BONDSPLITTING STRENGTH OF RC MEMBERS BASED ON LOCAL BOND STRESS AND SLIP BEHAVIOR

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ABSTRACT

This research presents a proposal of predicting formulas for bond splitting strength of RC members in case of no lateral reinforcement. The purpose of this study is to derive a simple calculation formula for bond splitting strength keeping the mechanical meaning considerations. Making the area of the equivalent bond stress block (EBSB) same as one of local bond stress versus slippage of reinforcement relationship proposed by the authors, the prediction formula for bond splitting strength is simply built. Assuming that bond stress is a constant value, strain and slippage of reinforcement are obtained from solving a simple differential equation. A new prediction formula is expressed separately in case of longer bond length than effective bond length and shorter bond length. The predicted bond splitting strengths show a good agreement with experimental results observed in previous studies.

1 INTRODUCTION

In many studies related to bond splitting behavior of reinforced concrete members, it is reported that many local bond stress versus slippage relationships are proposed. Authors have also quantified local bond stress - slippage of reinforcement relationships without lateral reinforcement from results of the bond splitting test which has been carried out using shorter bond length specimens [1]. From the results of the numerical analysis conducted to obtain average bond behavior using this model, analyzed bond splitting strengths showed a good correlation with experimental values observed in previous studies.

On the other hand, some bond splitting strength calculation formulas are proposed by the present [2,3,4]. However, these calculation formulas are proposed by regression analysis of experimental results and mechanical meaning considerations is not always done.

The purpose of this study is to derive a new prediction formula for bond splitting strength keeping the mechanical meaning considerations by simple method based on local bond stress versus slippage relationships.

2 BOND STRESS DISTRIBUTION OF REINFORCEMENT

2.1 Numerical analysis using local bond stress - slippage relationships

The authors have proposed local bond stress - slippage of reinforcement relationships as shown in the below [1].

$$\tau_b = 2 \cdot \sigma_t \cdot \beta \cdot s \cdot \frac{(r_u/d_b)^2 - (\beta \cdot s)^2}{(r_u/d_b)^2 + (\beta \cdot s)^2} \cdot \cot\alpha \tag{1}$$

where, τ_b = local bond stress, σ_t = tensile strength of the concrete, β = coefficient between the internal crack depth and slippage of reinforcement = 10.2 (1/mm), s = slippage of reinforcement, d_b = diameter of reinforcement, $r_u = C + d_b/2$ (C : thickness of concrete cover), α = angle between the principal bond stress and the axis of the reinforcement (= 34 degrees), note : $\tau_b = 0$ in $s > r_u/(\beta \cdot d_b)$.

Analytical results by numerical calculation of pull-out bond test using cantilever type specimens are shown in Figure 1 and 2. The dimensions and mechanical properties of specimens used in the analysis are as follows :

- Dimensions : Section = 300 x 400 mm (cantilever type specimens)
- Concrete : Tensile strength of concrete (σ_t) = 2.45MPa
- Reinforcement : Young's modulus (E_b) = 198GPa, 4-D19 of deformed bar
- Thickness of concrete cover : r_u/d_b =1.96 (from center of reinforcement)
- Bond length : l_b = 150, 300, 600mm (3 cases)

Figure 1 shows analytical average bond stress - slippage of loaded end slip relationship in case of bond length of 150, 300, 600mm. The circle plots indicate maximum average bond stress (bond splitting strength). Figure 2 shows analytical results for (a) bond stress distribution; (b) tensile load distribution; (c) slippage distribution along the axial direction with distance from the free end of specimen. The left, center and right side graphs correspond to the case of 150mm, 300mm, 600mm bond lengths, respectively. Solid lines indicate the results at maximum average bond stress using Eq.(1). Hatched boxes in (a) and dotted lines in (b) and (c) indicate the images of distributions considered by constant bond stress which defined as equivalent bond stress block (EBSB). Namely, bond stress distributes as constant, tensile load distribution is linear, and slippage distribution is parabolic.

In the cases of 150mm and 300mm bond length, bond stress is observed in the whole bonded region. However, in the case of 600mm bond length, bond stress is distributed on the limited region, and this region moves from loaded end to free end of specimens.

2.2 Equivalent bond stress block (EBSB)

Equivalent bond stress block (EBSB) is defined such that the constant bond stress distribution has the same area with actual bond stress distribution [5]. Bond stress of EBSB changes by bond length as shown in Figure 2. Bond stress of EBSB increases as bond length becomes shorter, and decreases as bond length becomes longer, even if any other variables are not changed. Therefore, in the case of longer bond length, it is necessary to define the effective bond length on which comparatively effective bond stress is observed. Since bond stress distribute in region of effective bond length, a lower limit of bond stress of EBSB could be defined.

3 PREDICTING FORMULA FOR BOND SPLITTING STRENGTH

3.1 Basic equations about bond analysis

The following second differential equation expresses the bond between reinforcement and concrete neglecting concrete deformation :

$$\frac{d^2 s}{dx^2} = \frac{\phi_b}{E_b \cdot a_b} \cdot \tau_b \tag{2}$$

where, s = slippage, ϕ_b = perimeter of reinforcement, a_b = area of reinforcement, E_b = Young's modulus of reinforcement, τ_b = bond stress

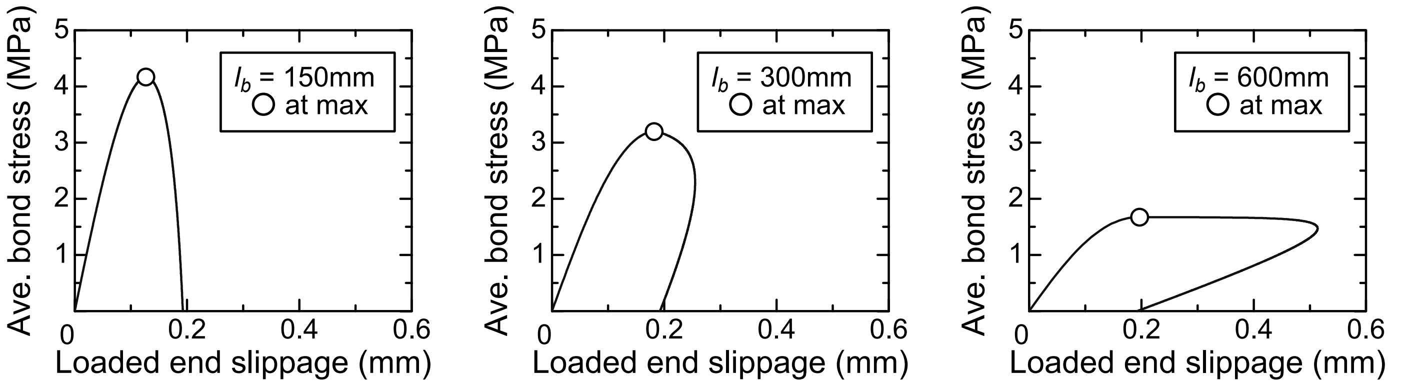

Figure 1 Analytical results of average bond stress - slippage relationships

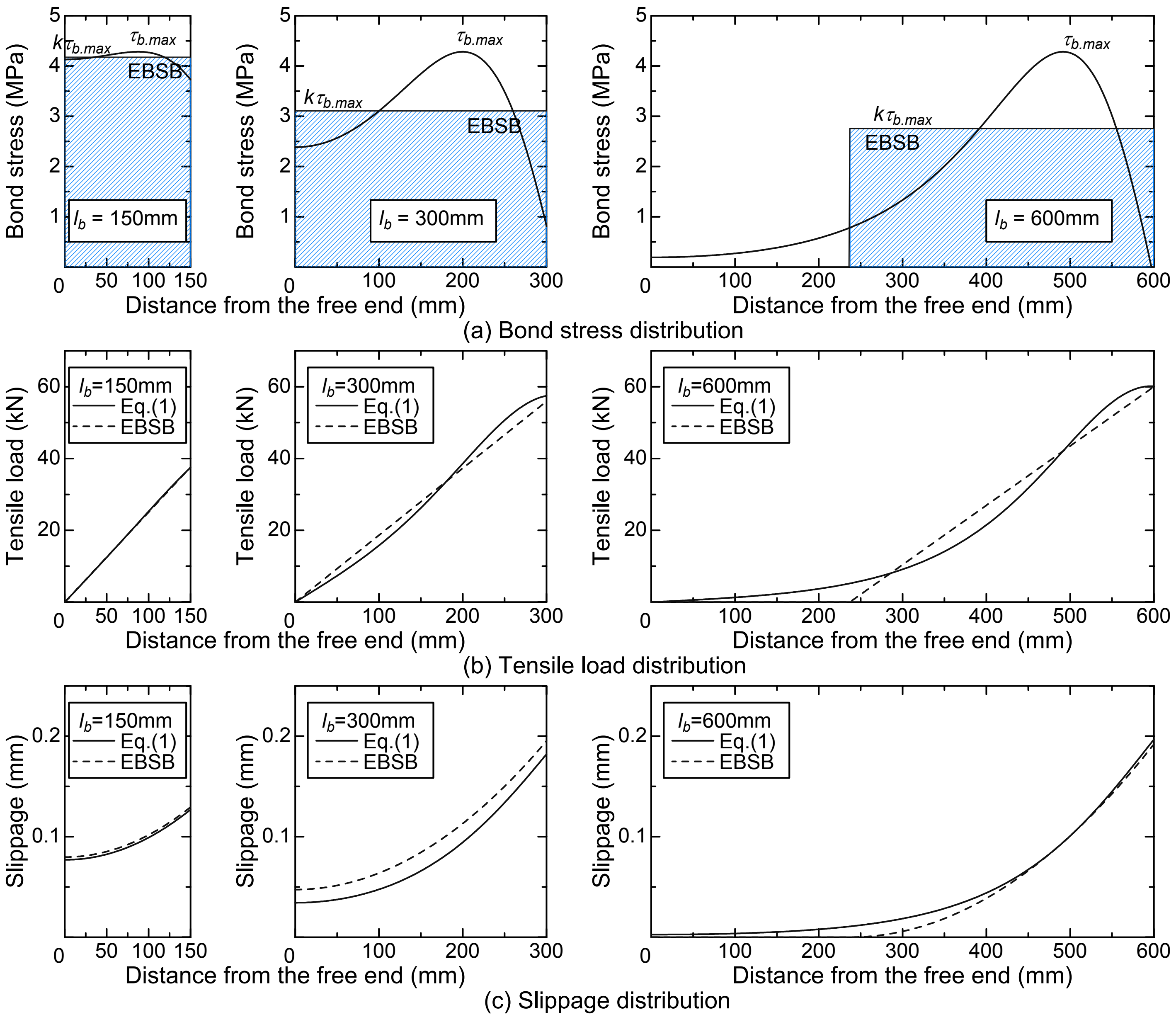

Figure 2 Analysis results of bond stress, tensile load and slippage distribution

If τ_b is expressed as Eq.(1) that is the function of s, the bond strength can be calculated by solving Eq.(2). However, solving Eq.(2) is not possible mathematically. When τ_b is assumed to be constant, that is, using EBSB, Eq.(2) can be solved easily. If the bond stress of EBSB is defined as the product of maximum local bond stress which is given by Eq.(3), and constant k ($0 \leqq k \leqq 1$), then

$$\tau_{b,max} = \left(\sqrt{5}-1\right)\sqrt{\sqrt{5}-2}\cdot\sigma_t\cdot\frac{r_u}{d_b}\cdot\cot\alpha \tag{3}$$

$$\frac{d^2s}{dx^2} = k\cdot\frac{\phi_b}{E_b\cdot a_b}\cdot\tau_{b,max} \tag{4}$$

Defining a "bond length index", λ_b, as shown in Eq.(5), Eq.(4) can be expressed as Eq.(6).

$$\lambda_b \equiv \frac{E_b\cdot a_b}{\phi_b\cdot\tau_{b,max}} \tag{5}$$

$$\frac{d^2s}{dx^2} = \frac{k}{\lambda_b} \tag{6}$$

Integration of Eq.(6) gives Eq.(7), from which Eq.(8) can be obtained, with C_1, C_2 as integration constants. Eq.(7) gives the tensile strain of reinforcement, ε_b.

$$\frac{ds}{dx} = \frac{k}{\lambda_b}x + C_1 = \varepsilon_b \tag{7}$$

$$s = (1/2)\cdot(k/\lambda_b)\cdot x^2 + C_1\cdot x + C_2 \tag{8}$$

When the origin in the axial direction is defined at the free end of cantilever type specimens, applying the boundary condition that tensile strain of reinforcement $\varepsilon_b = 0$ at $x = 0$ leads to $C_1 = 0$, hence

$$\frac{ds}{dx} = \frac{k}{\lambda_b}x \tag{9}$$

$$s = (1/2)\cdot(k/\lambda_b)\cdot x^2 + s_f \tag{10}$$

where s_f is defined as slippage at the free end of specimen and is an arbitrary value. Slippage at the loaded end of specimen, s_l, can be calculated by Eq.(10) by substituting bond length l_b for x, that is :

$$s_l = (1/2)\cdot(k/\lambda_b)\cdot l_b^2 + s_f \tag{11}$$

Δs is defined as subtraction of s_l and s_f, and is calculated as Eq.(12).

$$\Delta s \equiv s_l - s_f = (1/2)\cdot(k/\lambda_b)\cdot l_b^2 \tag{12}$$

3.2 Constant bond stress of EBSB

Defining s_e as slippage when bond stress on Eq.(1) become equal to 0, as shown by Eq.(13), the area of bond stress distribution is obtained by integration of Eq.(1) giving Eq.(14)

$$s_e \equiv \frac{r_u}{\beta\cdot d_b} \tag{13}$$

$$T(s) \equiv \frac{2\cdot s_e}{(\sqrt{5}-1)\sqrt{\sqrt{5}-2}}\cdot\left\{\log\left(1+(s/s_e)^2\right)-\frac{1}{2}\cdot\left(s/s_e\right)^2\right\} \tag{14}$$

For the purpose of calculating constant k, which is ratio of bond stress of EBSB to maximum bond stress $\tau_{b,max}$, two cases are considered. One is the case of a bond length longer than effective bond length, and another is the case of a shorter bond length.

(1) In case of longer bond length

In case of a long bond length, the actual bond stress is distributed within a limited region. Because bond stress on Eq.(1) become equal to 0 at the slippage bigger than s_e, slippage which corresponds to effective bond length becomes from 0 to s_e. Hence, k_e, which is the constant bond stress ratio in case of longer bond length, can be given as Eq.(15).

$$k_e = \frac{T(s_e) - T(0)}{s_e - 0} = \frac{2\log 2 - 1}{(\sqrt{5}-1)\sqrt{\sqrt{5}-2}} = 0.643 \tag{15}$$

When bond length of specimen, l_b, is equal to effective bond length l_e, slippage at the free end of specimen s_f and at the loaded end of specimen s_l, become equal to 0 and s_e, respectively. Therefore, Eq.(11) can be written as Eq.(16), and Eq.(17) gives the effective bond length.

$$s_e = (1/2)\cdot(k/\lambda_b)\cdot l_e^2 \tag{16}$$

$$\therefore l_e = \sqrt{\frac{2\cdot\lambda_b\cdot s_e}{k_e}} = \sqrt{\frac{2\cdot E_b\cdot a_b}{(2\log 2 - 1)\cdot\sigma_t\cdot\beta\cdot\phi_b\cdot\cot\alpha}} \tag{17}$$

(2) In case of shorter bond length

In the case of shorter bond length, the actual bond stress is distributed in whole bonded region, and the constant bond stress of EBSB changes with bond length. Defining $s_{f,c}$ as slippage of the free end and $s_{l,c}$ as slippage of the loaded end at maximum tensile load, the constant k, which is ratio of bond stress of EBSB to maximum bond stress in case of shorter bond length, can be expressed as Eq.(19) by Eq.(12) and Eq.(14).

$$k = \frac{T(s_{l,c}) - T(s_{f,c})}{\Delta s} \tag{18}$$

$$k^2 = \frac{2}{(\sqrt{5}-1)\sqrt{\sqrt{5}-2}}\cdot\frac{k_e}{(l_b/l_e)^2}\cdot\log\frac{1+(s_{l,c}/s_e)^2}{1+(s_{f,c}/s_e)^2} + \frac{1}{(\sqrt{5}-1)\sqrt{\sqrt{5}-2}}\cdot\frac{k_e}{(l_b/l_e)^2}\cdot\left\{(s_{f,c}/s_e)^2 - (s_{l,c}/s_e)^2\right\} \tag{19}$$

Eq.(19) represents the equation about bond length. However, Eq.(19) can not be solved mathematically. Figure 3 shows results of numerical analysis solving Eq.(19), for which an approximate formula expressed as Eq.(20) could be defined :

$$k = \frac{1-k_e}{2}\cdot\cos\left((l_b/l_e)^2\pi\right) + \frac{1+k_e}{2} \qquad (0 < l_b < l_e) \tag{20}$$

4 PREDICTING FORMULA FOR BOND SPLITTNG STRENGTH

The following formulas for predicting bond splitting strength between reinforcement and concrete are obtained base on the discussion in the earlier sections :

$$l_e = \sqrt{\frac{2\cdot\lambda_b\cdot s_e}{k_e}} = \sqrt{\frac{2\cdot E_b\cdot a_b}{(2\log 2 - 1)\cdot\sigma_t\cdot\beta\cdot\phi_b\cdot\cot\alpha}} \tag{21}$$

$$\tau_{b,max} = 0.601\cdot\sigma_t\cdot\frac{r_u}{d_b}\cdot\cot\alpha \tag{22}$$

$$\tau_{co} = k_e\cdot\tau_{b,max}\cdot\frac{l_e}{l_b} \qquad (l_b \geqq l_e) \tag{23}$$

$$\tau_{co} = k\cdot\tau_{b,max} \qquad (l_b < l_e) \tag{24}$$

$$k = \frac{1-k_e}{2}\cdot\cos\left((l_b/l_e)^2\pi\right) + \frac{1+k_e}{2} \tag{25}$$

where, l_e = effective bond length, λ_b = bond length index ($\lambda_b = E_b\cdot a_b/(\phi_b\cdot\tau_{b,max})$), s_e = slippage in effective bond length ($s_e = r_u/(\beta\cdot d_b)$), k_e = bond stress ratio in effective bond length (= 0.643), E_b = Young's modulus of reinforcement, a_b = area of reinforcement, σ_t = tensile strength of concrete, β = coefficient between the internal crack depth and slippage of reinforcement (= 10.2 (1/mm)), ϕ_b = perimeter of reinforcement, α = angle between the principal bond stress and the axis of the reinforcement (= 34 degrees), d_b = diameter of reinforcement, $r_u = C + d_b/2$ (C : thickness of concrete cover), l_b = bond length, τ_{co} = bond splitting strength, $\tau_{b,max}$ = local maximum bond splitting strength, k = bond stress ratio

Figure 4 shows the comparison between experimental bond strength and predicting values calculated by Eq.(21) - (25). Experimental values are obtained by bond test of cantilever and beam type specimens done in previous studies. The ratio of experimental values to calculated values is 1.22 in average, and the coefficient of variation is 14 percent.

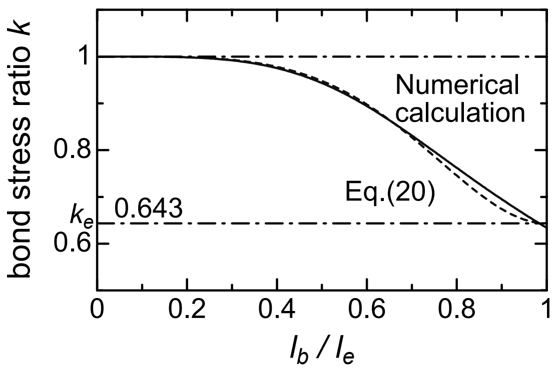

Figure 3 Bond stress ratio of EBSB

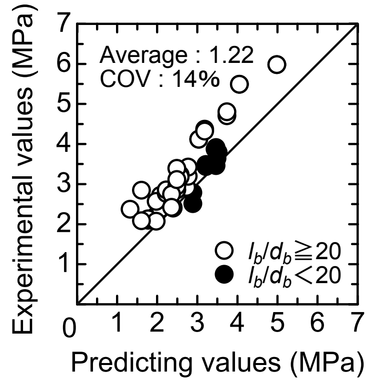

Figure 4 Comparison of calculation

5 CONCLUSIONS

New formulas predicting bond splitting strength between reinforcement and concrete is proposed by solving second differential equation of bond problem using EBSB, which is defined as the area of EBSB has the same area of local bond stress versus slippage. The predicted values show a good agreement with experimental results reported previously.

References

[1] T. Sakai, T. Kanakubo, K. Yonemaru and H. Fukuyama : Bond Splitting Behavior of Continuous Fiber Reinforced Concrete Members, Fiber Reinforced Polymer for Reinforced Concrete Structures, ACI SP-188, pp.1131-1144, 1999.10

[2] S. Morita and S. Fujii : Bond Capacity of Deformed Bars Due to Splitting of Surrounding Concrete, Proceedings of the International Conference on Bond in Concrete, 1982

[3] T. Kaku, J. Zhang, S. Iizuka and M. Yamada : A Proposal of Equation for Bond Splitting Strength of Reinforced Concrete Members Including High Strength Concrete Level, JCI Concrete Research and Technology, Vol.3, No.1, pp.97-108, 1992 (in Japanese)

[4] M. Maeda, S. Otani and H. Aoyama : A Proposal of A Formula for Bond Splitting Strength of Reinforced Concrete Members, AIJ Journal of Structural Engineering, Vol.38B, pp.293-306, 1992 (in Japanese)

[5] T. Kanakubo, T. Furuta and H. Fukuyama : Bond Strength between Fiber-Reinforced Polymer Laminates and Concrete, Proceedings of 6th International Symposium on FRP Reinforcement for Concrete Structures, Vol.1, pp.133-142, 2003