Fractal Finite Element Method for Thermal Stress Intensity Factor Calculation

A.Y.T. Leung

Department of Building and Construction, City University, Tatchee Avenue, Hong Kong
Andrew.leung@cityu.edu.hk

Abstract Fractal Finite Element Method (FFEM) is one of the finite element methods which can determine SIF directly. The FFEM has been proved to be very efficient and accurate in two-dimensional static and dynamic crack problems. In this paper, we extend our previous study to include the thermal effect for two-dimensional thermal crack problems. The temperature distribution is first found, which is imposed thereafter as a thermal load in the elastic problem. The global transformation function has been found analytically. The effects of different thermal loading on the thermal SIF are presented.

Key words: finite element method, stress intensity factor, crack

INTRODUCTION

The fundamental postulate of linear elastic fracture mechanics is that the behaviour of cracks can be determined solely by the value of the stress intensity factor (SIF). It is well known that the singular point occurs at the crack tip, the temperature flux becomes infinity in the linear theory of steady-state heat conduction and so are the stresses in the linear theory of elasticity [1]. The SIFs are the first two coefficients of the William's eigenfunction expansion of the stresses in the vicinity of the singularity, and the strength of the stress singularity characterises how fast the stress tensor approaches infinity in the vicinity of the singular point. Use of the SIF in examining crack stability requires an accurate knowledge of the stress field in the vicinity of the crack tip for the given geometry, loading and boundary conditions.

The Fractal Finite Element Method (FFEM) is one of the finite element methods which can determine stress intensity factor directly. The fractal finite element method has been proved to be very efficient and accurate in two-dimensional static [2, 3] and dynamic [4] crack problems. The method separates the overall cracked elastic body into a finite singular stress region near the crack tip and a regular region away from the crack tip by an artificial boundary. By using the William's eigenfunction expansion for crack problems, the displacement unknowns within the singular region are transformed to the generalized coordinates (the coefficients in the William's eigenfunction expansion). Thus the unknowns within the singular region are reduced significantly. Also the SIFs become primary unknowns after the transformation. Hence the SIFs can be determined directly from the generalized coordinates without the need for post-processing. Since the order of the final stiffness matrix after the transformation is much smaller than the original matrix, the computer storage and solution times can be reduced significantly.

The FFEM utilizes the William's eigenfunction expansion to do the transformation. The William's eigenfunction can be found analytically for most two-dimensional crack problems. In the case that an analytically solution can not be found, a numerical method has been developed to evaluate the eigenfunction numerically [5]. In this study, we extend our previous study to include thermal effect for two-dimensional thermal crack problems. The temperature distribution is first found, which is imposed thereafter as a thermal load to the elastic problem. The William's eigenfunction has been found analytically. The effects of different mechanical and thermal loading on the thermal SIF are presented.

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