AN ANALYSIS OF DISTINCTION BETWEEN LOADING MODE AND FRACTURE MODE

Zongqi Sun[1], Qiuhua Rao[2] and Feng Chen[1]

[1]College of Resources and Safety Engineering, Central South University, Changsha, 410083, P.R. China

[2]School of Civil and Architecture Engineering, Central South University, Changsha, 410083, P.R. China

E-mail: sunzongqi@sina.com, Raoqh@mail.csu.edu.cn

ABSTRACT

The study of stress state under shear load indicated that shear load applied to crack induces tensile fracture instead of shear fracture. It leads to consider that loading mode does not always correspond to fracture mode. Under different mixed loading, fracture belongs to single tensile fracture in most cases, only in certain special case shear fracture can occurs. Thus fracture mode should be distinguished from loading mode.To judge what fracture mode will occur under arbitrary loading prerequisites for mode I and mode II fracture become vital important. The criterion to define mode I or mode II fracture under any plane loading can be summarized as: prerequisites for mode I fracture are: 1), $f_{\theta max} > f_{r\theta max}$ or $f_{r\theta max} / f_{\theta max} < 1$, or 2), $f_{r\theta max}/ f_{\theta max} > 1$, but $f_{r\theta max}/ f_{\theta max} < K_{IIC}/K_{IC}$; prerequisite for mode II fracture is $f_{r\theta max}/ f_{\theta max} > 1$ and $f_{r\theta max}/ f_{\theta max} > K_{IIC}/K_{IC}$, where $f_{\theta max}$ is the dimensionless maximum circumferential stress intensity factor and $f_{r\theta max}$ is the dimensionless maximum shear stress intensity factor.

1. INTRODUCTION

Classical fracture mechanics always distinguish three basic modes from the point of crack surface displacement view [1]: mode I (opening mode) corresponds to normal separation of the crack walls under the action of tensile stress; mode II (sliding mode) corresponds to mutual sliding of the crack walls in a direction normal to the crack front; mode III (tearing mode) corresponds to mutual shearing parallel to the crack front. The stress intensity factors and their critical values (as called fracture toughness) associated with each mode are labeled by K_I, K_{II}, K_{III} and K_{IC}, K_{IIC}, K_{IIIC} respectively. Unfortunately, classical fracture mechanics does not pay sufficient attention to judge what fracture mode could occur under arbitrary loading and only simply relates three basic fracture modes to loading mode, i.e. the mode of loading is identical to the mode of fracture as shown in Fig.1. In fact, mode I loading causes mode I fracture, but mode II loading does not cause mode II fracture.

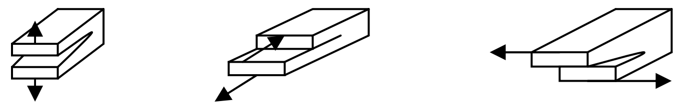

Fig.1 The three basic modes

2. WHY SHEAR FRACTURE DOES NOT TAKE PLACE WHEN CRACK IS SUBJECTED TO SHEAR STRESS?

A number of specimens providing pure shear stress on crack plane proposed by different authors are available. The common method used for shear fracture testing is anti-symmetric four point bending test[2-3], in which only shear stress exists on the notch. In addition, to conduct shear test, circular tubes of PMMA[4] or 4340 steel[5] were loaded in torsion. Richard[6], Banks-Sills[7] and Watkins[8] developed compact tension-shear specimen, edge cracked Arcan specimen and short beam compression specimen respectively. All results obtained by these specimens showed crack branching and 'K_{IIC}', being less than K_{IC}, Table 1. It had to conclude that 'in ideal brittle materials, the so-called "sliding" and "tearing" modes of crack extension do not take place. The mode of fracture seems to be always a crack opening' [9], with which many researchers agree.

To explain no shear fracture occurred under shear loading the dimensionless circumferential and shear stress intensity factors, f_θ and $f_{r\theta}$, of infinite plate with central crack are employed[1] as:

$$f_\theta = \sigma_\theta \sqrt{2\pi \cdot r} / K_{II} = -3\sin\frac{\theta}{2}\cos^2\frac{\theta}{2} \qquad f_{r\theta} = \sigma_{r\theta}\sqrt{2\pi \cdot r} / K_{II} = \cos\frac{\theta}{2}(1 - 3\sin^2\frac{\theta}{2}) \quad (1)$$

Table 1 Some test results of fracture angle and 'shear fracture toughness' data under shear loading

Materials	Test methods	Fracture angles	'K_{IIc}' $Mpam^{1/2}$	'K_{IIc}' / K_{Ic}	References
plexiglass	A plate with a central crack subjected to opposite concentrated shear loads	~70	0.464	0.89	Endogan & Sih[9]
PMMA	A circular tubes under torsion	~70	1.0	0.89	Ewing&Williams [4]
4340 steel	A circular tubes under torsion	70-75	74.2	0.92	Shah [5]
Westerly granite	Antisymmetric 4-point bending specimen		2.16	1.05	Ingraffea [2]
Marble	Antisymmetric 4-point bending specimen	58-68	0.76	0.61	Wang [3]
concrete	Short beam compression specimens		0.53	0.75	Watkins & Liu[8]
PMMA	Compact tension-shear specimen		1.52	0.93	Richard [6]

Variation of f_θ and $f_{r\theta}$ with angle θ are shown in Fig.2. It is shown when a crack is subjected to pure shear loading, both circumferential and shear stresses exist around the crack tip. The dimensionless circumferential stress here has different sign, i. e. positive represents tensile stress, negative--compressive stress. The maximum tensile stress is larger than maximum shear stress: the dimensionless tensile stress intensity factor has maximum value of 1.1546, while maximum shear stress intensity factor is equal to 1.The tensile stress reaches its maximum at $\theta = -70.5^o$, while shear stress is equal to zero. The shear stress reaches its maximum value at $\theta = 0^o$ (original crack plane) while circumferential stress is equal to zero.

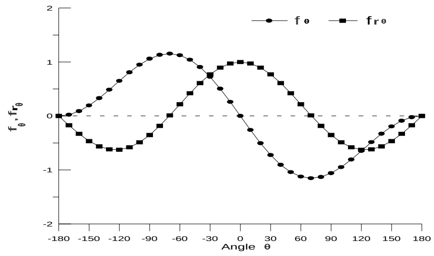

Figure.2 Angular variation of circumferential f_θ and shear $f_{r\theta}$ in pure shear loading

Now the question is where fracture will occur, at θ = -70.5 or at θ = 0? Many fracture tests [2-9] under mode II loading proved that the fracture was deviated from original crack plane with angle 60^o~75^o. It clearly indicates that the fracture was caused by tensile stress, not caused by shear stress. Thus fracture should be related to mode I (opening mode)rather than to mode II(sliding mode). Recognizing the fracture at θ = -70.5^o being mode I, the stress intensity factor of mode I in this plane under shear loading will be expressed as:

$$K_I^{II}=1.1546\ \tau\ (\pi\ a)^{1/2}. \quad (2)$$

Where superscript II denotes mode II loading.

Then shear stress intensity factor at θ = 0 is expressed as:

$$K_{II}=\tau\ (\pi\ a)^{1/2}. \quad (3)$$

3. WHY THE SHEAR FRACTURE TOUGHNESS, 'K_{IIC}', CALCULATED FROM SHEAR TEST IS ALWAYS LESS THAN K_{IC}?

Classic fracture mechanics usually considers the fracture caused by shear loading as a shear fracture. Equations of shear stress intensity factor for different specimens given in literatures and handbooks were derived at θ = 0^o where shear stress intensity factor has maximum value and tensile stress intensity factor vanishes. Note that all these equations for different specimens under shear loading are correct. The question is that no fracture really takes place along original crack plane, i.e. at θ=0^o. The shear stress, as well as shear stress intensity factor always equals to zero at θ= -70^o where fracture does occur under shear loading. Therefore, after shear tests the fracture toughness calculated by the present equations of shear stress intensity factor is meaningless and 'K_{IIC}' calculated by these equations will inevitablely be less than K_{IC}. For example, when a central cracked infinite plate is subjected to shear loading, the fracture toughness calculated by equation (3) will be certainly less than that calculated by equation (2).

Unfortunately up to now all literatures and handbooks have not provided the equations similar to equation (2) to calculate tensile stress intensity factor, K_I, under shear loading at angle where K_I reaches its maximum value.

4. HOW TO OBTAIN SHEAR FRACTURE?

As mentioned above, when a crack is subjected to shear loading, f_θ reaches its maximum value of 1.1546 at θ=-70.5^o and it is larger than 1 in the range of angles -90< θ< -50. To restrain or eliminate high tensile stress around crack tip at angles θ >|50|, the compressive stress applied normal to crack plane is insufficient. For example, knowing tensile stress around crack tip obtained by numerical study of beam, Swartz and Taha[10] applied axial force during four point bending test and Petit[11] applied uniaxial compressive stress on a plate with central crack of 30^o to loading axis, crack branching was discovered again. Thus compressive stress parallel to crack plane should be applied in opposite direction above and below crack plane. This parallel compressive stress should be almost uniformly distributed on the lateral sides of the block, instead of concentrated shear force and will be effective to restrain or eliminate high tensile stress at angles θ >|50|. Meantime it serves as a shear stress on the crack plane.

In addition, Fig.2 shows that f_θ with lower value exists at small angles θ < |50|. Thus compressive stress normal to crack plane with smaller magnitude should be also applied to the specimen. Thus the shear model shown in Fig.3 is proposed[12]. This model is distinguished from general compression-shear load (inclined crack under uniaxial compressive load) for shear fracture while the latter only leads to mode I fracture during tests.

Izum [13], Rao[14] and Wang[3] conducted tests on concrete and rocks using compression-shear box, corresponding to a shear model (Fig.2) keeping $\sigma / \tau < 0.5$. It causes co-plane shear fracture and K_{IIC} is larger than K_{IC}. It is worthy noting that to calculate K_{IIC} the effective shear stress was adopted instead of shear stress due to normal compressive stress applied on crack plane.The ratio of K_{IIC} /K_{IC} is in the range of 2.5-3.5 for rock material[3,14], 1.7 for concrete[13] and 1.73 for 4340 steel[15]. Shear fracture tests could be found in literatures[3,14-15] in details.

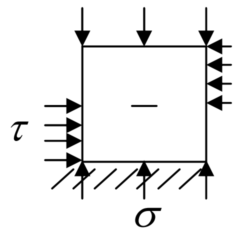

Fig.3 Shear model

5. HOW TO JUDGE WHAT FRACTURE MODE, MODE I OR MODE II, WILL OCCURUNDER ARBITRARY LOADING?

In order to judge fracture mode under arbitrary loading, it is vital important to study the stress state around crack tip (0--±180) and to compare maximum circumferential tensile stress intensity factor with maximum shear stress intensity factor. Thus, a parameter, $f_{r\theta max}$ / $f_{\theta max}$, identical with K_{II}

$(\tau_{rmax})/K_I(\sigma_{\theta max})$, is proposed[19].

A prerequisite of possible occurrence of mode II fracture under arbitrary loading could be described as: maximum dimensionless shear stress intensity factor, $f_{r\theta max}$ around crack tip should be larger than maximum dimensionless tensile stress intensity factor, $f_{\theta max}$,. It is, however, still insufficient for mode II fracture. Mode II fracture could occur under a given loading condition if an inequality, $f_{r\theta max} / f_{\theta max} > K_{IIC}/K_{IC}$, is satisfied.

When $f_{r\theta max}/ f_{\theta max} < 1$, the mode I fracture is preferred to occur for brittle material. It happens under many loading conditions such as tensile, shear as well as mixed (tensile-shear) loading, Fig.4.

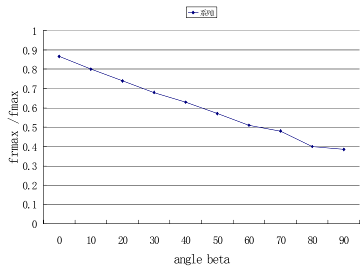

Figure. 4 Variation of $f_{r\theta max}/ f_{\theta max}$ with β under tensile-shear mixed loading

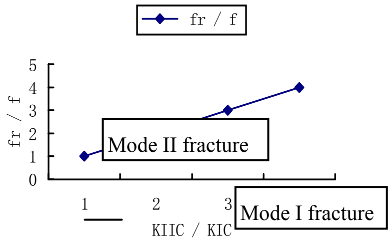

Figure.5 Fracture locus for occurrence of mode I or mode II fracture

Fig.4 indicates that the ratio, $f_{r\theta max}/ f_{\theta max}$,, has the lowest value of 0.385 under pure tensile loading. In tensile—shear loading it continuously increases and reaches 0.866 under pure shear loading. It is, however, always less than 1, which will result in mode I fracture only. If $f_{r\theta max} / f_{\theta max} < K_{IIC}/K_{IC}$, mode I fracture will certainly take place even if $f_{r\theta max} > f_{\theta max}$. The prerequisite for possible occurrence of mode I and mode II fracture can be illustrated in Fig.5.

6. CONCLUSION

Following conclusions can be drawed from above analysis:

1) Note that the loading mode does not always correspond to fracture mode. Under tensile, shear and tension-shear loading only pure mode I fracture could occur.

2) Shear load applied to crack induces only tensile fracture instead of shear fracture because the maximum tensile stress is larger than maximum shear stress around crack tip.

3)After shear tests the fracture toughness calculated by the present equations of shear stress intensity factor is meaningless and 'K_{IIC}' calculated by these equations will inevitably be less than K_{IC}.

4) To obtain shear fracture the initial crack should be loaded under special mixed loading as shown in Fig.3. Fracture toughness of mode II is larger than that of mode I, if experiment is

conducted correctly.

5) To define what fracture mode will occur under arbitrary loading is vital important. The criterion to define mode I or mode II fracture under any plane loading can be summarized as follows:

Preferred conditions to occurrence of *pure mode I fracture* are:

a), $f_{\theta max} > f_{r\theta max}$ or $f_{r\theta max} / f_{\theta max} < 1$, and $f_{r\theta} = 0$ when f_θ reaches its maximum at certain θ,or

b), $f_{r\theta max}/ f_{\theta max} > 1$, but $f_{r\theta max}/ f_{\theta max} < K_{IIC}/K_{IC}$

Preferred conditions for occurrence of *mode II fracture* are:

$f_{r\theta max}/ f_{\theta max} > 1$ and $f_{r\theta max}/ f_{\theta max} > K_{IIC}/K_{IC}$

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