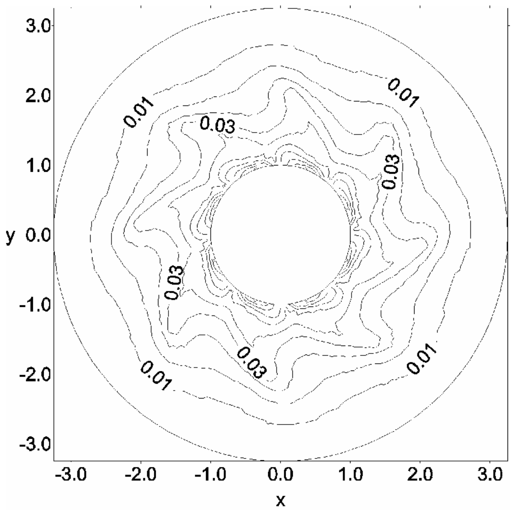

Figure 2: Isolines of maximum tangential stress.

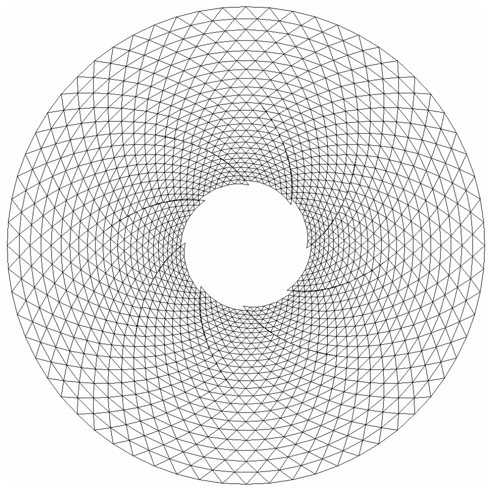

Figure 3: Finite elements mesh.

Figure 3 shows finite elements mesh obtained by summing coordinates of the nodes of the initial mesh and the corresponding deformation induced displacements of the nodes for $n = 8$. Discontinuities of tangential displacement occur along the cuts – initially coinciding nodes get shifted relative to each other, and "steps" develop at the internal boundary.

The discontinuities are most pronounced at the perimeter of the hole and gradually vanish towards the elastoplastic boundary. The more cuts are generated, the weaker are the discontinuities at each single cut. The shape of the cuts along which gliding with constant tangential stress takes place reproduces the shape of shear lines in the continual elastoplastic problem. Numerical solution with a large number of cuts is rather close to the analytic elastoplastic solution. The effect of slipping along the cuts on the stresses is confined to small neighborhoods of the cuts. For $n = 2$ and $n = 1$ the solution converges to the elastic solution for a ring without cuts when moving away from the shear lines. The presence of the given number of cuts in an elastic ring reduces the average radial stress building up at the external boundary.

When the distances between the shear lines at the perimeter of the hole is less than 0.2 of the characteristic scale of the deformed region the obtained solution practically reproduces the continual solution, but the discrepancy between the two becomes substantial as the inter-line distances grow larger. The suggested approach allows transition to continual plasticity and elasticity.

5 CONCLUSION

The suggested approach can be used to advantage to describe the intermediate state of material, between the classical elasticity with no shear lines, and continual plasticity when the shear lines are infinitely close to each other.

The developed numerical algorithm can be applied to solve contact problems with various boundary conditions at contacts, to study fatigue strength of material, and fragmentation of the loaded material at the stage of failure.

REFERENCES

1. Physical Mesomechanics and Computer-Aided design of Materials [in Russian], 2 vols, Ed: Panin V.E., Novosibirsk, Nauka, Vol. 1 – P. 298, Vol. 2 – P. 320, 1995.
2. Nadai A. Theory of Flow and Fracture of Solids, McGraw-Hill, New York, Vol. 2, 1950.
3. Sokolovsky V.V. Theory of plasticity [in Russian], M., Vysshaya Shkola, P. 608, 1969.
4. Revuzhenko A. F. Mechanics of Elastoplastic Media and Nonstandard Analysis [in Russian], Izd. Novosib. Univ., Novosibirsk, P. 429, 2000.
5. Bushmanova O. P. “Application of the finite element method for modeling discontinuity lines in elastoplastic problems”, in: Numerical Methods for Solving Problems of the Theory of Elasticity and Plasticity, Proc.XVI Conf. (Novosibirsk, 6-8 July, 1999), Izd. Sib. Otd. Ross. Acad. Nauk, Novosibirsk, pp. 46-50, 1999.
6. Bushmanova O. P. “Modeling of shear localization, Journal of Applied Mechanics and Technical Physics”, Vol. 44, No. 6, pp. 885-889, 2003.
7. Timoshenko S.P., Goodier J.N. Theory of Elasticity, McGraw-Hill, New York, 1970.