Meshless Analysis of Crack Propagation In Multiphase Material

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ABSTRACT

The material body considered in this work consists of multiphases, such as composite material, polycrystalline solid, and concrete, etc. Digital imaging data are taken as the input to specify the configuration and composition of the specimen. In this work, meshless method, which constructs the approximation of field variables entirely in terms of nodes without the need of a highly structured mesh as required in finite element method, is demonstrated as a superior numerical tool to analyze crack propagation in multiphase material. Crack propagation can be viewed as a process of moving discontinuities. Meshless analysis of crack propagation does not involve the formidable task of constantly remeshing the cracking specimen. In this work material forces and Eshelby stresses, due to the existence of material inhomogeneity, are calculated and can be employed as the indicator for the location of crack initiation. A fracture criterion, based on the ratio of the opening stress over the material toughness distributed in front of the crack tip, is proposed to determine the direction of crack propagation of mixed mode fracture problem in multiphase material. Numerical results, including deformation, stresses, path of crack propagation, failure process and the ultimate strength of the multiphase material, are presented and discussed.

1 MULTIPHASE MATERIALS

The material body considered in this work consists of multiphases. For example, concrete consists of aggregates, pastes, and voids. The boundaries between phases are usually irregular and random. Therefore, to say the least, it is very difficult to model and to perform numerical simulation of the detailed feature of multiphase material. On the other hand, digital imaging data from CT, ultrasound, MRI, etc. are abounding.

CT, also known as CAT scanning or X-ray computed topography, is a completely nondestructive technique that enables one to visualize detailed features in the interior of opaque solid objects and to obtain information on their 3-D geometry and composition. In CT, cross sectional images are generated by projecting a thin beams of X-ray through one plane of an object from many different angles. A 2-D image of a section or a slice of a 3-D object usually has 512×512 pixels. The value of each pixel is a measure of the reduction in X-ray intensity and energy, which in turn is a measure of the density of the material at that point. Therefore, the values at the pixels can be taken as the input to specify the configuration and composition of the specimen.

2 MESHLESS METHODS

Meshless methods can be constructed solely in terms of nodes without the need of a highly structured mesh as required in finite element (FE) method. For a variety of problems with large deformation, moving boundary discontinuities, or in optimization problems where re-meshing may be required, meshless methods are very attractive [1-3]. The meshless methods are based on the moving least squares technique in which the approximation of any scalar-valued function, $\tilde{U}(\mathbf{x})$, can be expressed as an inner product between a vector of shape functions, $\mathbf{\Phi}(\mathbf{x})$, and a vector of nodal values, $\mathbf{U}$, as

$$\tilde{U}(\mathbf{x}) = \mathbf{\Phi}(\mathbf{x}) \cdot \mathbf{U} \quad , \tag{1}$$

which has the same form as in the FE method. However, there is a characteristic difference between FE method and meshless method: eq. (1) is an approximation rather than an interpolation, i.e., in meshless method, $\tilde{U}(\mathbf{x}_i) \neq U_i$. This character requires special and careful treatments of essential boundary conditions, mirror symmetries, and moving discontinuities, such as crack propagation [3,4].

3 MATERIAL FORCES

The gravitational forces, the Lorentz force on a charged particle, and a radiation force that causes damping are all physical forces in the usual Newtonian view of mechanics. They are the contributors to Newton's equation of motion (balance of linear momentum) or Euler-Cauchy equations of motion when we pass from discrete model to continuum field theory. Physical forces are generated by displacements in physical space. For a continuous body, this means a change in its actual position in its physical configuration at time t [5].

On the other hand, the concept of material forces was first introduced by Eshelby [6], elaborated and further developed by Maugin [5,7]. Material forces are generated by displacement, not in physical space, but on material manifold. For example, they can be generated by (a) an infinitesimal rigid displacement of a finite region surrounding a point of singularity in an elastic body [6], (b) an infinitesimal displacement of a dislocation line [8], (c) an infinitesimal increase in the length of a crack [9-10]. This characteristic property of material forces also leads to their christening as inhomogeneity forces. Material inhomogeneity is defined as the dependence of properties (not the solution), such as density, elastic coefficients, viscosity, plasticity threshold, on the material point. These inhomogeneities may be more or less continuous such as in metallurgically superficially treated specimens or in a polycrystal observed at a mesoscopic scale, or it may change abruptly such as in laminated composite or in a body with foreign inclusions or cavities.

For thermoelastic material, the governing equations of material forces may be expressed as

$$B_{KL,K} + F_L = \dot{P}_L \quad , \tag{2}$$

where the pseudomomentum $\mathbf{P}$, Eshelby stress $\mathbf{B}$, and material force $\mathbf{F}$ are derived to be [5, 11]:

$$P_L \equiv -\rho^o v_k x_{k,L} \quad , \tag{3}$$

$$B_{KL} = -(K - W)\delta_{KL} - T_{KM} C_{LM} \quad , \tag{4}$$

$$\begin{aligned} F_L = &-\rho^o f_l x_{l,L} + \tfrac{1}{2} v_k v_k (\rho^o)_{,L} + (\rho^o \gamma T / T^o + a_{KM} E_{KM}) T_{,L} \\ &+(\rho^o \gamma)_{,L} \frac{T^2}{2T^o} + a_{KM,L} E_{KM} T - \tfrac{1}{2} A_{IJMN,L} E_{IJ} E_{MN} \end{aligned} \quad . \tag{5}$$

It is seen that the material force in thermoelastic solid is due to (1) body force $\mathbf{f}$, (2) temperature gradient ∇T, and (3) the material inhomogeneities in density ρ^o and all the thermoelastic coefficients γ, $\mathbf{a}$, and $\mathbf{A}$. In eqs. (3-5) $\mathbf{v}, \mathbf{C}, \mathbf{E}, K, W, T^o$ are the velocity, Green deformation tensor, Lagrangian Strain, kinetic energy, strain energy, reference temperature, respectively. It should be emphasized that it is almost impossible and even erroneous to calculate the derivatives of the material properties through FE method, and, on the other hand, it is natural and easy to do so through meshless method.

Also, for 2-D problems in the presence of propagating crack, the material force associated with the crack tip is obtained as

$$\mathbf{F} = \lim_{\Gamma \to 0} \int_{\Gamma} \mathbf{N} \cdot (\mathbf{V} \otimes \mathbf{P} - \mathbf{B}) d\Gamma \quad , \tag{6}$$

where Γ denotes the cross-sectional circuit around the crack tip; $\mathbf{N}$ is the unit vector normal to Γ pointing away from the crack tip; $\mathbf{V}$ is the velocity of crack propagation. Notice that crack propagation is a movement on material manifold, not in physical space, therefore, $\mathbf{V}$ is not equal to the material time rate of change of the position vector (velocity) of any particle. It can be shown that, in a very special case, the projection of $\mathbf{F}$ in the direction tangent to the crack path behind the crack tip is reduced to the J-integral, which is path-independent if the material within Γ is homogeneous.

4 CRACK PROPAGATION

In two dimensional fracture problems, Mode I fracture may lead to self-similar crack extension due to symmetry. In general case, especially in case of multiphase material, we encounter mixed mode fracture problems. Therefore, to determine the direction of crack extension is an unavoidable task. Usually, we use the maximum opening stress criterion or the maximum energy release rate criterion to determine the direction of crack propagation. For example, using maximum opening stress criterion, the current crack tip will extends to $\{r_c,\ \theta\}$ if the opening stress $t_{\theta\theta}$ is maximum at $\{r_c,\ \theta\}$, where $r_c > 0$ is small and finite constant. One may consider that $t_{\theta\theta}(r_c, \theta)$ is the driving force distributed along an arc with a radius r_c with respect to the current crack tip. If the material is homogeneous, the maximum opening stress criterion is reasonable, i.e., the information of driving force is enough to determine the direction of crack extension. However, if the material is inhomogeneous, one has to consider the resistance, i.e., the toughness, distributed in front of crack tip. In this work, we propose that the current crack tip will extend to $\{r_c,\ \theta\}$ if the ratio

$$R(r_c, \theta) \equiv \frac{t_{\theta\theta}(r_c, \theta)}{t_c(r_c, \theta)} \quad , \tag{7}$$

reaches a maximum at $\{r_c,\ \theta\}$, where t_c is the toughness associated with the opening stress. Crack propagation process can be viewed as a changing of crack tip with a moving barrier following the advancing of the crack tip. It is noticed that meshless analysis of crack propagation does not involve the formidable task of constantly remeshing the cracking specimen. It only needs the updating of the barrier and the sprinkle of additional nodes in front of the current crack tip to enhance the solution accuracy.

5 NUMERICAL RESULTS

In this work material forces and Eshelby stresses, due to the existence of material inhomogeneity, are calculated and can be employed as the indicator for the location of crack initiation. The fracture criterion, based on the ratio of the opening stress over the material toughness distributed in front of the crack tip, is proposed to determine the direction of crack propagation of mixed mode fracture problem in multiphase material. Numerical results, including deformation, stresses, path of crack propagation, failure process and the ultimate strength of the multiphase material, are presented and discussed.

ACKNOWLEDGEMENT

The support to this work by National Science Foundation under Award Number CMS-0301539 and by the Federal Highway Administration of US Department of Transportation is gratefully acknowledged.

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