A COHESIVE INTERFACE MODEL BASED ON DAMAGE AND FRICTION

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ABSTRACT

A new cohesive-zone model is presented based on the combination of an elastic-damage formulation, which simulates the interface decohesion, and a friction contact model. The classical mesomechanic assumption is made whereby a representative area of the interface can be additively decomposed into an undamaged part and a damaged part, so that the ratio between the damaged part and the total area represents a damage parameter ranging from 0 to 1. The main idea consists of adding a Coulomb-like friction term for the stress only on the damaged part of the interface, while the decohesion model takes into account mixed-mode fracture and specializes to piecewise linear interface laws for pure-mode problems. Numerical results are presented for the simulation of a benchmark test for the analysis of concrete dams, in which the proposed model is used to simulate the behaviour of the soil-concrete interface at the base of the dam.

1. INTRODUCTION

Cohesive-zone models have been widely used, in the last decades, to model fracture and debonding problems in many areas of computational mechanics. For many of such problems friction can significantly increase the energy dissipated in mode II and should be suitably taken into account in a reliable numerical model. This has often been done in the literature by introducing a non-associative plasticity formulation, with softening, in the interface relationship used (see Cocchetti et al. [1], or Bolzon and Cocchetti [2]).

In this paper, a new approach for taking into account the influence of friction on the interface behaviour is described. It is based on the observation that, using a Kachanov interpretation of damage, an additive decomposition of a representative elementary area (REA) A of the interface into a damaged part A_d and an undamaged part A_u can be assumed at a mesoscale level, whereby a damage parameter D can be introduced. In accordance with this assumption, it is supposed that friction occurs only on the damaged part A_d of the REA, in accordance with a Coulomb-like friction law. The evolution of the damage parameter D is obtained by applying the interface model derived by Crisfield and his co-workers in [3], in the modified form presented by Alfano and Crisfield in [4].

The interface model is implemented in the finite element code LUSAS and the results of a numerical simulation of a benchmark test for concrete dams analysis are presented in order to assess the reliability and the efficiency of the proposed model.

2. INTERFACE MODEL

Two-dimensional problems are considered in this paper, whereby the interface is a line of the initial configuration, and a local reference system is point wise introduced with axis 1 normal to the interface and axis 2 direct along its tangent [4]. A relative-displacement vector **s** is defined at each point of the interface which is null in the initial configuration. Accordingly, the relative displacement can be decomposed into its components along the axes, s_1 and s_2.

At a mesoscale level, a representative elementary area (REA) of the interface is partitioned into an 'undamaged' part A_u and a 'damaged' part A_d ; in the undamaged part the interface is fully bonded while in the 'damaged' part a unilateral fiction contact occurs. Denoting by A the area of REA and by D the ratio $D = A_d /A$ the following relationship holds (see figure 1):

$$A_u + A_d = A. \tag{1}$$

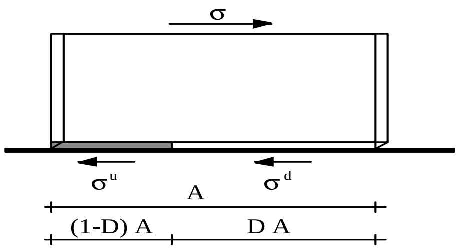


Figure 1: Representative elementary area of the interface.

Further hypotheses concerning the kinematics of the interface model are introduced. Firstly, the relative displacement is assumed to be constant over the entire REA. Hence denoting by $\mathbf{s}^u$ and $\mathbf{s}^d$ the relative displacement vectors on the two parts A_u and A_d, respectively, it follows that $\mathbf{s}^u = \mathbf{s}^d = \mathbf{s}$. Secondly, the relative displacement in the 'damaged' part is additively split into an elastic part $\mathbf{s}^{de}$ and an inelastic part $\mathbf{s}^{di}$, so that $\mathbf{s}^d = \mathbf{s}^{de} + \mathbf{s}^{di}$, whereas on the undamaged part the relative displacement is assumed to be totally elastic: $\mathbf{s}^u = \mathbf{s}^{ue} = \mathbf{s}$.

On either part, the interface stress is supposed to be constant, although it is generally different on the undamaged and on the damaged parts. On the undamaged part of the REA the interface stress is denoted by $\boldsymbol{\sigma}^u$ and is related to $\mathbf{s}^{ue} = \mathbf{s}$ by the linear elastic relationship:

$$\boldsymbol{\sigma}^u = \mathbf{K}\,\mathbf{s}\,, \tag{2}$$

where $\mathbf{K} = \text{diag}\,[K_i]$ is a diagonal matrix which collects the stiffness values in all of the modes. These can be chosen in the range which ensures a good prediction of the undamaged behaviour of the interface and avoids ill conditioning.

On the damaged part, the interface stress is denoted by $\boldsymbol{\sigma}^d$ and is related to $\mathbf{s}^{de} = \mathbf{s} - \mathbf{s}^{di}$ by the following linear elastic relationship:

$$\boldsymbol{\sigma}^d = \mathbf{CH}\left(\mathbf{s} - \mathbf{s}^{di}\right), \tag{3}$$

where $\mathbf{H} = \text{diag}\,[(1 - h(s_1))H_1, H_2]$, with the stiffness values H_i being possibly different from K_i, and the symbol $h(\bullet)$ denoting the Heaviside function. The term $1 - h(s_1)$ is then equal to unity in the case $s_1 < 0$, and equal to zero in the case $s_1 \geq 0$, so as to take into account the unilateral nature of contact.

The total (homogenised) value of the interface stress over the REA will be indicated by $\boldsymbol{\sigma}$ and is obtained by weighting the two values $\boldsymbol{\sigma}^u$ and $\boldsymbol{\sigma}^d$ as follows:

$$\boldsymbol{\sigma} = (1 - D)\boldsymbol{\sigma}^u + D\boldsymbol{\sigma}^d\,. \tag{4}$$

The two components of the stress $\boldsymbol{\sigma}$ represent the normal and the tangential interface stresses, respectively, and are accordingly denoted by σ and τ, so that :

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma \\ \tau \end{bmatrix} \qquad \boldsymbol{\sigma}^u = \begin{bmatrix} \sigma^u \\ \tau^u \end{bmatrix} \qquad \boldsymbol{\sigma}^d = \begin{bmatrix} \sigma^d \\ \tau^d \end{bmatrix} \tag{5}$$

The inelastic relative displacement $\mathbf{s}^{di}$ physically represents the inelastic sliding which has occurred on the damaged part of the REA and is accompanied by dissipation due to friction. Hence, the following 'friction' yield function is introduced:

$$\phi\left(\boldsymbol{\sigma}^d\right) = \mu\left\langle \sigma^d \right\rangle_- + \left|\tau^d\right|, \tag{6}$$

where μ is the friction coefficient and $\langle \bullet \rangle_-$ denotes the negative part of $\bullet$.

The evolution of $\mathbf{s}^{di}$ is governed by the following non-associative relationship:

$$\dot{\mathbf{s}}^{di} = \dot{\lambda}\begin{bmatrix} 0 \\ \dfrac{\partial \phi}{\partial \tau} \end{bmatrix}, \tag{7}$$

with the additional Kuhn-Tucker conditions:

$$\dot{\lambda} \geq 0 \qquad \phi\left(\boldsymbol{\sigma}^d\right) \leq 0 \qquad \dot{\lambda}\,\phi\left(\boldsymbol{\sigma}^d\right) = 0 \tag{8}$$

In order to fully define the interface model, the formulation initially proposed by Crisfield and his co-workers in [3] is used to obtain the evolution law for the damage parameter *D*. This model can be described starting from the two pure-mode relationships depicted in figure 2, with figure 2.a relating to mode I and figure 2.b relating to mode II. Input parameters of the laws are, for each mode, the values G_{ci}, s_{oi} and σ_{oi}, which represent the fracture energies, the first 'cracking' relative-displacement components and the peak values of the related traction components, which are denoted in the figures as σ_o and τ_o, respectively.

Since the areas enclosed by each bilinear law of figure 2 are equal to the fracture energies G_{ci}, the 'critical' relative-displacement values are then equal to $s_{ci} = 2G_{ci} / s_{oi}$.

Following Alfano and Crisfield [4], the hypothesis that $s_{o1} / s_{c1} = s_{o2} / s_{c2}$ is made. This can be justified by considering that the parameters s_{o1} and s_{o2} are input parameters which are related to the initial pure-mode stiffness values K_i by the relationship $s_{oi} = \sigma_{oi} / K_i$. It has been earlier observed that K_i can be viewed as numerical 'penalty' stiffness values which can be chosen in a certain range. For most of the problems of engineering interest, in such range there exist two stiffness values which imply fulfilment of $s_{o1} / s_{c1} = s_{o2} / s_{c2}$. Accordingly, the following parameter η is introduced:

$$\eta = 1 - \frac{s_{o1}}{s_{c1}} = 1 - \frac{s_{o2}}{s_{c2}}, \tag{9}$$

and, for a general mixed-mode decohesion process, a damage parameter *D* is introduced as a function of the history of the total relative displacement as follows:

$$D = \max_{history} \tilde{D}, \tag{10}$$

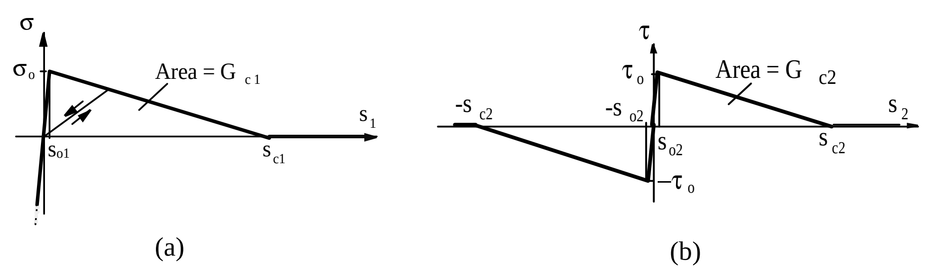


Figure 2: pure-mode interface relationships.

where:

$$\tilde{D} = \max\left\{0, \min\left\{1, \frac{1}{\eta}\left(\frac{\beta}{1+\beta}\right)\right\}\right\}, \quad \text{and} \quad \beta = \sqrt{\left(\frac{\langle s_1 \rangle_+}{s_{o1}}\right)^2 + \left(\frac{s_2}{s_{o2}}\right)^2} - 1 \tag{11}$$

with $\langle \bullet \rangle_+$ denoting the positive part of $\bullet$.

Further details on the algorithmic implementation and on the consistent linearization of the model can be found in [5] ad [6].

3. NUMERICAL RESULTS

The interface model which has briefly been described in Section 2 has been implemented in the finite-element code LUSAS [7] as a new constitutive law for 2D interface elements and some numerical results for the benchmark problem of concrete dams analysis proposed by the ICOLD in [8] are briefly reported in this section.

The geometry of the concrete dam is described in figure 3.a, and a non-proportional loading process is assumed. The initial loads are given by the sum of the specific weight of the concrete, $\gamma_c = 24000$ N m^{-3}, and of the hydrostatic pressure, $q_w(y) = 1000 \cdot 9.81 \cdot (80 - y)$ N m^{-2}. The additional load consists of an overpressure, constant along y, equal to a reference value $q_o = 1.0$ MPa multiplied by a loading factor α. Hence, the external water pressure is given by: $q(y) = q_w(y) + \alpha\, q_o$.

Concrete behaviour is assumed to be linearly elastic, with Young modulus E = 24 GPa and Poisson ratio $\nu = 0.15$, and the hypothesis of small deformations is made.

A predefined 'week' concrete-soil interface has been considered at the base of the dam, while no concrete joint-interfaces resulting by the typical step-wise construction process have been modelled, for the sake of simplicity. The influence of the uplift water pressure within the open crack, which is also neglected in the analysis presented here, is investigated in [5].

The interface properties have been chosen in accordance with [2], as follows:

$$G_{c1} = 90 \text{ J m}^{-2} \quad \sigma_o = 0.3 \text{ MPa} \quad G_{c2} = 350 \text{ J m}^{-2} \quad \tau_o = 0.7 \text{ MPa} \quad \mu = \tan 30°$$

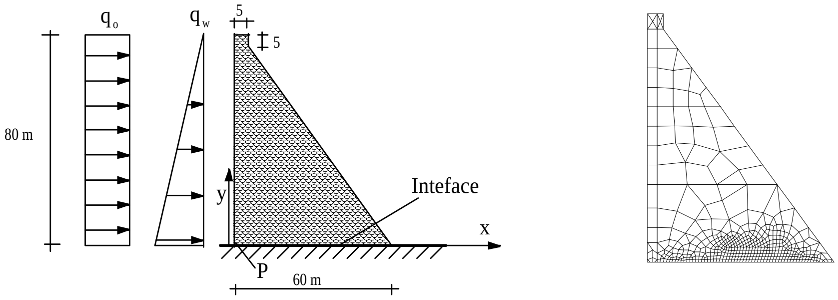
while the parameter η of formula (16) is set equal to 0.9.

The finite-element mesh depicted in figure 3.b is created with a view to having a sufficiently refined discretization in the vicinity of the concrete-soil interface, and a coarser mesh away from it.

For the concrete bulk material, 434 4-node and 50 3-node plane-strain interface elements with enhanced modes [7] are used, while 64 4-node interface elements are placed on the concrete-soil interface.

An incremental, quasi-static analysis is conducted. In the first increment, the dead load and the hydrostatic water pressure are assigned with their entire value, with the load multiplier α set to zero. Then, starting from the second increment, α is increased following an automatic incrementation procedure [7].

In figure 4 the crack-sliding-displacement (CSD) (the horizontal displacement component u_x of point P in figure 3.a) and the crack-opening-displacement (COD) (the vertical displacement component u_y of point P in figure 3.a) are plotted against the load multiplier α.



(a) (b)

Figure 3: (a) geometry and loading of the dam; (b) finite-element mesh.

Both curves begin with a steep, increasing and almost linear branch, which does not exactly start from the origin of the axes because of the initial elastic deformation of the interface for $\alpha = 0$ due to the dead load and the hydrostatic water pressure.

It can be observed that the CSD monotonically increases with α, with a decreasing slope of the curve, until the maximum value $\alpha = 0.492$ is reached for a value u_x of about 0.8 mm. This point represents a limit point in the equilibrium path, which is followed by a softening, unstable part of the load-displacement curve, until the value $\alpha = 0.423$ is attained. At this point the crack has reached the end of the concrete-soil interface and the whole dam slides at a constant value of the applied overpressure. In this part of the process the interface adhesion is completely lost and the interface tangential tractions are only due to friction.

The corresponding COD-α curve for $\alpha = 0$ is characterised by an increasing part until the maximum value of α is attained, at which a snap back occurs. The COD then decreases back to a value of about 1.8 mm, which represents the constant COD value during the final sliding phase.

4. CONLUSIONS

A novel approach to the inclusion of friction effects in a cohesive-zone model has been proposed. Based on the additive decomposition of a representative area of the interface into a damaged and an undamaged part, a unilateral, friction contact model has been adopted on the latter part, while the evolution of the damage is governed by a mixed-mode interface relationship which specialises to piecewise linear laws for pure mode cases.

Numerical results have been presented for a quasi-static, incremental simulation of a benchmark problem of dams analysis, concerning a concrete dam subjected to gravity load, hydrostatic pressure, and an additional overpressure depending on a load factor. They show the ability of the model of well capturing the increase in the mode-II interface strength due to friction.

Further research will involve a sensitivity analysis to some of the material parameters of the model, like the friction coefficient and the initial interface stiffness.

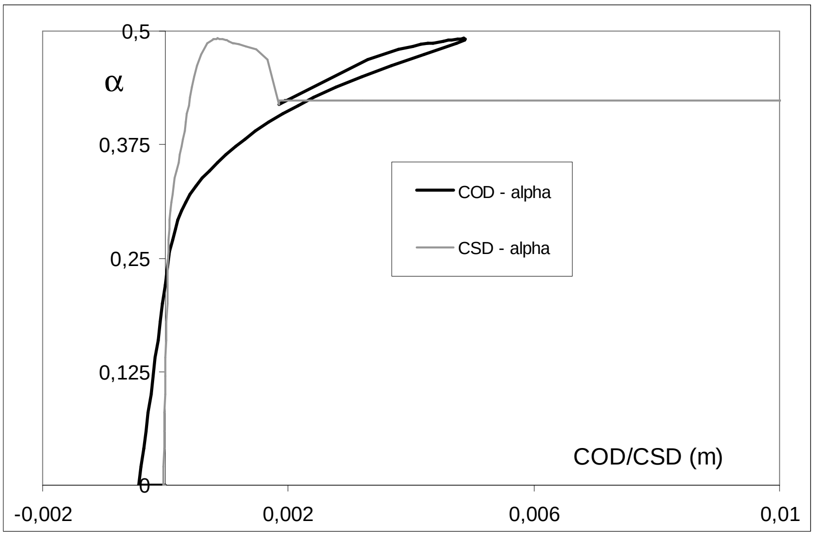


Figure 4: COD-α and CSD-α curves.

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