SIZE EFFECT EXPERIMENTS ON CONCRETE UNDER MULTI-AXIAL COMPRESSION

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ABSTRACT
In this paper, we present the results of an experimental/numerical investigation on the influence of specimen and aggregate size on concrete fracture under multiaxial compressive loading conditions. Scaled thick-walled cylindrical specimens in a size range 1:4 have been tested under an external hydrostatic pressure. The stress-induced deformations at the inner opening were monitored and the fractures around it were preserved. Two concrete mixtures of high similitude were utilized in the investigation with the main variable being their maximum aggregate size, viz. 2 and 4 mm. Complementary, numerical simulations using the discrete particle code *PFC*^{2D} have been employed for analyzing the experimental findings. Observed experimental results indicate a decrease of stress with an increase in specimen size for the two tested materials. The linear size-stress relationship in a bi-logarithmic scale suggested a size effect in the form of a power relation for both materials. In terms of fracture behaviour, observations suggest a splitting mode as the initial failure mechanism for the tested materials regardless to specimen size. Both inter- and intra-granular crack types are observed that are aligned all around the inner-wall mostly parallel to it. Preliminary outcome of the simulations using PFC^{2D} have revealed its ability in simulating macroscopic material behaviour observed in the laboratory experiments in terms of realistic fracture patterns and reproducing observed damage zone.

1 INTRODUCTION

Studies on size/scale effects in fracture of concrete and rock materials attained prominent interest in the past three decades. Geometrically homothetical samples of the same rock formation or concrete batch, when submitted to the same load and test conditions, show responses that are function of specimen size. This size dependency is a salient consequence of the heterogeneous nature of the material over its scales from micro to meso. Because of its intricacy, studies on size/scale effects have focused on uniaxial tensile or compressive stress domains. Less is known when passing in to biaxial (compression-compression or compression-tension) domains and a lack of data emerges when the triaxial compressive range is considered. Conversely, most of the loading states of concrete and rock structures lie in the biaxial and triaxial compressive domains and therefore the importance for quantifying size effects in this loading range.

Size effect tests in multiaxial load domains implicitly involve three-dimensional scaling and are experimentally challenging, especially if cubical specimens are considered for poly-axial (true triaxial) testing. Alternatively, solid cylindrical specimens could present a less demanding solution and are advantageous in terms of boundary restraint and load application. Application of fluid pressure along the circumference of a cylinder is almost unrestrained (van Mier [1]). However, a solid-cylinder test is not true triaxial but biaxial, as two of the applied principal stresses are equal. For true triaxial studies, a hollow cylinder geometry is more suitable as it lends it self for providing permutations of various multiaxial states of stress around its inner-hole depending on the stress path applied to its external boundaries. Such test geometry is almost routinely used as a model test in stability studies of underground tunnels and deep boreholes. Few studies concerning size effect in hollow cylinder tests were conducted (e.g. van den Hoek et al. [2]) and a trend of decreasing hollow cylinder strength with increasing specimen size was reported.

The aim of this research is to carefully observe and simulate fracture processes and size dependency in hollow cylinder tests on cementitious materials. To this end, experiments using scaled thick-walled cylinders of two different concrete mixtures are performed. In this paper, experimental results in terms of size effect relation and fracture mechanisms are presented and discussed on two mixtures of 2 and 4 mm maximum aggregate size. Complementary to the experimental results, the discrete "Particle Flow Code in Two Dimensions" PFC^{2D} (Itasca Consulting Group [3]) is used as an analysis tool for test interpretation. The discrete nature of the model allows large displacements and unrestricted cracks to develop at the contacts between the particles.

2 EXPERIMNTAL WORK

2.1 Materials and test methods

Two cement-based materials have been developed in this investigation as model testing materials (Elkadi et al. [4]). In order to investigate the effect of aggregate grain size (heterogeneity scale) on size effect and fracture behaviour, the mixtures were designed in similarity with only varying their maximum aggregate size, namely 2 and 4 mm. Both mixtures were set to a target compressive strength of ≅ 10 MPa with analogous load-deformation characteristics. Ordinary Portland cement of type CEM I 32.5R is used in the mixtures at a ratio by weight of 10-14% with water/cement (w/c) ratio of 0.8. Uniaxial and triaxial compression tests have shown the mixtures to behave in a quasi-brittle manner under uniaxial load and plastic-strain-hardening when subject to confinement. Generally, samples failed gradually in a quasi-plastic manner and could sustain considerable permanent deformations prior to collapse.

A high-pressure hollow-cylinder test cell was developed for this study. The cell is capable of accommodating three different specimen sizes in a size range 1:4. Dimensions of the largest specimen are: 200, 300, and 50 mm for the external diameter, length, and internal diameter respectively. Maximum safe pressure capacity of the cell is 40 MPa—comparable to an underground geostatic stress at about 2200 m depth—and both axial and radial pressures could be applied independently. During testing, the radial deformation at the inner-wall is continuously monitored using a specially developed measuring device. In addition, fracture patterns remaining after testing are preserved by impregnating a fluorescent epoxy resin into the vicinity of the inner-hole under vacuum. After hardening and sectioning, post-mortem inspection takes place using uv-optical microscopy.

All tests are performed under an external hydrostatic pressure, while the inner-hole pressure is kept at atmospheric. This hydrostatic pressure translates into a triaxial stress gradient over the thickness of the cylinder wall with a maximum principal stress (σ_1) = tangential stress, an intermediate principal stress (σ_2) = axial stress, and a minimum principal stress (σ_3) = radial stress. Linear elastic solution for stresses in a hollow-cylinder suggest a maximum stress concentration to develop around the inner-wall with $\sigma_1>\sigma_2>\sigma_3$ (compressive stresses are positive) and hence failure is expected to initiate at the inner-wall and propagate outwards under increased stress.

2.2 Experimental results and analysis

Eight hollow-cylinder test series were performed, four series per material. Each test series comprises three hollow-cylinder specimens of three different sizes. The specimens are of inner-diameters *D* equal to 12.5, 25, and 50 mm, which correspond to a cylinder wall-thickness *W* equal to 18.75, 37.5, and 75 mm respectively. The wall-thickness *W* represents the characteristic size in a hollow-cylinder. In this paper, specimens with W value of 18.75, 37.5, and 75 mm are referred to as type A, B, and C respectively.

In a thick-walled cylinder subject to an external hydrostatic pressure, the geometry imposes a true triaxial stress state around its inner-hole, which gradually transforms into a hydrostatic stress state when approaching the outer-boundary. This stress gradient results virtually in concentric zones of different failure and fracture behaviour. Adjacent to the inner-hole, high stress gradient exists with a large deviatoric stress component that results in material failure with basically the development and propagation of (micro-)cracks. Conversely, near the outer boundary with a nearly zero deviatoric component, bulk material compression is the main failure mechanism. This complex stress situation should be considered when interpreting the hollow-cylinder test results

In Figure 2, bi-logarithmic plots of the size-stress relation from the performed test series are presented. The log (σ) – log (W) is given wherein the stress value corresponds to a tangential strain value equal to 0.5% strain. In the size effect plots (Figure 2), both the mean values of test results and their standard deviation are shown. The presented results illustrate a size effect on the strength of the tested hollow-cylinders. The mean stress values, for both mixtures, show a stress difference between specimen size A and specimen size C of about 23%. The size effect nearly vanishes, for mean values, when considering specimens size B and C in the 2 mm mixture. However, for both mixtures a linear fit could fairly well represent the data indicating a power size effect relation to exist between the three sizes. Accordingly, the statistical Weibull size effect theory was investigated if representative of these results. The theory in its simplest form is the weakest link model, most widely applied, in which the specimen fails through extensile fracturing in a chain reaction-like process following the failure of one of its constituting elements. The application of this model in fracture of concrete subject to triaxial stress-state is commonly represented in the following form (Bažant et al. [6]): $\sigma_N \propto D^{-n_d/m}$, where σ_N = nominal material strength, D = characteristic dimension, n_d = scale number, and m = Weibull modulus. In the present study n_d = 3 for three-dimensional scaling, and m = 12 as commonly applied for concrete. This results in the following equation for three-dimensional similarity:

$$\sigma_N \propto D^{-1/4} \tag{1}$$

The relation in (1) is shown graphically in Figure 2 as well, where a good agreement with the test results for both mixtures is evident. This could suggest that the observed size effect is pure statistical size effect and that it suffice to use the Weibull theory without need for curve-fitting exercises.

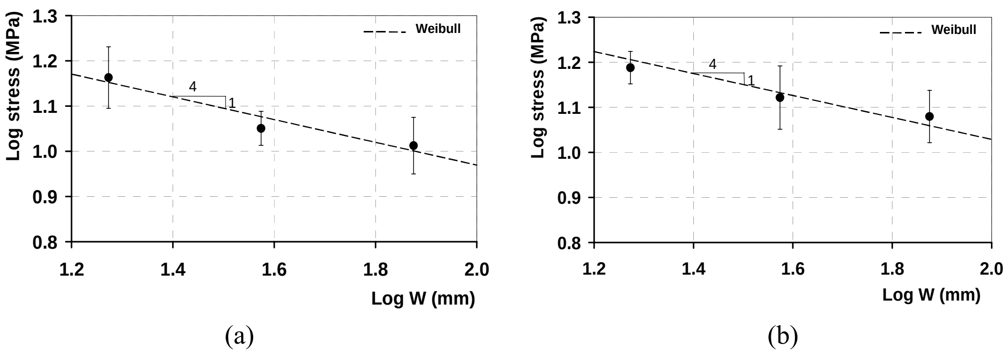

Figure 2. Bi-logarithmic plot for stress versus wall thickness, specimens A, B, and C. Shown are mean values and standard deviation for the 2 mm mixture (a), and the 4 mm mixture (b).

The justification for applying the weakest link theory to our test results could be questionable as it assumes tensile/extensile failure under uniform stress. As it is elucidated next, the primary fracture mechanism observed in our tests is extensile and therefore does not contradict the assumption of tension failure. As regards stress gradients, stress in-homogeneity could be taken into account in Weibull's basic equation for the probability of failure (eqn (8) in Bažant [6]) by assuming the stress as dependent on position (radius in our case) and integrating over the volume of the hollow-cylinder. This results in stress values for each size that typically obey the size relation given in eqn (1) with inverse linear relation of gradient 1:4. Noteworthy, however, is the effect of stress redistribution taking place during failure, which is not included in Weibull's theory. Accordingly, the statistical size effect does not suffice to describe the observed size effect, although fits best the results, and structural size effects should be accounted for as well.

Post-mortem examination of impregnated specimens, suggested a principal failure mechanism in the tested materials to be a splitting-type failure mode (Elkadi et al. [5]). Wherein, fracture initiation is characterized by small splitting cracks encircling the inner-hole in a concentric pattern. Those cracks are often oriented parallel to the inner-wall in a direction coinciding with the direction of the maximum principal stress σ_1 (see Figure 3a). Depending on the grain size and stiffness with respect to the orientation of crack propagation, and grain/matrix relative strengths, cracks would propagate either inter- or intra-granularly parallel to the σ_1 direction. For the 2 mm mix, inter-granular crack propagation seemed preferable as cracks could meander around grain boundaries with much less energy as compared to the 4 mm mixture. In the latter, if grain strength is close to or less than the matrix strength cracks could propagate intra-granularly under increasing stress levels. In both materials, however, matrix strength is relatively low due to its porous structure resulting from the high w/c ratio and low cement content used, which in turn results in grain boundaries as being favourable sites for crack nucleation and propagations.

3 NUMERICAL MODEL

PFC^{2D} is a two-dimensional discrete element code capable of simulating the mechanical behaviour of bonded and un-bonded granular materials. Cohesive-frictional granular materials such as concrete and rocks are suited for modelling using the PFC^{2D} for their heterogeneous and discrete nature at the meso-scale. The code models solids as an assembly of distinct and arbitrarily sized circular particles. The particles are regarded as rigid bodies and their contacts are characterized through a contact-stiffness, slip condition, and bonding models. In this study, a linear contact-stiffness model is chosen together with a contact-bond model, which assumes a particle contact over a vanishing area, a point, and transmits forces only. The bond model can be envisaged as a pair of elastic springs of specified constant normal and shear stiffness and normal and shear strengths. Bond breakage—crack formation—occurs when either the normal or shear component of the contact force exceeds its corresponding contact-bond strength. If breakage takes place in shear, the contact forces are not altered and the residual shear contact force obeys a frictional law.

To simulate the mechanical behaviour of our model material, some numerical calibration tests are necessary. Calibration is the term used to describe an iterative procedure of determining and modifying the micro-properties for a PFC^{2D} model. In this calibration process, responses of the model are compared to those of laboratory tests, e.g. uniaxial compression and Brazilian splitting tests, and the model micro-parameters are iteratively modified to achieve good agreement with the material's macroscopic parameters. The micro-parameters in PFC^{2D} are: normal and shear stiffness of the contact k_n and k_s, the average radii of the particles $\tilde{R}$, the normal and shear strength of the bonds T_n and T_s, and the friction coefficient between the particles μ. The parameters obtained from the calibration process and used in the hollow-cylinder analysis are summarized in Table 1.

Table 1. Micro-parameters for the PFC material model

PFC^{2D} micro-parameter		Value
Normal stiffness	k_n	9.75 GPa
Shear stiffness	k_s	1.95 GPa
Normal strength	T_n	4.00 MPa
Shear strength	T_s	15.0 MPa
Average radius	$\tilde{R}$	13.3 mm
Ratio	R_{max}/R_{min}	1.66

3.1 Analysis of the hollow-cylinder test

In order to model the hollow-cylinder test, the option of general walls was utilized in PFC^{2D}, which enables full circular or arc walls to be generated. In this paper, preliminary results of the failure analysis during a hollow-cylinder simulation on a model comprising 4451 particles and 9582 contacts are presented in terms of damage and crack patterns.

In Figure 3b, the hollow-cylinder model is presented at the completion of a numerical experiment up to an external equivalent stress of 38 MPa, which is the stress used in the physical experiments. Model dimensions are 200 mm external diameter, 50 mm inner-diameter, and $\tilde{R}$ = 13.3 mm. After creating the sample, the external walls were slowly moved inwards to simulate the loading procedure. In the model, damage is represented by cracks that develop at broken bonds either in tension or shear. When comparing the crack pattern from the PFC^{2D} simulation (Figure 3b) and the impregnation experiment (figure 3a), a reasonable similarity in terms of crack initiation and crack pattern is illustrated. The model appears to capture the damage zone being concentric with the inner-hole and the cracks being aligned in the direction of σ_1.

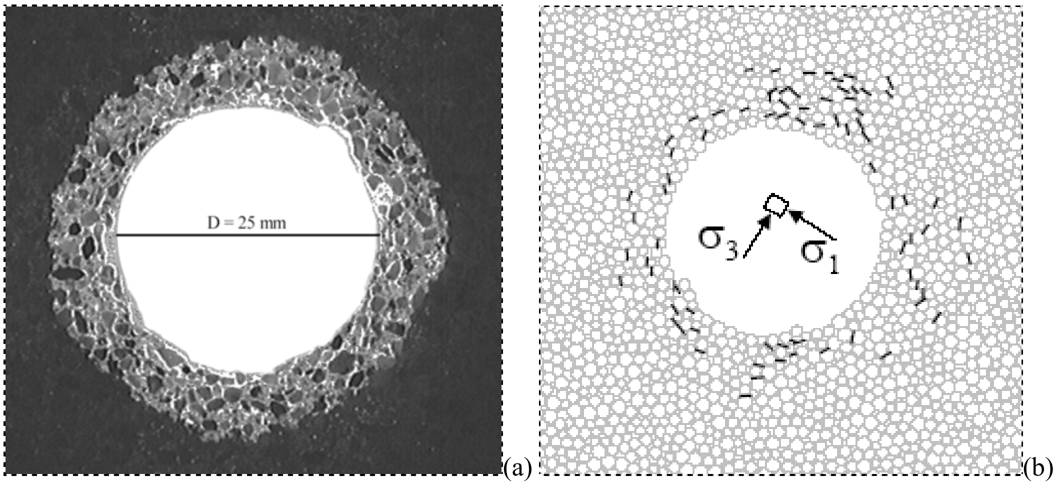

Figure 3: Crack pattern from a 2-mm specimen type B after testing to an external pressure of 38 MPa. Shown is the damage zone being lighter in color than surrounding intact material and the cracks tinted in white (a). Section from the PFC^{2D} hollow-cylinder model of 4451 arbitrarily sized particles and 9582 contacts. Shown is the crack pattern developed at an external pressure of 38 MPa as short black lines (b).

In terms of the dimension of the damage zone, the average thickness of the damage zone relative to the wall-thickness (W) was compared from both the experimental and numerical results. From a visual examination of the specimen in Figure 3a, the damage zone is ≅ 14% of the wall thickness, whereas from the numerical simulation (Figure 3b), this percentage is ≅ 16%.

4 CONCLUSIONS

Series of size effect experiments on scaled hollow-cylinders from two concrete mixtures were performed and have illustrated a decrease of strength with increasing specimen size. In terms of mean values, the observed size effect was less pronounced for the 2-mm mixture as compared to the 4-mm as it tends to go asymptotic from the second size onward. The test results showed a good agreement with the classical size effect theory, Weibull theory, for three-dimensional similarity. Nevertheless, structural size effects due to stress gradients and stress redistribution are of important role under such complex stress situations and should be accounted for as well.

In terms of fracture behaviour, the laboratory observations suggested a splitting mode as the principal failure initiation mechanism for both tested materials regardless of specimen size. Inter- and intra-granular crack types were observed, which were often aligned parallel to the cylinder's inner-wall and typically encircling it.

Preliminary computations using PFC^{2D} have revealed its ability in reproducing comparable macroscopic material behaviour observed in our hollow-cylinder experiments. The discrete nature of this meso-mechanical model has shown to yield realistic fracture patterns and damage development. More work, however, is needed to understand the calibration process for simulating a complex test as the hollow-cylinder, since it appeared insufficient calibrating only using simpler tests as the uniaxial compression. Defining extra parameters could be necessary or more involved test types might appear necessary for the calibration. Lastly, next phase of this computational work is to try to analyse the experimentally observed scale effect in the hollow-cylinder tests.

5 ACKNOWLDGEMENTS

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