IDENTIFICATION OF LAMINATE MECHANICAL PROPERTIES VIA EXTENDED KALMAN FILTER

A. CORIGLIANO[1], S. MARIANI[2]
Dipartimento di Ingegneria Strutturale, Politecnico di Milano, Milano (ITALY)
[1]alberto.corigliano@polimi.it, [2]stefano.mariani@polimi.it

ABSTRACT

Delamination, i.e. interlaminar debonding, is believed to be one of the main sources of laminate failure under both quasi-static and dynamic loading conditions. This process can be numerically simulated through interface models apt to progressively reduce the adhesion between laminae; calibration of interface constitutive laws has been shown to represent a difficult task, mainly because structural effects can partially shadow the local mechanical properties in the process zone.

In this work an inverse analysis, whose engine is the dual extended Kalman filter, is explored to identify dynamic strength and toughness properties of the interlaminar phases, where all the dissipative phenomena are assumed to take place.

1 INTRODUCTION

Delamination and debonding phenomena can be advantageously described and simulated by making use of interface models (see e.g. [1] and the reference therein for a recent review). One major obstacle in the effective practical use of interface models is related to the difficulty in identifying model parameters. In fact, no direct tests can be done on the interfaces and indirect parameter identification procedures should in general be used.

The Authors have recently experienced the use of the extended Kalman filter (see e.g. [2]) for interface model identification in composite materials both in the quasi-static [3, 4] and in the dynamic regime [5, 6, 7]. It is the purpose of the present paper to discuss the performance of the dual extended Kalman filter (see [8]) in the context of explicit structural dynamics, thus extending previous results presented in [5, 6, 7]. The particular case of composite laminates subject to impacts (see e.g. [9]) is here considered, where softening interface models can be used in order to progressively simulate impact induced delamination.

The outline of the paper is as follows. In Section 2 the dynamics of structures containing softening interfaces is formulated; space-discretization by means of finite elements and time-discretization by means of the explicit central difference algorithm are briefly discussed. Section 3 is devoted to the illustration of numerical results concerning state estimation and parameter identification for a one-dimensional model of impact tests on a 5-layer composite.

2 NONLINEAR COMPOSITE DYNAMICS

Let us consider a two-dimensional (2D) solid Ω, which is subdivided by a set of interfaces Γ_i, $i = 1, ..., n_\Gamma$, into $n_\Gamma + 1$ disjoint portions Ω^i (Fig. 1). The bulk material in $\Omega \backslash \Gamma$, being $\Gamma = \cup_{i=1}^{n_\Gamma} \Gamma_i$, is assumed to behave elastically, while a nonlinear softening model is adopted to simulate the progressive failure along each Γ_i.

After space discretization, the equation of motion of the laminate in the small strain regime is:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{D}\dot{\boldsymbol{u}} + \boldsymbol{K}_\Omega \boldsymbol{u} + \int_\Gamma \boldsymbol{B}_\Gamma^{\mathrm{T}} \boldsymbol{\tau} \, d\Gamma = \boldsymbol{q}, \tag{1}$$

where: $\boldsymbol{M}$ and $\boldsymbol{D}$ are the mass and viscous damping matrices of the laminate; $\boldsymbol{K}_\Omega$ is the stiffness matrix of the elastic bulk $\Omega \setminus \Gamma$; $\ddot{\boldsymbol{u}}$, $\dot{\boldsymbol{u}}$ and $\boldsymbol{u}$ are, respectively, the vectors gathering nodal accelerations, velocities and displacements; $\int_\Gamma \boldsymbol{B}_\Gamma^{\mathrm{T}} \boldsymbol{\tau} \, d\Gamma$ is the vector of internal forces linked to the nonlinear behavior along

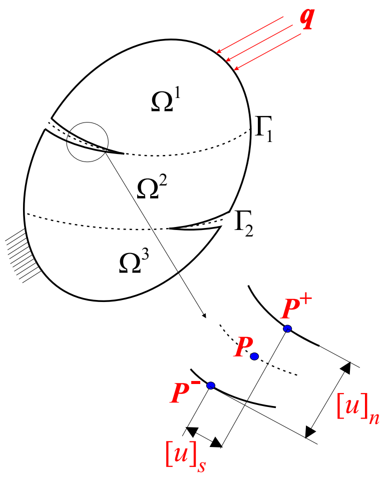

Figure 1: 2D layered solid. Geometry and notation in the case of two debonding interfaces ($n_\Gamma = 2$).

Γ, $\boldsymbol{B}_\Gamma$ being the interface compatibility matrix of the space discretized model, which relates displacement discontinuities $[\boldsymbol{u}]$ to the vector of nodal displacements $\boldsymbol{u}$; $\boldsymbol{\tau}$ is the vector of tractions transmitted across the interfaces Γ_i; $\boldsymbol{q}$ is the vector of external loads. Eqn. (1) is supplemented with the initial conditions at time t_0:

$$\boldsymbol{u}\,(t_0) = \boldsymbol{u}_0; \qquad \dot{\boldsymbol{u}}\,(t_0) = \dot{\boldsymbol{u}}_0. \tag{2}$$

In the local tangential-normal reference frame along each Γ_i, opening (mode I) and sliding (mode II) components of $[\boldsymbol{u}]$ are respectively denoted by $[u]_n$ and $[u]_s$ (see Fig. 1). The traction vector $\boldsymbol{\tau}$ is related to the displacement discontinuities $[\boldsymbol{u}]$ and to the vector $\boldsymbol{\vartheta}$ of model parameters to be calibrated through:

$$\boldsymbol{\tau} = \boldsymbol{\tau}\,([\boldsymbol{u}]; \boldsymbol{\vartheta}) = \boldsymbol{\tau}\,(\boldsymbol{B}_\Gamma \boldsymbol{u}; \boldsymbol{\vartheta})\,. \tag{3}$$

After partitioning the time interval of interest according to $[t_0 \quad t_N] = \cup_{i=0}^{N-1} [t_i \quad t_{i+1}]$, the central difference algorithm is used to advance the solution of the governing relation (1) in time. Nodal displacements at the end of the generic time step $[t_i \quad t_{i+1}]$ are [6]:

$$\boldsymbol{u}_{i+1} = \left(\frac{\boldsymbol{M}}{\Delta t^2} + \frac{\boldsymbol{D}}{2\Delta t} \right)^{-1} \left[\left(\frac{2\boldsymbol{M}}{\Delta t^2} - \boldsymbol{K}_\Omega \right) \boldsymbol{u}_i - \left(\frac{\boldsymbol{M}}{\Delta t^2} - \frac{\boldsymbol{D}}{2\Delta t} \right) \boldsymbol{u}_{i-1} - \int_\Gamma \boldsymbol{B}_\Gamma^{\mathrm{T}} \boldsymbol{\tau}_i \, d\Gamma + \boldsymbol{q}_i \right], \tag{4}$$

$\Delta t = t_{i+1} - t_i$ being the time step size.

To simulate interlaminar debonding, the constitutive model for the resin-enriched interphase between plies has to account for possible strength degradation, up to complete failure; this is achieved through a strain softening regime that follows the initial hardening phase. For mode I debonding a holonomic, rate-independent interface constitutive law is formulated according to [10, 11] (Fig. 2):

$$\begin{cases} \tau_n = K[u]_n & \text{if } [u]_n < 0; \\ \tau_n = K[u]_n \exp\left(-\dfrac{[u]_n}{[\bar{u}]_n} \right) & \text{if } [u]_n \geq 0, \end{cases} \tag{5}$$

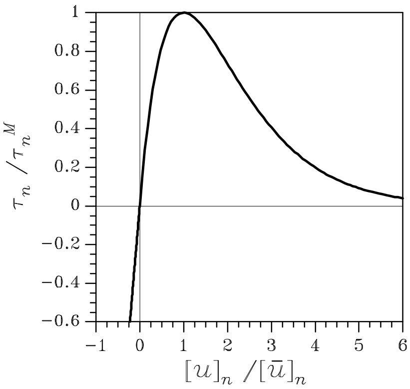

Figure 2: nonlinear interface constitutive model.

where K and $[\bar{u}]_n$ are, respectively, the interface stiffness for $[u]_n \leq 0$ and the opening displacement discontinuity corresponding to the peak strength. This constitutive model is characterized by a strength τ_n^M (see Fig. 2) and a fracture energy G_c, respectively given by:

$$\tau_n^M = \frac{K[\bar{u}]_n}{\exp(1)}; \qquad G_c = \int_0^{\infty} \tau_n \, d[u]_n = K[\bar{u}]^2. \tag{6}$$

In the forthcoming numerical experiments only dilatational waves propagating inside the laminate in the through-the-thickness direction are considered. It is thus beyond the scope of this work to generalize the constitutive law (5) to mixed mode loading conditions (see e.g. [1, 4]).

3 A TEST CASE: IMPACTS ON A 5-LAYER COMPOSITE

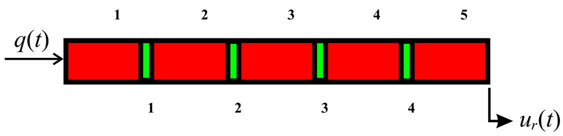

Figure 3: 1D model of the impact test on a 5-layer composite.

Assuming that the elastic properties of the bulk $\Omega \backslash \Gamma$ are known, to identify model parameters τ_n^M and G_c the use of the dual extended Kalman filter (EKF) is explored. This filter has been recently proposed in [8] to take into due account nonlinearities in the equations governing system evolution (Eqn. (4) in this case); indeed, it was shown to be superior with respect to the usual joint EKF as far as stability and convergence issues are concerned. Because of length constraint, we do not furnish details of the algorithm; readers can find a thorough presentation of the subject in [7, 8].

As a test case to assess the capability of this filter, the model problem of Fig. 3 is analyzed. This 1D system approximately describes the effects of an impactor striking from the left the external surface of the layered specimen. Below the contact zone, just after the impact event a dilatational stress wave starts propagating inside the laminate in the through-the-thickness direction; every interface Γ_i, $i = 1, .., 4$, causes a partial reflection of the wave according to its constitutive law (see e.g. [11, 12]), so that the free-surface displacement u_r contains information about the evolving delamination process. Anyway, u_r is weakly

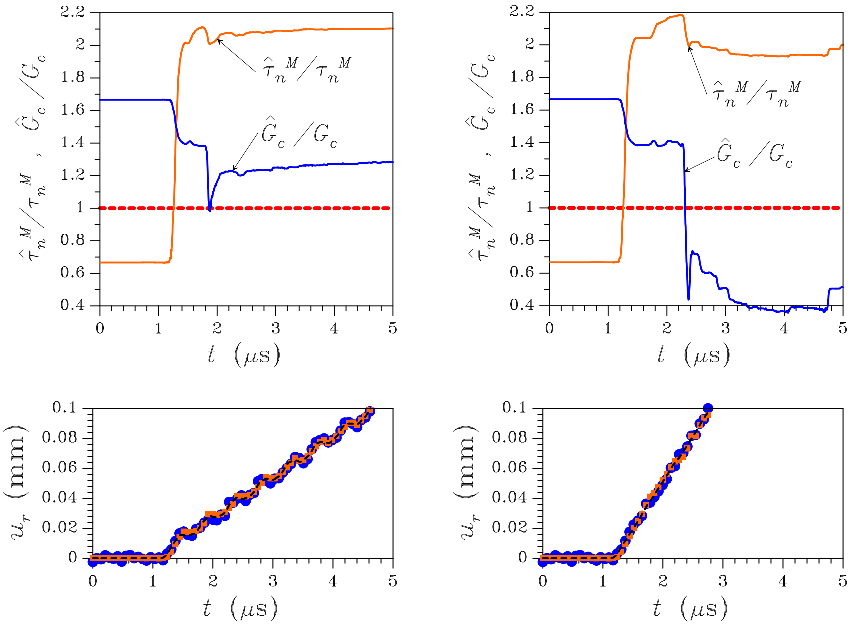

Figure 4: impacts on a 5-layer composite, $\bar{q} = 150$ (N/mm^2), $e_u = 0.005$ (mm). Effect of the loading condition (left column: $\bar{t} = 0.25$ μs; right column: $\bar{t} = 1.00$ μs) on the current estimated τ_n^M and G_c (top row) and free surface displacement u_r (bottom row).

sensitive to law parameters [7]; it is thus expected that only a rough model calibration can be achieved in the analyses.

The impact pressure $q(t)$ is assumed:

$$q(t) = \begin{cases} 0 & \text{if } t < 0; \\ \bar{q} & \text{if } 0 < t < \bar{t}; \\ 0 & \text{if } t > \bar{t}, \end{cases} \tag{7}$$

with $\bar{q} = 150$ (N/mm^2) and letting $\bar{t}$ to vary between 0.25 (μs) and 1.00 (μs). Target values of model parameters are: $\tau_n^M = 75.0$ (N/mm^2); $G_c = 0.15$ (N/mm). Pseudo-experimental data are synthetically generated through direct analyses corrupted with a Gaussian noise, whose standard deviation is proportional to the parameter e_u.

Results of the identification procedure are shown in Fig. 4 and 5 in the case of observed u_r and for only one set of initialization data; they are anyway representative of the results achievable from any initial data sets, both in terms of accuracy and convergence rate.

Fig. 4 shows, in the time interval $0 \leq t \leq 5$ (μs), the current estimated values of τ_n^M and G_c (highlighted with a hat) and the tracked free surface displacement u_r, in the two instances characterized by $\bar{t} = 0.25$ (μs) (left) and $\bar{t} = 1.00$ (μs) (right). As far as u_r is concerned, in the plots the dark circles represent the noisy pseudo-experimental data, sampled at constant time intervals, the dashed line stands for the true noise-free system response, and the light squares represent the filtered response of the laminate, as furnished by the dual EKF. It can be noticed that the filter leads to an optimal performance in tracking u_r, while partially satisfactory results are obtained concerning model calibration.

Fig. 5 illustrates the typical accuracy of the dual EKF as for state estimation (notice that interface openings are one order of magnitude smaller than the observed u_r). Variations of the displacement discontinuities are well captured, with an increasing level of precision as time goes by. Furthermore, the fracturing interface, where $[u]_n^i$ exceeds the threshold value $[\bar{u}]_n$ ($[\bar{u}]_n$ being the dashed horizontal line in the plots), and the time of failure are both correctly captured.
The right column in Fig. 5 details also an instability of the filter at $t \cong 2.3$ (μs), after the laminate has failed. This is still an open issue for coupled model calibration and state estimation in damaging structures, where softening phenomena affect the performance and even the stability of the filters.

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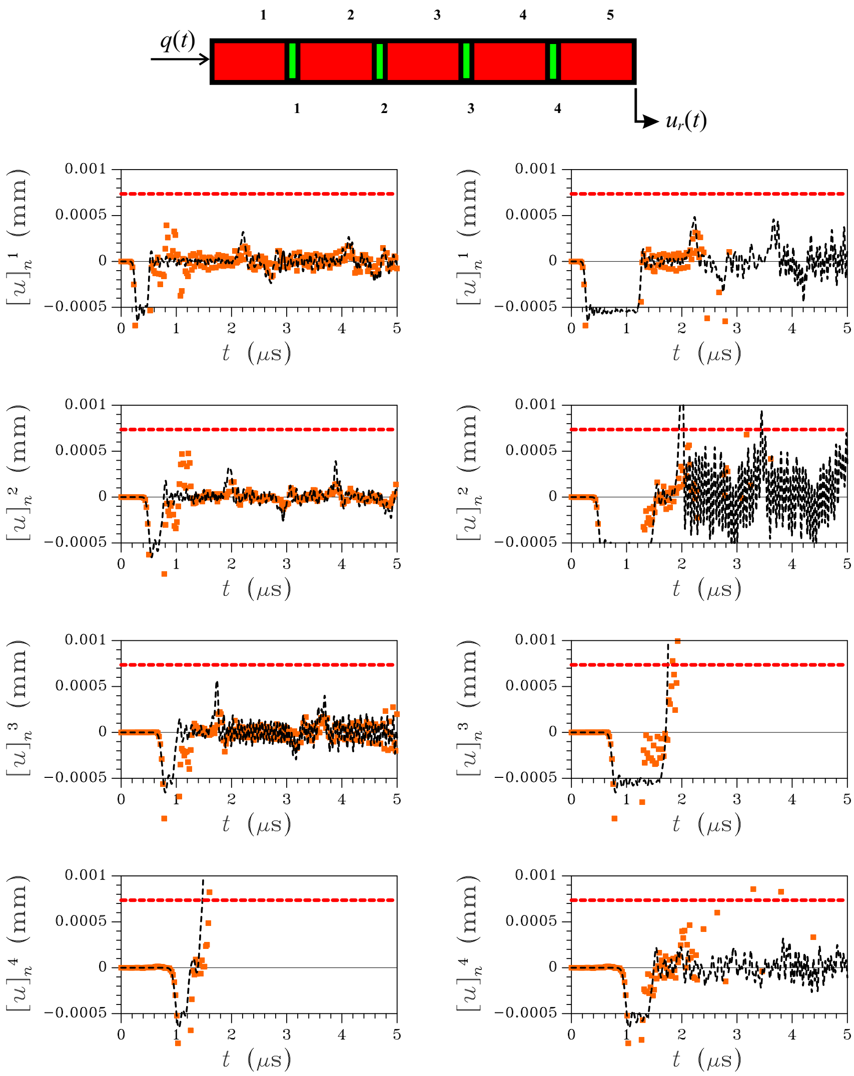

Figure 5: impacts on a 5-layer composite, $\bar{q} = 150$ (N/mm^2), $e_u = 0.005$ (mm). Effect of the loading condition (left column: $\bar{t} = 0.25$ μs; right column: $\bar{t} = 1.00$ μs) on the current estimated interface openings $[u]_n^1$, $[u]_n^2$, $[u]_n^3$, $[u]_n^4$.