SINGULAR STRESS FIELDS ALONG A STRESS SINGULAR LINE AND AT A VERTEX IN THREE-DIMENSIONAL JOINTS

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ABSTRACT

Stress singularity at a vertex in three-dimensional joints is closely related to the strength of joints. It is very important to analyze for evaluating the reliability in a three-dimensional joint. However, the stress singularity field near a vertex and along a stress singularity line is not made still clear. In the present study, we investigated the characteristics of stress singularity field at a vertex and a point located on the stress singularity line in three-dimensional joints using a three-dimensional BEM with a fundamental solution for two-phase materials and an eigen analysis based on FEM. A 3-ple root and a 5-ple root of eigen value, p=1, occur at the vertex and a point on the singularity line, respectively. The variation of stress distribution in a cylindrical coordinate system concerned with a point on the singular line with a distance from the vertex is examined using BEM. The order of stress singularity deduced from the stress distribution is compared with that deduced from the eigen analysis. The slope of the stress distribution reduced the logarithmic singularity terms was fairly agreed with the order of stress singularity calculated by the eigen analysis. It was shown that the order of stress singularity at the vertex was larger than that at the point on the stress singularity line, and the effect of logarithmic singularity along the singularity line on the stress distribution is larger than that at vertex. The value of coefficient in a power-law singularity increased as approaching to the vertex along the singularity line. In particular, all coefficients of terms in logarithmic singularity vary in the same way with the angle from an interface.

1 INTRODUCTION

There are a lot of investigations on stress singularity in joints. Stresses increase to infinity as approaching to a point with stress singularity, however, displacement at the point bounds a finite. An intersection of an interface in a joint and a free surface becomes a stress singularity line. In two-dimensional joints, the intersection of an interface and a free surface is a stress singular point. A lot of studies on joints were concerned with two-dimensional ones. It is found that the order of stress singularity at a vertex in three-dimensional joints is greater than that at edge in two-dimensional ones (Koguchi [1]). A crack and the delamination of interface at the vertex occur easily in three-dimensional joints. However, the characteristic of stress distribution near the vertex in three-dimensional joints is not make clear until now. In the previous study on three-dimensional joints, the authors showed that power law singularity and logarithmic singularity occur at the vertex from an eigen value analysis (Koguchi [2]). In the present study, the characteristic of stress distribution at the vertex and along the stress singularity line will reveal using BEM with the fundamental solution for two-phase materials and eigen analysis by FEM.

2 STRESS SINGULARITY ANALYSIS IN TWO-PHASE MATERIALS

In the present study, the following equation in BEM is used for determining the stress distribution in a three-dimensional joint.

$$c_{ij}(P)u_j(P) = \int_\Gamma U_{ij}^*(P,Q)t_j(Q)dS(Q) - \int_\Gamma T_{ij}^*(P,Q)u_j(Q)dS(Q) \tag{1}$$

where U_{ij}^* and T_{ij}^* are fundamental solutions for displacements and tractions which are derived from Rongved s solution. Observation point, P, and source point, Q, are located on the boundary of domain. t_j and u_j are traction and displacement vectors, respectively. When Rongved s solution is applied for the fundamental solution in BEM, mesh division on the interface is not needed and displacements and stresses at any points in the domain are determined accurately. Furthermore, the order of stress singularity is deduced from an eigen analysis of FEM as follows.

$$\left(p^2[A] + p[B] + [C]\right)\{u\} = 0 \tag{2}$$

where [A], [B] and [C] are matrices composed of elastic moduli, and p is the eigen value, which is related with the order of stress singularity, λ, as following λ=1-p.

Model for analysis is demonstrated in Fig.1. The joint is 20mm in width, and 20mm in height. Tensile load, P=1.0GPa, is applied on the upper surface and the displacement in the z-direction at the lower surface is fixed. Minimum length of mesh near the vertex is 0.8μm, and total node

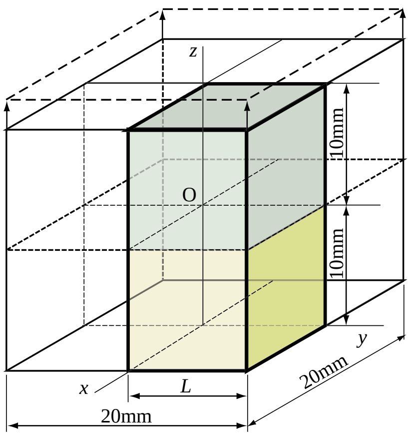

Fig.1 Joint model for analysis

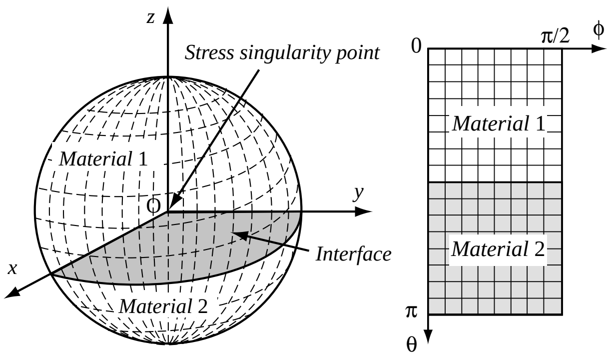

Fig.2 Model for eigen analysis in FEM

number and total element number are 3067 and 1370, respectively. Materials used in the analysis are resin and ceramics(Al_2O_3). Elastic moduli are shown in Table 1. Hereafter, the analyses at the vertex and along the stress singularity line are referred to as 3D-corner analysis and 3D-line analysis, respectively.

3 RESULTS OF 3D-CORNER ANALYSIS

Figure 2 is a model used in the eigen analysis, in which the domain for analysis is a quarter or a half of sphere with a origin located at the vertex or a point on stress singularity lines. Mesh division unfolded in a $\phi\times\theta$ plane is shown in Fig.2. Size of mesh is $\phi\times\theta$=9.0°×9.0°, and the total numbers of node and element are 669 and 200, respectively. Result of eigen analysis is shown in Table 2. We can obtain the eigen values for the freedom number of analytical model. The smaller value of eigen value is listed in turn up to 6. The minimum value of eigen value is p=0.5982 and the next eigen value is a 3-ple root of p=1. The stress distribution near the vertex obtained by using BEM is shown in Figs. 4 and 5. Stress component, $\sigma_{\theta\theta}$, is derived by transforming stress components in a Cartesian coordinate system to those in a spherical coordinate system shown in Fig.3. Plots are almost straight in a log-log scale for several different angles, ϕ and θ. Figure 4 represents a plot of stress on the interface. It is found that stress increases as the angle, ϕ, approaches to 0 and $\pi/2$. Figure 5 shows the distribution for different angles θ at angle of ϕ=45°. In this case, the power-law singularity is governs the stress fields at the vertex, furthermore the logarithmic singularity also occurs. Then, the stress can be expressed as follows.

$$\sigma_{\theta\theta}/P = C_1(\theta)\bar{r}^{-\lambda} + C_2(\theta) + C_3(\theta)\log\bar{r} + C_4(\theta)(\log\bar{r})^2 \tag{3}$$

where $\bar{r}$ represents r/L. Coefficients of logarithmic singularity terms are determined using a least square method. The coefficients, C_1, for the power-law singularity and $C_i(\theta)$ (i=2,3,4) for the loga-

Table 1 Material properties used in the analysis

	Young s modulus (GPa)	Poisson s ratio
Resin	2.97	0.38
Al_2O_3	260.0	0.24

Table 2 Eigen values in Al_2O_3-Resin joint

	Real	Imaginary	λ
1	0.5982423	0.0000000	0.4017577
2	1.0000002	0.0000000	-0.0000002
3	1.0000116	0.0000000	-0.0000116
4	1.0000056	0.0000000	-0.0000056
5	1.3258479	0.5675051	-0.3258479
6	1.3258479	0.5675051	0.3258479

rithmic singularity terms are shown in Figs.6 and 7, respectively. You can see from Fig.7 that all coefficients vary in the same manner for the angle θ. Then, eqn(3) can be modified as

$$\sigma_{\theta\theta}/P = C_1(\theta)\bar{r}^{-\lambda} + C_2(\theta)\left\{1 + \bar{C}_3 \log\bar{r} + \bar{C}_4\left(\log\bar{r}\right)^2\right\} \tag{4}$$

where $\bar{C}_3$=1/86.36 and $\bar{C}_4$=-1/14.30.

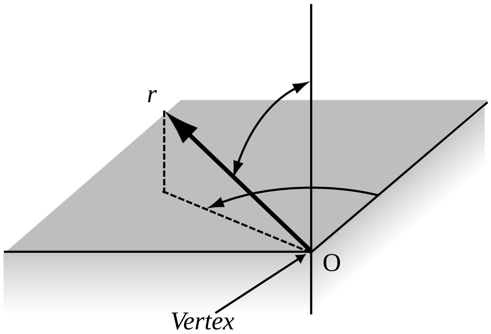

Fig.3 Spherical coodinate system with an origin at the vertex

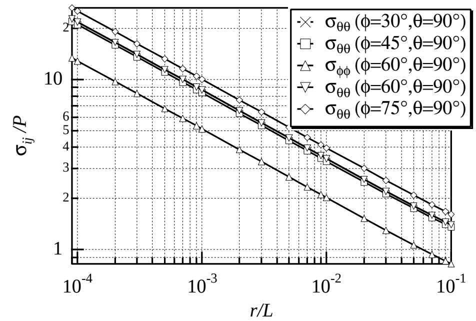

Fig.4 Distribution of stress σ_{ij} against r/L

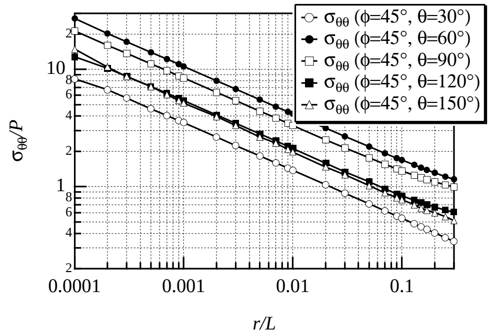

Fig.5 Distribution of stress $\sigma_{\theta\theta}$ against r/L

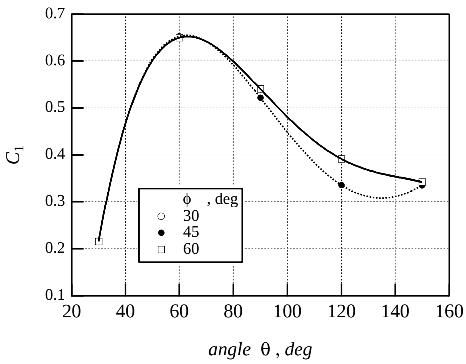

Fig.6 Variation of coefficients in power-law with angle θ

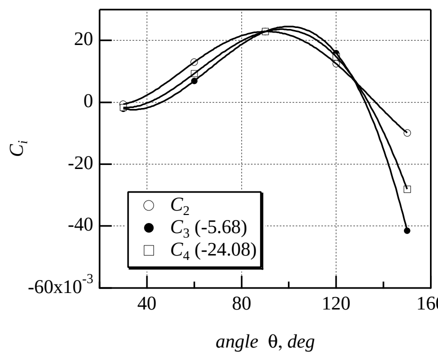

Fig.7 Variation of coefficients in logarithmic terms with angle θ

4 RESULTS OF 3D-LINE ANALYSIS

Model for BEM used in 3D-line analysis is the same as that in the 3D-corner analysis. Model used for eigen analysis is shown in Fig. 8. Numbers of node and element are 341 and 100, respectively. Result of eigen analysis is listed in Table 3. Minimum value of eigen value is 0.677, then the order of stress singularity is 0.323. This value is fairly agreed with the order of stress singularity in plane strain condition for two-dimensional joint with the same material combination. Furthermore, a 5-ple root of eigen value of p=1 occurs in the 3D-joint, although single root of p=1 only occurs in the two-dimensional joint with the same material combination as the 3D joint. The distribution of

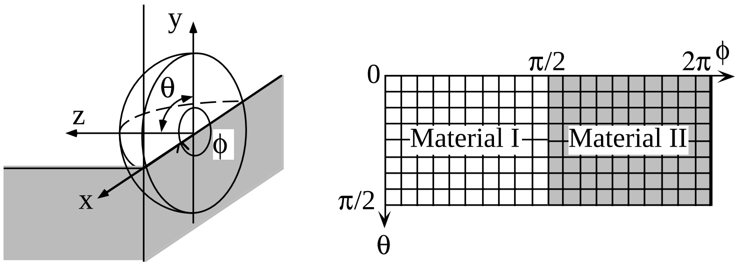

Fig.8 Model for eigen analysis (3D-line analysis)

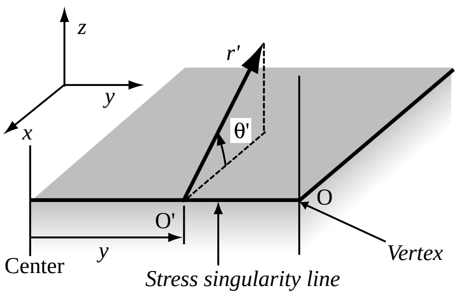

Fig.9 Cylindrical coordinate system

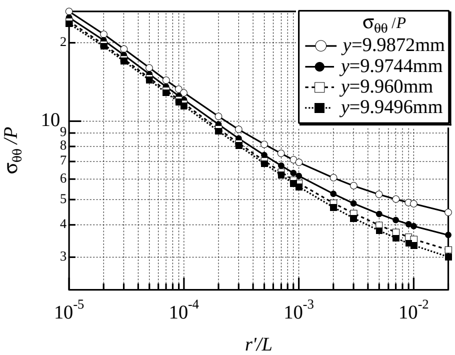

Fig.10 Stress distribution, $\sigma_{\theta\theta}$, against r /L

Table 3 Eigen value in 3D-line analysis

	Real	Imaginary	λ
1	0.6774580	0.000000	0.3225420
2	1.0017762	0.000000	-0.0017762
3	1.0009080	0.000000	0.0009080
4	1.0000696	0.000000	-0.0000696
5	1.0001129	0.000000	-0.0001129
6	0.9999974	0.000000	0.0000026
7	1.7090509	-0.637774	-0.7090509

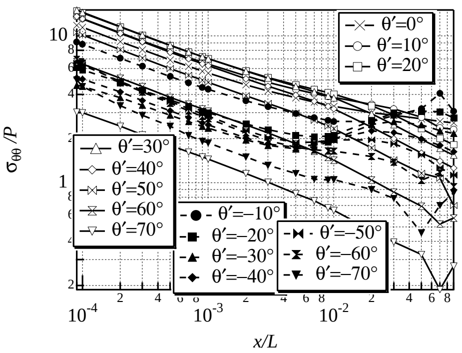

Fig.11 Distribution of stress, $\sigma_{\theta\theta}$, against r /L

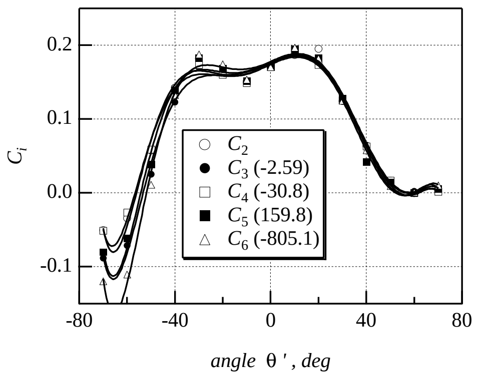

Fig.12 Variation of coefficients for angle θ

stress, $\sigma_{\theta\theta}$, in the planes shown in Fig. 9, is shown in Fig. 10. Where r represents the distance from an inner point to the origin. Figure 11 represents the stress distribution against r /L for various angle θ (see Fig.8) at y=9.9496mm. Stress along stress singularity lines can be expressed considering the result of eigen analysis.

$$\sigma_{\theta\theta}/P = C_1^*(\theta)\,\bar{r}'^{\,-\lambda} + C_2^*(\theta) + C_3^*(\theta)\log \bar{r}' + C_4^*(\theta)(\log \bar{r}')^2 + C_5^*(\theta)(\log \bar{r}')^3 + C_6^*(\theta)(\log \bar{r}')^4 \quad (5)$$

Coefficients, $C_i^*(\theta)$, for logarithmic singularity terms are determined using a least square method, and they are shown in Fig.12 against angle θ. It is found that all coefficients in logarithmic singularity terms vary with the angle θ in the same manner. Finally, stress, $\sigma_{\theta\theta}$, can be expressed as follows.

$$\sigma_{\theta\theta}/P = \bar{C}_1^*(\theta)\,\bar{r}'^{\,-\lambda} + \bar{C}_2^*(\theta)\{1 + \bar{C}_3^*\log \bar{r}' + \bar{C}_4^*(\log \bar{r}')^2 + \bar{C}_5^*(\log \bar{r}')^3 + \bar{C}_6^*(\log \bar{r}')^4\} \quad (6)$$

5 CONCLUSION

In the present paper, characteristics of stress distribution in the stress singularity fields at the vertex and on the stress singularity line were investigated using a three-dimensional BEM with Rongved s fundamental solution and an eigen value analysis using FEM. Stress, $\sigma_{\theta\theta}$, at the vertex can be expressed as a sum of the power-law singularity, $r^{-\lambda}$, $(\log r)^2$, $\log r$ and constant terms. On other hand, stress, $\sigma_{\theta\theta}$, at a point on the stress singularity line can be expressed as a sum of $r^{-\lambda}$, $(\log r)^4$, $(\log r)^3$, $(\log r)^2$, $\log r$ and constant terms depending on the multiplicity of p=1.

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