GROWTH OF CRACKS BRIDGED BY NANOFIBERS

MIKHAIL PERELMUTER
Institute for Problems in Mechanics of RAS,
Pr. Vernadskogo 101-1, 119526, Moscow, Russia
perelm@ipmnet.ru

ABSTRACT

The two-parametric fracture criterion for a problem of quasi-static growth of a crack with bridged zone is proposed. The energetic characteristics of the large scale bridged crack (the strain energy release rate (ERR) and the rate of the energy dissipation by the bonds (RED)) are introduced. The condition of the crack tip limit equilibrium is the equality of the ERR and the RED values (the first necessary condition of fracture). The second condition of fracture is the condition of the bond limit stretching at the trailing edge of the bridge zone. Based on these two fracture conditions the regimes of the bridged zone and the crack tip equilibrium and quasi-static growth are considered. Analytical application of the proposed criterion for the problem of the straight crack in homogeneous plane with the rectilinear law of the bonds stresses was performed. In the general case of the bond stress dependent on the crack opening the problem of the bond stress and the energetic characteristics determination is transformed to the numerical solution of the singular integral-differential equations system. The estimations of the equilibrium size of the bridged zone, the adhesion fracture energy and the external fracture stress depending on the crack size are found.

1 INTRODUCTION

Composites based on polymers or ceramics matrix and filled by nanosized particles or nanotubes are materials with strong and tough mechanical properties. The mechanisms of toughening these materials by nanoparticles are investigated experimentally and theoretically (*Qian at al.*, [1], *Xia at al.*, [2], *Srivastava et al.*, [3], *Wanga and Pyrzb*, [4]). From the experimental observations ([1-2], [5]) was found that the crack bridging mechanisms is very important during nanocracks formations and fracture of nanocomposites. Noted that in the most observed cases the size of the nanocrack bridged zones were comparable with the whole crack size. In these cases need the special consideration during the bridged zone and crack tip growth. Below the two-parametric fracture criterion for a problem of quasi-static growth of a crack with large scale bridged zone is proposed and considered.

Let us consider a straight crack of length 2ℓ at an interface of two dissimilar elastic half-planes such that the crack is placed at $|x| \le \ell, y = 0$. Assume that the uniform tensile stresses, σ_o, are applied at infinity normal to the interface. Consider segments of length d (end zones) adjacent to the tips of the crack, $(\ell - d) \le |x| \le \ell$. In these zones the surfaces of the crack interact with each other, which suppresses the crack opening. The physical nature of the crack surfaces interaction is generally changed depending on the crack scale and distance from the crack tip. The interatomic and intermolecular forces are limiting mechanisms of the surfaces interaction at the small distances from the crack tips (where the crack opening does not exceed the size of the region of the molecular forces action) while "mechanical" forces prevail at relatively larger distances. For polymers and nanocomposites with polymers or ceramics matrix these mechanical bonds are chains of molecules, nanotubes and nanoparticles. For these materials, as a rule, the size of the bridged zone is comparable to the size of the whole crack (large scale bridging). To describe mathematically the interaction between the surfaces of the crack, we assume that there exist bonds between the surfaces of the crack at the end zone. The law of deformation of these bonds, which is generally nonlinear, is given.

Under the action of external loads, σ_0, the stresses $Q(x)$ appear in the bonds between the surface of the interface crack at the boundary between different materials. These stresses have the normal $q_y(x)$ and tangential $q_x(x)$ components

$$Q(x) = q_y(x) - iq_x(x), i^2 = -1 \tag{1}$$

The surfaces of the crack are loaded by the normal and tangential stresses which are numerically equal to these components.

The opening of the interface crack, $u(x)$ at $|x| \le \ell, y = 0$, can be written as follows

$$u(x) = u_y(x) - iu_x(x) \tag{2}$$

where $u_y(x) = u_y^+(x) - u_y^-(x)$ and $u_x(x) = u_x^+(x) - u_x^-(x)$ are the projections of the crack opening on the coordinate axes, u_x^+, u_y^+ and u_x^-, u_y^- denote the components of the displacements of the upper and lower crack surfaces.

The relation between the crack opening and the bond tractions (the bond deformation law) depends on the physical origin of the bonds and their properties. The general form of spring-like bonds deformation law can be written as follows, (*Goldstein, Perelmuter,* [6])

$$u_i(x) = c_0(x,\sigma)q_i(x)\,, c_0(x,\sigma) = \gamma_0(x,\sigma)\frac{H}{E_B} \tag{3}$$

where γ_0 is a dimensionless function, H is a linear scale proportional to the bonding zone thickness, E_B is the effective Young modulus of the bonds and the function c_0 can be considered as the effective bond compliance, $\sigma = \sqrt{q_x^2 + q_y^2}$ is the modulus of the traction vector, $i = x, y$.

The bonds stresses and the crack opening along the crack end zone are determined from numerical solution of the singular integral-differential equations system in the case of the bond deformation law with displacements depends on bonds stresses [1, 2].

2 TWO PARAMETRIC FRACTURE CRITERION

Supposing that the bonds stresses and the crack opening along the crack end zone are known, the total potential energy of a body containing a crack with bridged zone (in the absence of body forces) is

$$\Pi = \int_v w(\varepsilon_{ij})dv - \int_{s_e} t_i u_i ds + \int_{s_i} \Phi(u)ds, \tag{4}$$

where $w(\varepsilon_{ij})$ is the density of the deformation energy in the body volume v, ε_{ij} are the components of the strain tensor; t_i, u_i are the tractions and displacements at the body boundary and (or) crack surfaces s_e; $\Phi(u)$ is the density of the strain energy of the bonds in the crack end zones, u is the crack opening in the end zones of area s_i.

The crack limit equilibrium corresponds to the following condition

$$-\frac{\partial \Pi}{\partial \ell} = \underbrace{-\frac{\partial}{\partial \ell}\left[\int_v w(\varepsilon_{ij})dv - \int_{s_e} t_i u_i ds\right]}_{G_{tip}(d,\ell)} \underbrace{-\frac{\partial}{\partial \ell}\int_{s_i} \Phi(u)ds}_{G_{bond}(d,\ell)} = 0 \tag{5}$$

The terms in the brackets represent the strain energy release rate at creation of a new crack surface and the last term is the rate of the energy absorption in the crack end zone and is associated with the energy necessary to create a unit of its new surface. Note, that within the framework of the model the rate of the energy absorption depends on the end zone size and bond characteristics. The equilibrium end zone size is not assumed to be constant. It can be determined from condition (5) while searching for the critical load needs additional conditions of the bond rupture.

In the general case the strain energy release rate can be defined through the stress intensity factors

$$G_{tip}(d,\ell) = \mathbb{Z} K_B^2, \tag{6}$$

where the parameter $\mathbb{Z}$ depends on the elastic properties of materials and $K_B = \sqrt{K_{\mathrm{I}}^2 + K_{\mathrm{II}}^2}$ is the modulus of the stress intensity factors due to the external loads and the stresses in the crack end zone.

The stress intensity factors (SIF) $K_{I,II}$ for the interface bridged crack are determined in [6].

Let us calculate the rate of the energy absorption for the interface crack with bonding. Denote by $U_{bond}(d,\ell)$ the work of the deformation of bonds and by $G_{bond}(d,\ell)$ the rate of the energy absorption per unit thickness of the body. Then

$$U_{bond}(d,\ell) = b\int_{\ell-d}^{\ell} \Phi(u)dx\,, \quad G_{bond}(d,\ell) = \frac{\partial U_{bond}(d,\ell)}{b\;\partial \ell} \tag{7}$$

where b is the body thickness.

The density of the strain energy of the bonds is equal to

$$\Phi(u) = \int_0^{u(x)} \sigma(u)du\,, \quad u(x) = \sqrt{u_x^2(x) + u_y^2(x)}, \quad \sigma = \sqrt{q_x^2 + q_y^2} \tag{8}$$

After differentiation in formula (7) with respect to the upper and the bottom limits of the integral we can get

$$\frac{\partial U_{bond}(d,\ell)}{b\partial \ell} = \int_{\ell-d}^{\ell}\left(\frac{\partial u(x)}{\partial \ell}\sigma(u)\right)dx + G_c - G_b, \tag{9}$$

where

$$G_c = \int_0^{u(\ell)} \sigma(u)du;, \quad G_b = \int_0^{u(\ell-d)} \sigma(u)du \tag{10}$$

If we consider the model of the crack with zero opening at the crack tip $(u(\ell)=0)$ then $G_c = 0$ and it is necessary to add in the left part of (9) the value of the intrinsic toughness of the matrix material

$$G_c = 2c_m\gamma_m$$

where c_m is the volume fraction of the matrix material and $2\gamma_m$ is the matrix toughness.

Finally, we obtain the following expression for the rate of the energy absorption

$$G_{bond}(d,\ell) = \int_{\ell-d}^{\ell} \left(\frac{\partial u_y(x)}{\partial \ell} q_y(u) + \frac{\partial u_x(x)}{\partial \ell} q_x(u) \right) dx - \int_{0}^{u(\ell-d)} \sigma(u) du + G_c \tag{11}$$

where the second term is the density of deformation energy allocated at break of the bond at the trailing edge of the crack end zone.
For a homogeneous material or an adhesion layer connecting different materials the following relations are held

$$G_c = G_b = \int_{0}^{u(\ell-d)} \sigma(u) du \tag{12}$$

In this case the expression (11) completely coincides with similar expressions from [6, 7].
For a weak matrix material $(G_c \ll G_b)$ we suppose $G_c = 0$ in (11). In this case $G_{bond}(d,\ell) \to 0$ if $d/\ell \to 0$ and therefore this approach coincide with Barenblatt's model in this limit [6].
The condition of the crack tip limit equilibrium (5) can be rewritten as follows taking into account the notation from the formulae (6) and (12)

$$G_{tip}(d,\ell) = G_{bond}(d,\ell) \tag{13}$$

Condition (13) is necessary but insufficient for searching for a limit equilibrium state of the crack tip and the end zone. This condition enables us to determine the end zone size, d_{cr} , such that the crack tip is in an equilibrium at the given level of the external loads. To search for the limit state of both the crack tip and end zone within the framework of the model one should introduce an additional condition, e.g., the condition of bond limit stretching at the trailing edge of the end zone $x_0 = \ell - d_{cr}$

$$u(x_0) = ([u_x(x_0)]^2 + [u_y(x_0)]^2)^{1/2} = \delta_{cr} \tag{14}$$

where δ_{cr} is the bond rupture length.
If

$$G_{tip}(d,\ell) \ge G_{bond}(d,\ell) \tag{15}$$

at a certain end zone size, d , and

$$u(\ell - d) < \delta_{cr} \tag{16}$$

then the crack length increases with the end zone growth up to the size d_{cr} without bond rupture. This stage of the crack growth can be treated as the system shakedown to the given level of the external loads (subcritical crack growth).
The crack tip advance with simultaneous bond rupture at the trailing edge of the end zone occurs if both conditions

$$u(\ell - d) \ge \delta_{cr} \tag{17}$$

and (15) are fulfilled.
The regime of bond rupture at the trailing edge of the end zone without the crack tip advance is observed then conditions

$$G_{tip}(d,\ell) < G_{bond}(d,\ell) \tag{18}$$

and (17) are fulfilled. In this case the size of the end zone decreases and tends to the limit value d_{cr} at the given load.

The end zone size and crack length are reserved within the framework of the model if the inequalities (16) and (18) hold. Thus, the bond rupture characteristics and load level determine the fracture regimes: 1) the crack tip advance with the end zone growth; 2) end zone shortening without the crack tip advance; 3) the crack tip advance and bond rupture at the trailing edge of the end zone.

Solving jointly eqs. (13-14) we can determine the critical external loads σ_0, the end zone size d_{cr} and the adhesion fracture resistance at the crack limit equilibrium state for the given crack length and bond characteristics.

3 CRACK WITH UNIFORM BRIDGED STRESSES

At first, the analytical consideration of the proposed criterion is performed for the problem of the straight crack in a homogeneous plane with the rectilinear law of the bond stress. In this very simple case the normal bridged stresses in the crack end zone are prescribed $(Q(x) = P_0)$, uniformly distributed along the end zone and independent on the crack opening. The normal displacements of an upper crack surface for this problem $u_0(x)$ are given by (*Panasyuk,* [8]).

In the case small scale bridging condition we obtain from eqs. (13-14) of the two-parametric criterion the critical end zone size which is *independent on the crack size in a small scale bridging limit* (see details in *Perelmuter,* [9])

$$d_{cr} = d_\infty = d_0\left(\sqrt{\eta+1}-\sqrt{\eta}\right)^2, \quad d_0 = \frac{\pi E\delta_{cr}}{8P_0}, \quad \eta = \frac{G_c}{G_b} \qquad (19)$$

and the critical external stress (E is Young modulus of material)

$$\sigma_{cr} = \sqrt{(1+\eta)\frac{EP_0\delta_{cr}}{\pi\ell}} = \sqrt{\frac{E\left(G_b+G_c\right)}{\pi\ell}} \qquad (20)$$

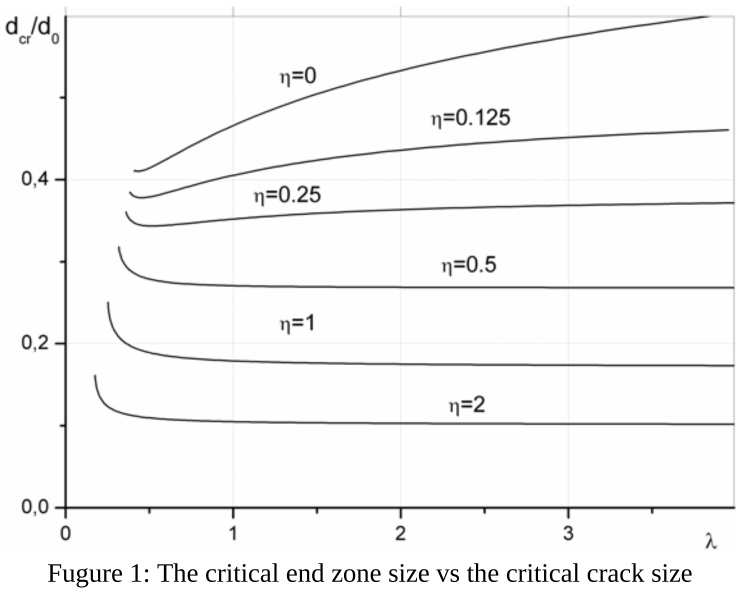
The size and the shape of the crack end zone do not change in the case of small scale bridging, therefore, the condition of autonomy of the end zone is satisfied and the energy absorbed to bonds in the end zone is equal to the energy released while breaking the bonds at the edge of the end zone. Thus, the total flow of the energy to the crack tip is spent on formation of a new surface of the crack. For this reason relationships (19-20) coincide with results which was obtained in (*Cox, Marshall* [10]) on the basis of the two-parametric fracture criterion with the first force condition of fracture $K_0 - K_b = K_{Ic}$, where K_0 is the SIF due to an external loading, K_b is the SIF due to bonds and K_{Ic} is the matrix toughness. Noted that in the force fracture criterion [10] the work of bonds in the crack bridged zone is neglected and for the large scale bridging the noticeable difference is observed because, for example, nanotubes are rather deformable and stiff in compare to matrix material and their strain energy must not be ignored.

In the case of the uniform bridge stress it's possible to get the analytical solutions for the critical end zone size and the external stresses also for the large scale bridging case [9]. The dependencies of the critical end zone size d_{cr}/d_0 vs the critical crack size $\lambda = \ell_{cr}/d_0$ is given in fig. 1. It's interesting to not note, that for composites with a weak matrix material $(\eta < 0.5)$ the critical end zone size is increasing during the crack growth (nanocraze formation), see the experimental results in [1].

Work was partly supported by RFBR, grant number is 02-01-000116 and by grant for leading scientific school of the Russian Federation, the grant number is 1849-2003.1.

References

1. Qian D., Dickey E.C., Andrews R. and Rantell T., Load transfer and deformation mechanisms on carbon nanotube-polystyrene composites, *App. Phys. Let.* 76 (20) 2868 (2000)
2. Xia Z. at al. Direct observation of toughening mechanisms in carbon nanotube ceramic matrix composites, *Acta Materiala*, vol. 52, pp. 931-944 (2004)
3. Srivastava D., Wei C., Cho K. Nanomechanics of carbon nanotubes and composites, *App. Mech. Rev.*, vol. 56, no 2, pp.215-230 (2003)
4. Wanga J., Pyrzb R. Prediction of the overall moduli of layered silicate-reinforced nanocomposites, part I, II: *Composites Science and Technology,* 64, 925–9544 (2004)
5. Watts P.,C.P., Hsu W.K. Behaviours of embedded carbon nanotubes during film cracking, *Nanotechnology*, vol. 14, L7-L10 (2003)
6. Goldstein R. V., Perelmuter M. N. Modeling of bonding at an interface crack, *International J. of Fracture*, (1-2), 53-79 (1999)
7. Perelmuter M. N. Polymer interface crack and adhesion resistance In: *Proc. of Workshop 'Mechanical Reliability of Polymeric Materials and Plastic Packages of IC Devices'*, 231-236, Paris, France. (1998)
8. Panasyuk V. V. *Limiting Equilibrium of Brittle Solids with Fractures*. Michigan Information Service, Detroit, 284 pp. (1971)
9. Perelmuter M. N. Fracture criterion for cracks with bridged zone, pp. 261-270, In: *IUTAM Symposium on Asymptotics, Singularities and Homogenisation in Problems of Mechanics*, A.B. Movchan (Ed.), Kluwer Academic Publishers (2003)
10. Cox B N and Marshall D B Concepts for bridged cracks in fracture and fatigue, *Acta metal mater.* (2), 341-363 (1994)



Fugure 1: The critical end zone size vs the critical crack size