PROBABILISTIC ANALYSIS OF THE FRACTURE TOUGHNESS, K_{IC}, OF BRITTLE AND DUCTILE MATERIALS DETERMINED BY SIMPLIFIED METHODS USING THE WEIBULL DISTRIBUTION FUNCTION

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ABSTRACT

Simplified methods were developed to determine the critical stress intensity factor or fracture toughness K_{IC} for brittle and ductile materials. The methods are the simplest for both kinds of materials. Moreover, they are low-cost and allow for the obtention of a great number of samples in a short time. The results of K_{IC} were analyzed using the Weibull distribution function determining the respective mean values and the Weibull parameters. The results of K_{IC} are quite similar to those obtained by traditional methods. Glass, cement, steel and copper were the materials used.

1 INTRODUCTION

Even with the development of high technology in the manufacturing of materials, there are certain defects in all of them, such as flaws, cracks, etc. In materials such as natural rock, those defects came from their genesis process, and in manufactured materials, defects came from the manufacturing process. In consequence, many structures,being brittle or ductile, present catastrophic collapse, due to sudden fracture, in which the stress that allows for the collapse is lower than design stress. In order to prevent such castastrophic failure it is necessary to determine the critical stress intensity factor in mode I or fracture toughness, K_{IC}. For metallic materials the determination of K_{IC} has been standardized by ASTM [1] and measurements of K_{IC} values are determined by using conventional samples, for example, compact tension specimens and three-point loaded bend specimens. After the notch is machined in the specimen, the sharpest possible crack is produced at the notch root by fatiging the specimen in a low-cycle, high-strain mode. This method involved considerable time in the preparation of sufficient specimens. Much work has been carried out on the study of K_{IC} in ductile materials. When dealing with brittle materials the problem becomes even more complex. In such cases an alternative method to determine K_{IC} using the indentation test from the measuring of Vickers indentation and obtaining K_{IC} through empirical relationship has been developed. Other formulas have also been used to determine K_{IC} employing Hertzian indentation fracture. There has been a constant pursuit of simpler-geometry specimens that meet routine quality control requirements as well as being low-cost and time-efficient. For these reasons, simplified methods to determine K_{IC} in brittle and ductile materials have been developed by us and succesfully applied to glass, compacted neat cement paste cement mortar Díaz et.al.[2], Kittl et.al.[3], applied to steel Artigas et.al.[4] and applied to copper. The methods allows us to use a probabilistical analysis of the measurements values of K_{IC} using the Weibull distribution function Weibull [5] and Kittl and Díaz [6]. Such analysis is justified considering the existence of cracks which unchain the brittle fracture in both types of materials, brittle or ductile, without warning and in a catastrophic manner. The objective of this of work is to discuss the simpler method to determine K_{IC} in brittle and ductile materials which requires a much less complex experimentation, applied to glass, one of the most brittle materials; cement, steel and copper which is one of the most ductile materials, and determining the mean value of K_{IC} and the Weibull parameters.

2 WEIBULL DISTRIBUTION OF K_{IC}

According to probabilistic strength of materials Weibull [5] and Kittl and Díaz [6] if fracture toughness K_{IC} follows a Weibull distribution function, it can be pointed out as follows:

$$F\left(K_{IC}\right) = 1 - \exp\left\{-\frac{b}{b_0}\phi\left(K_{IC}\right)\right\}. \tag{1}$$

where F is the cumulative probability, ϕ is the Weibull specific risk function, b_0 is the unit length and b is the crack width. The Weibull specific risk function can be expressed as follows:

$$\phi\left(K_{IC}\right) = \begin{cases} \left(\dfrac{K_{IC} - K_{ICL}}{K_{IC0}}\right)^m & K_{ICL} \le K_{IC} < \infty \\ 0 & 0 \le K_{IC} \le K_{ICL} \end{cases}. \tag{2}$$

where m and K_{ICO} are the Weibull parameters depending on the crack-generating process and on the material manufacturing and K_{ICL} is the lowest limit value for fracture toughness under which no crack propagation occurs. When $K_{ICL} = 0$ the two-parameter Weibull specific function, m and K_{ICO}, is obtained. There is also another expression for function ϕ, named Kies-Kittl function Díaz and Morales [7] and Kittl et.al. [8], and given by:

$$\phi\left(K_{IC}\right) = \begin{cases} K\left(\dfrac{K_{IC} - K_{ICL}}{K_{ICS} - K_{IC}}\right)^m & K_{ICL} \le K_{IC} < K_{ICS} \\ 0 & 0 \le K_{IC} \le K_{ICL} \\ \infty & K_{ICS} \le K_{IC} \le \infty \end{cases}. \tag{3}$$

where K_{ICS} is the highest limit value for fracture toughness over which the crack propagation always occurs and K is the Kittl constant. In order to make the respective Weibull diagrams, ln [ln 1/(1-F(K_{IC}))] against ln K_{IC} was made and the following cumulative probability estimation was employed: $F(K_{IC}) = (n - 0.5)/N$, where the experimental values of K_{IC} have been ascendantly ranked, n is the number of samples having a fracture toughness less than or equal to K_{IC} and N is the total number of samples tested.

3 SIMPLIFIED METHODS TO DETERMINE K_{IC} IN BRITTLE MATERIALS

The three cases presented here as follows were developed by using three-point loaded bend specimens where the notchs were manufactured and the crack was not induced by fatigue. Then, the method consist in making a pseudocrack in rectangular-cross-sectional-area sample which are later subjected to the three-point bending test.

The first material was glass samples in the form of beams of length S, width b, height W and crack size a which were made from three pieces of glass sheet extracted from a given commercial glass sheet. For all samples the cracks were made in the same manner. From the pieces of glass sheet, rectangular bits of length S, width b and height W/2 were cut with a diamond saw. Then, with a glazier's diamond a scratch was made at a distance S/2 from the bit ends and extending to all its width. Also, from the same piece of glass sheet, rectangular bits of length S/2, width b and height W/2 were cut. Afterwards, the transversal edges of these last rectangular bits were polished by using a mechanical polishing disk and the same ones were employed as the profile of the crack induced. Finally, two of these bits of length S/2 were stuck together with epoxy resin to every bit of length S in order to obtain samples having a pseudocrack formed by a sractch, crack tip, and the polished transversal edges, profile of the crack. The scratch was situated on the side opposite to the one of the application of load. The epoxy resin was left to polimerize during 24 hour at room temperature, and then the samples were tested. The samples manufactured in this manner allowed us to obtain 30 samples which had the following dimensions: length S = 50 mm, width b= 20mm, height W=8.6 mm and crack size a = 3.8 mm. For all the samples the testing span L was 40 mm.

The second material was compact neat cement paste. In the manufacturing of the samples commercial portland cement devoid of admixtures was employed. The cement was mixed with 4%, in weight, of water and was introduced in a steel rectangular mould with 55 x 20 mm in dimension and then a compacting pressure of 35 MPa was applied to it in order to obtain samples 4 mm high. The samples were introduced into the moist room during 7 days. The hydration process was stopped by putting the samples into a furnace at 110 °C for 24 hours. Later they were stored in a dessicator until the moment of preparing them for testing. In this way, 50 samples were made. Half of the samples were transversely cut at the mid-length by using a diamond disk. The following process for the generation of the crack was analogous to the one employed in the preparation of the glass samples, except that in the samples of compact neat cement paste it was not necessary to make a transversal scratch because in the internal constitution of hardening cement there is a great number of cracks. One of the transversal edges of the samples obtained was polished in order to generate the profile of the crack, and then two little pieces were stuck to a complete sample by using epoxy resin. This method allowed us to obtain 25 samples of length S = 50 mm, width b = 20 mm, height W = 10 mm and crack size a = 5 mm. For all samples the testing span L was 39 mm.

The third material was mortar cement. In this case the samples of portland cement mortar were fabricated according to the RILEM's procedure obtaining parallelepiped samples of length S, width b and height W. The crack, a transversal crack, was generated by introducing a steel lamina into the samples, in the fresh state, immediately after they were manufactured. Afterwards, the samples were stored in a moist room. The steel lamina was removed after the first day of curing in a moist room. After that the samples were stored again in a moist room until completing 7 days of hydration. This method allowed us to obtain 25 samples of length S = 160 mm, width b = 40 mm, height W = 25 mm and crack size a = 9.4 mm with a steel lamina of 0.5 mm thickness, and the testing span L was 100 mm.

For the three cases fracture toughness K_{IC} was determined by employing $K_{IC} = \sigma Y a^{1/2}$ where a is the crack length and Y is a function depending on the geometry of the samples which, after Chermant [9], can be obtained from the tables. As σ is the maximum fracture stress obtained in the three-point bending test, the expression of K_{IC} is transformed into:

$$K_{IC} = \frac{3}{2}\frac{PL}{bW^2} Y\sqrt{a} \ . \tag{4}$$

where P is the fracture load, and L, b and W are the testing span, width and height of the beam, respectively. For each material and in all samples tested, the fracture always occurs due to a crack tip growing across the transversal section of the beams in the three-point bending test. For example, for glass and for compact neat cement paste samples the fracture never occurs in the interface between the pieces stuck with epoxy resin. In the case of the compact portland cement mortar samples the cracks grows in the same direction where the crack was induced with the steel sheet.

4 SIMPLIFIED METHOD TO DETERMINE K_{IC} IN DUCTILE MATERIALS

The method developed here considers cylindrical notched round samples subjected to a tension test, starting with an arbitrary notch depth and increasing it until brittle fracture occur. Such method was applied to steel and copper, one of the most ductile materials. The steel samples were AISI 1020 and AISI 1045 and the copper samples had 99.9% of copper. Both materials were obtained from long commercial wire drawing. The method consists of putting the round sample in a lathe and making an arbitrary small notch with a cutting tool around the middle of its length. The notched sample was later subjected to a tension test in an Instrom machine and the load-displacement diagram was simultaneously drawn. When a small quantity of nonlinear deformation appeared, the test was sttoped and the sample was put in the lathe again in order to increase the notch slightly with a cutting tool. Afterwards, the sample was subjected again to the tension test until a small quantity of nonlinear deformation was observed in the load-displacement diagram. The test was sttoped again and the process was successively repeated until only linear deformation was observed. The final conditions correspond to brittle fracture and allow for the determination of K_{IC}. Note that a material should not change its properties due to a small non linear deformation in a material previously subjected to a very high percentage of plastic deformation by lamination. This means that the material is even in the elastic range and there is total springability, i.e. elastic limit is not exceeded.

For the case of both kinds of steel, AISI 1020 and AISI 1045, the initial diameter of the samples was D = 9 mm and the util length was 150 mm. The total length of the samples was higher in order to consider that the samples must be put between the jaws of the Instrom machine. 30 samples were manufactured for each kind of steel by using this method. The respective load-displacement diagram of unnotched samples for both kinds of steel showed clearly the ductility of the materials.

For the case of copper two sizes of cylindrical bars were employed and the diameters were D = 9.53 mm and D = 12.70 mm, respectively. The util length necessary was 400 mm. In the notch-making process and due to the great ductility of copper the penetration of the cutting tool into the cylindrical samples was very deep, near 80% of initial diameter D. In order to avoid the tip of the cutting tool breaking, a special dimension was considered for it. In some opportunities, due to the deep penetration of the cutting tool to make the notch some samples were broken in the lathe because they suffered flexure due to the strength applied by the cutting tool, then these samples couldn´t be taken into account. 22 samples of copper bars 9.53 mm in diameter and 21 samples for copper bars 12.70 mm in diameter were obtained by using this method.

In both ductile materials a number of samples were used to determine the minimum penetration, crack tip, to produce brittle fracture. After that, the notched round samples were manufactured with an aleatory deep notch from the minimum penetration. For determining K_{IC} the secant method was used ASTM [1] and Knoot and Withey [10]. The K_{IC} was determined by employing the following equation after Koiter and Benthem [11]:

$$K_{IC} = \sigma\sqrt{\pi a}\left(\frac{D}{d} + \frac{1}{2} + \frac{3}{8}\frac{d}{D} - 0.36\frac{d^2}{D^2} + 0.73\frac{d^3}{D^3}\right)\frac{1}{2}\sqrt{\frac{D}{d}}\ ;\ \frac{a}{D} \in \left(0, \frac{1}{2}\right)\ ;\ t = \pi d = \pi\left(D - 2a\right). \quad (5)$$

where a is the crack penetration, σ is the stress of fracture, D is the sample diameter, d = D – 2a is the effective diameter or the diameter of the notch crack of length t. SEM observations of the fracture surface at the final stage of the process of tension test and the load-displacement diagrams showed that failure occurs by brittle fracture, which means decohesion, and a sudden braking into the elastic linear zone. Due to different sample diameters being employed, the values of K_{IC} are changed and a decrease in fracture toughness is observed when increasing the notch diameter.

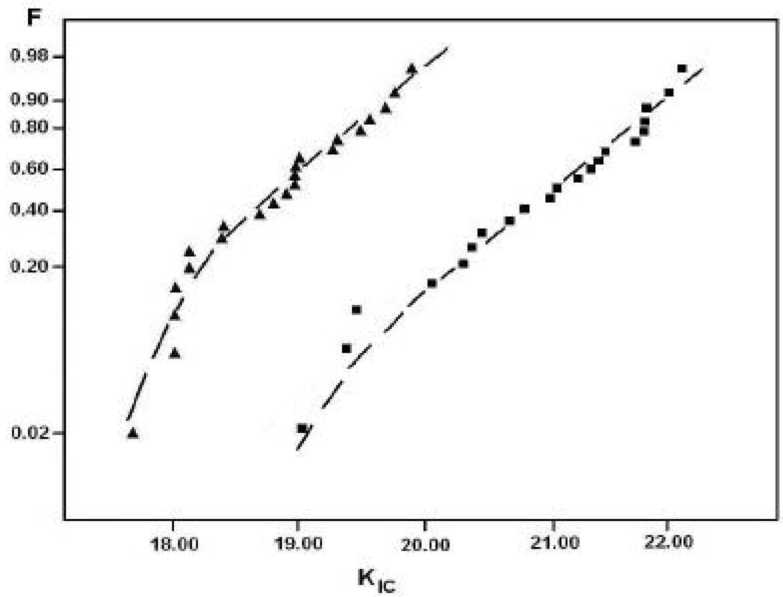

Figure 1: Weibull diagram of fracture toughness K_{IC} in $MNm^{-3/2}$ for copper bars. : D = 9.53 mm; : D = 12.70 mm. D = bar diameter.

5 ESTIMATION OF WEIBULL PARAMETERS

For each material the Weibull diagrams and Weibull parameters were made and were estimated, respectively. In addition, the size effect was considered too and its respective Weibull parameters were obtained, but not shown here. Different Weibull specific risk functions were obtained. In Figure 1 the Weibull diagram for K_{IC} in two bars of wire drawing copper is shown. The Weibull parameters and the mean value of fracture toughness appear in Table 1. The cases where the Weibull specific risk function has two parameters has only been considered, i.e. K_{ICL} =0 in eqn (2). In other cases, not shown here, the specific risk function follows one of Kies-Kittl´s function and Weibull´s function with K_{IC} different from zero.The results for fracture toughness obtained for the simplified method applied to brittle materials, glass, compact neat cement paste and compact cement mortar are in accordance with those obtained for traditional methods. The validity of fracture toughness obtained for the simplified method applied to ductile materials, steel and copper, is sustainable in the observation with SEM, to the surface fracture and with the evidence of brittle fracture registered in the load-displacement diagrams made for the samples tested.

Table 1: Weibull parameters and mean value of fracture toughness for brittle and ductile materials.

Material	Weibull parameters		$\bar{K}_{IC}$ $MNm^{-3/2}$
	m	K_{IC0} $MNm^{-3/2}$	
Glass	1.53	0.27	0.99
Neat cement paste	6.60	0.79	0.97
Cement mortar	3.98	0.13	0.33
Steel AISI 1020	29.28	0.83	50.81
Steel AISI 1045	23.64	0.81	55.40
Copper D = 9.53 mm	35.47	18.92	18.86
Copper D = 12.70 mm	27.17	21.22	20.99

6 CONCLUSIONS

The simplified methods developed here to determine fracture toughness in brittle and ductile materials were successfully applied to different materials. For glass, one of the most brittle materials, compact neat cement paste, compact cement mortar, the values for fracture toughness determined by the methods proposed are in accordance with the measurements made by the traditional method. On the other hand, for steel and copper, the latter being one of the most ductile materials, the values determined by the methods proposed allowed us to assign validity to the values obtained for fracture toughness by means of observations with SEM of the surface fracture and with the evidence of brittle fracture registered in the load-displacement diagrams. The traditional method to determine fracture toughness K_{IC} involves the testing of notched specimens that have been precracked in fatigue by loading either in tension or three point bending. The methods proposed use the same geometry of the samples and the same test applied to the samples, the differences are in the manner the notch is induced and how the load is applied, it is not necessary for the crack to grow by fatigue. In accordance with this it is easy to manufacture a great number of samples to be tested and moreover at a low cost which is very important in the quality control process. The number of samples obtained allows for a statistical analysis of the experimental data obtained. Finally, the fracture toughness is a mechanical property which follows a Weibull probability distribution function and the measurement obtained has a mean value and a dispersion which allows for an accurate design of the materials used in structural applications.

REFERENCES

1.- ASTM E399-90. Standard Test Method for Plane-Strain Fracture Toughness of Metallic Materials. Annual Book of ASTM Standard, Vol. 03.01, 1998, pp. 413-443.

2.- DIAZ, G., KITTL, P. and MARTINEZ, V.: In 8th CIMTEC, World Ceramic Congress, Florence, Italy, 1994.

3.- KITTL, P., DIAZ, G. and MARTINEZ, V.: A simplified method for determining K_{IC} in glass with liquid solutions impregnation. Journal of Materials Science 31(1996)3675-3678.

4.- ARTIGAS, A., DIAZ, G., KITTL and MARTINEZ, V.: Determination of K_{IC} in AISI 1020 and AISI 1045 steels by means of a simple method. PACAM V. Applied Mechanics in the Americas. L. A. Godoy, M. Rysz and L. E. Suárez (eds.). The University of Iowa, IA, Vol. 4(1997) 103-106.

5.- WEIBULL, W.: A statistical theory of the strength of materials. Ing. Vetenskaps Akad. Handl., 151(1939)1-45.

6.- KITTL, P. and DIAZ, G.: Weibull's fracture statistics, or probabilistic strength of materials: state of the art. Res Mechanica 24(1988)99-207.

7.- DIAZ, G. and MORALES, M.: Fracture statistics of torsion in glass cylinders. Journal of Materials Science 23(1988)99-207.

8.- KITTL, P., DIAZ, G. and MARTINEZ, V.: Applicability of a Weibullian model of fracture by application of a slowly gradual load. In: Probabilities and Materials. D. Breysse (ed) Kluwer Academic Publisher, London, 1994, pp. 439-449.

9.- CHERMANT, J. L.: Léssai de K_{IC} pour les Máteriaux Fragiles. Bulletin Societé Française Céramique 108(1975)39-52.

10.- KNOOT, J. F. and WITHEY, P. A.: Fracture Mechanics. Worked Examples. The Institute of Materials. ISBN 0 901719 28 6.

11.- KOITER, W. Y. and BENTHEM, J. P.: Mechanics of fracture. In G. C. SIH (ed.), Noodhoff, Leyden, Vol I, 1973, pp. 131-178.