FORMULATION AND IMPLEMENTATION OF A STRONG DISCONTINUITY MODEL FOR GRANULAR ROCKS INCORPORATING THE EFFECT OF SLIP SPEED ON FRICTIONAL RESISTANCE

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ABSTRACT

Strain localization is a ubiquitous feature of geomaterials undergoing non-homogeneous deformation. In rocks, the zone of localized deformation is generally referred to as a fault, shear band, or rupture zone, and is commonly explained as the result of bifurcation of the material response from a homogeneous deformation pattern. A post-localization response follows right after bifurcation, allowing for the formation and evolution of a shear band. A common technique for tracking the evolution of a shear band is based on strong discontinuity concept, where the band thickness is assumed to be equal to zero and the displacement field is assumed to be discontinuous. In this paper, we utilize the advances in strong discontinuity modeling to track the evolution and propagation of a shear band in granular rocks. In modeling the post-localization response by strong discontinuity concept, it is necessary to prescribe a constitutive law on the fractured zone that captures the effect of frictional interaction of two irregular surfaces sheared on their planes of contact. In this work, we utilize a well-known friction law for fractured rock. This contact law allows a detailed description of the constitutive phenomena occurring in fracture zones, including the effect of slip speed on the coefficient of friction mobilized in the fracture zone. We then embed this interface constitutive law into the finite element interpolation to enhance the capability of the element to capture localized deformation. The formulation is general enough to allow the incorporation of other, more complex constitutive models on the surface of discontinuity.

1. INTRODUCTION

Strain localization is a ubiquitous feature of geomaterials undergoing non-homogeneous deformation. In rocks, the zone of localized deformation is generally referred to as a fault, shear band, failure surface, or rupture zone. The large deformations and associated loss of strength accompanying these localized regions make them an important topic of study. Some of the applications of localization failure analysis include faulting, failures of foundations and slopes, and other applications related to oil and gas reservoirs, seismic safety of embankment dams, and behavior of concrete structures.

Several methods have been proposed for modeling localized deformation. However, bifurcation theory is one analysis technique that has shown promise in recent years. The original concept was proposed by Rudnicki and Rice [7]. The theory proposes that the onset of localization is a material instability that can be detected by the appearance of a zero eigenmode in the acoustic tensor for a particular normal. This normal is the normal to the failure surface, and the eigenvector is the slip direction of the localized region. For some simpler constitutive models and geometric approximations (e.g. plane strain assumption), the normal can be determined analytically [1-3], but for general three- dimensional problems a numerical search algorithm must be performed for a normal that may produce an acoustic tensor that is not positive definite [6,10].

The bifurcation theory may be coupled with the strong discontinuity approach, where the localized region is approximated by a slip surface for three-dimensional problems, or a slip line for two-dimensional problems. In the case of rocks and even some clays, this is often physically justified as the formation of separate surfaces is observed. In the case where the localized deformation zone does have finite thickness, this technique can still be used provided that the constitutive behavior of the shear band is properly modeled. It is preferable to avoid modeling a band of finite thickness, as thickness of this band is often several orders of magnitude smaller than the problem of interest. The need for excessive mesh refinement to capture the displacement across the band could make the numerical problem difficult to solve. Capturing the physical dimension of the shear band also requires the introduction of a length scale parameter that must be experimentally determined and justified. From a numerical modeling standpoint, it is also important to point out that bifurcation condition results in the loss of ellipticity of the governing partial differential equations, which could lead to spurious and strongly mesh dependent results. Detecting and correcting this condition, then, is important for ensuring the validity of the solutions.

2. DESCRIPTION OF THE MODEL

In order correct the mesh dependence of finite element solutions, we have implemented a post-bifurcation constitutive model. While bifurcation theory imparts the normal to the slip surface as well as the slip direction, the post-bifurcation behavior is not automatically given. Post-bifurcation analysis is the subject of much current research, since for many applications the deformations are an important consideration. Seismic evaluation of embankment dams and penetrator applications are examples of such problems. We have utilized a well-known friction law for granular surfaces proposed by Dieterich and Linker [4] and Dieterich and Kilgore [5], among others. This is a rate- and state-dependent model that allows for an elaborate description of the constitutive phenomena occurring in fracture zones in rocks, including the effect of slip speed on the effective friction angle mobilized in the fracture zone, and the effect of wear on the frictional surface. This constitutive model has been shown to be valid under a variety of conditions and should create a realistic slip behavior along the surface.

Figure 1 portrays the effect of steps in slip speed on coefficient of friction for different materials, as reported by Dieterich and Kilgore [5]. Clearly, higher slip speeds increase the coefficient of friction, while lower slip speeds decrease it. It is important that this constitutive feature be reflected in the description of frictional resistance on the surface of discontinuity, since deformation at post-localization is concentrated to this surface. Fortunately, complex constitutive models required to capture the features shown in Figure 1 can easily be incorporated into finite element modeling of strain localization via embedded strong discontinuity technique, described below.

Numerical implementation of strong discontinuity within the framework of the finite element method has traditionally employed the so-called assumed enhanced strain (AES) formulation [8,9]. This method relies on the additive decomposition of the total displacement field into a coarse-scale part and fine-scale part. Consequently, the technique requires the introduction of two weighting functions, one for the coarse-scale field as required in standard finite element formulation, and another for the fine-scale field. The latter weighting function for the fine-scale field is arbitrarily chosen in the AES method. Since the final solution of the technique depends on the assumed weighting function for the fine-scale field, which in turn is arbitrarily chosen, then we can conclude that the solution from the AES method is not physically meaningful.

In recent publications [1-3], it has been pointed out that in the context of strain localization via embedded strong discontinuity the weighting function for the fine-scale field is closely linked to the elastoplastic constitutive behavior on the surface of discontinuity. Therefore, varying the weighting function for the fine-scale field is tantamount to altering the constitutive response, or the characteristics of the decohesion model on the surface of discontinuity. This improved understanding of the role of the weighting function on the predicted structural response is extremely important since it sheds much light onto the physical significance of the solution, as well as on the underlying physical justification for the use of different weighting functions in finite element formulations.

Figure 2 shows how the embedded strong discontinuity technique may be used to capture the post-localization response of Gosford sandstone. Different finite element meshes have been used, and the solutions exhibited absolute mesh-insensitivity [1]. All of the solutions have utilized a simple frictional law on the surface of discontinuity, in which the coefficient of friction is constant. However, more complex constitutive laws, such as that depicted in Figure 1, may be used to model the post-localized behavior of rocks as well.

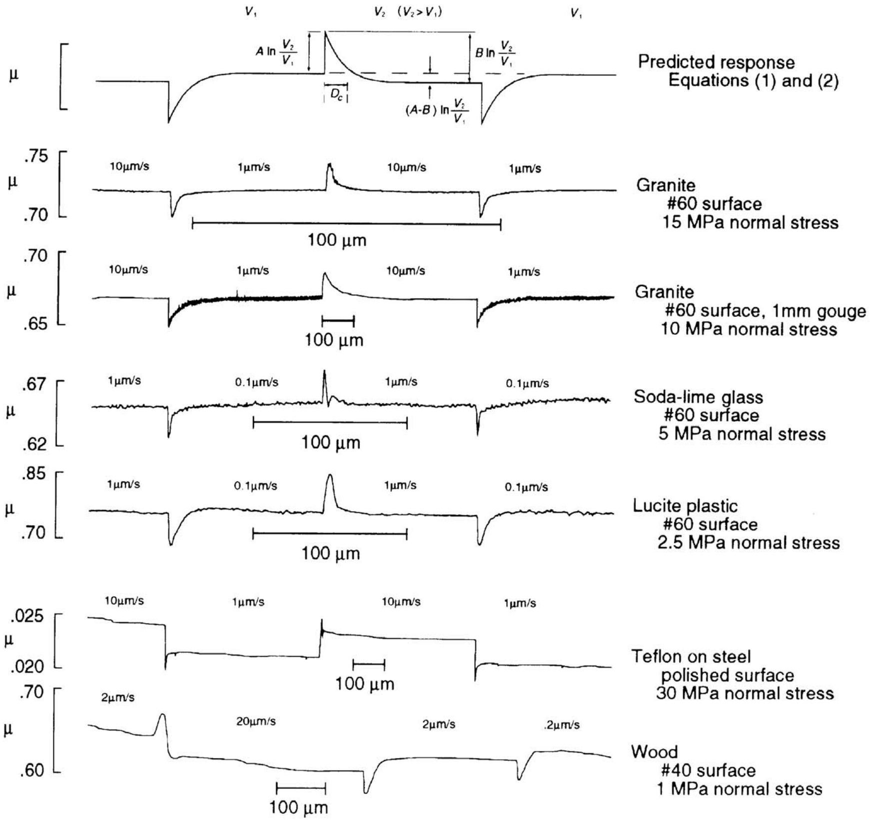

Figure 1. Effect of steps in slip speed on coefficient of friction μ (after Dieterich and Kilgore [5]).

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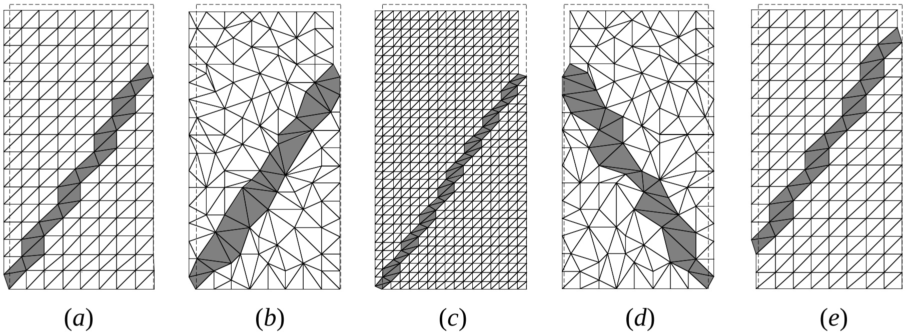

Figure 2. Deformed FE meshes for localization analysis of plane strain compression on Gosford sandstone tested at a confining stress of 20 MPa: (a) regular unstructured mesh; (b) irregular mesh; (c) regular unstructured finer mesh; (d) irregular mesh, second band orientation; (e) regular unstructured mesh, alternative band position (after Borja et al. [2]).

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