DISCRETE MODELING OF MIXED-MODE FRACTURE IN CONCRETE

Z. SHI
Research and Development Center, Nippon Koei Co., Ltd., Japan.

ABSTRACT

A numerical study of the mixed-mode (mode-I and mode-II) fracture in concrete is presented. For crack propagation the maximum principal stress criterion is adopted, assuming that the mode-I condition is dominant at the tip of a mixed-mode crack. As a main feature of this study, normal and tangential tractions are applied directly to the crack surface, following specific tension-softening and shear-transfer laws. To verify the approach, the single-notched shear beam test by Arrea and Ingraffea is studied. Varying the shear-resistance characteristics on the crack surface, the numerical model predicts distinct transitions from the mode-I fracture to the mixed-mode fracture, which show reasonable agreement with experimental observations. The main conclusions are as follows:

(1) The proposed shear-COD relation with a shear-lag mechanism serves as a simple shear-transfer function for a mixed-mode fracture that can be easily implemented in a numerical model.

(2) Within the scope of the study, the influence of the shear force on the maximum load is found to be small and negligible. The main effect is shown in the post-peak structural response and the crack path.

1 INTRODUCTION

This paper presents a numerical study of the mixed-mode (mode-I and mode-II) fracture in concrete, based on a recent work by Shi [1]. For crack propagation, the frequently adopted maximum principal stress criterion is used, which is based on the notion that the mode-I condition is dominant at the tip of a mixed-mode crack. Accordingly, the crack propagation direction is set perpendicular to the direction of the maximum tensile principal stress at the crack tip. As the tip stress reaches the tensile strength of concrete a mixed-mode crack propagates. A main feature of this study is that normal and tangential tractions are applied directly to the crack surface, following the tension-softening and shear-transfer relations specified below.

In the following, modeling of the cohesive forces in the FPZ of a mixed-mode crack is addressed first. Due to the space limitation, a numerical formulation for the mixed-mode fracture of concrete is omitted, which can be found elsewhere [1]. To verify the approach, the single-notched shear beam test by Arrea and Ingraffea [2] is studied. Varying the shear-transfer characteristics on the crack surface, the numerical model predicts distinct transitions from the mode-I fracture to the mixed-mode fracture in the obtained structural responses and crack paths, which show reasonable agreement with experimental observations.

2 MODELING OF COHESIVE FORCES IN MIXED-MODE CRACKING

Compared with the relatively clear mode-I crack propagation, shear transfer by aggregate interlock action is complex in mixed-mode fracture. Since there are no comprehensive mathematical models of the phenomenon, a functional approach to the problem is adopted. Firstly, it is assumed that the normal component of the cohesive forces is a function of the component of the displacement discontinuity normal to the crack surface, effectively imposing the tension-softening law of concrete on the normal traction and the

Crack-Opening Displacement (COD). As an example, a bilinear tension-softening relation is shown in Fig. 1(a), which is defined by the tensile strength of concrete f_t, the limit crack-opening displacement W_c, and the mode-I fracture energy G_F.

Secondly, for simplicity the tangential component of the cohesive forces, or shear, is also assumed to be a function of the COD, as shown in Fig. 1(b, c). The assumed bi-linear and tri-linear shear-COD relations possess the following characteristics. As seen, there is a delay in shear transfer at the initial stage of crack propagation, which is based on the analysis by Bazant and Gambarova [3] that the first displacement on the rough crack must be normal and the slip (causing frictional forces as shear) can occur only after some finite opening has already been achieved. As shear transfer begins at W_{s1}, the shear force builds up with increasing CODs and eventually attains its maximum value f_s at W_{s2}. Then, it decreases with further opening of the crack (passing the concave point at W_{s3} in the case of the tri-linear relation as shown in Fig. 1(c)) and vanishes at W_c, marking the formation of an open crack. This simple approach to the shear transfer that actually is related with the Crack-Sliding Displacement (CSD) and the COD, is based on experimental observations that within certain ranges, the two displacement discontinuities have an approximately linear relation.

3 NUMERICAL STUDIES OF SINGLE-NOTCHED SHEAR BEAM

The single-notched shear beam test by Arrea and Ingraffea [2] is selected as a mixed-mode fracture problem to be solved. The geometry and the loading arrangement of the selected test series can be found in the FE discretization of the beam specimen, as shown in Fig. 2. In the experiment the load was applied at point C of the steel beam AB and was controlled by a feedback mechanism with the Crack Mouth Sliding Displacement (CMSD) as a control parameter.

In the following studies, the bi-linear tension-softening relation in Fig. 1(a) is applied to the normal traction and the COD in the FPZ. Figure 3 illustrates four types of the shear-COD relations used for the mixed-mode fracture analysis; cases 1 to 3 are bi-linear types, and case 4 is a tri-linear type. As seen, two types of shear strength are assumed: for cases 1, 3 and 4 it is one half of the tensile strength, $f_s = f_t/2$, and for case 2 it is one third, $f_s = f_t/3$. For cases 1, 2 and 4, the shear transfer begins at $W_{s1} = 0.1W_c$, and reaches its maximum at $W_{s2} = 0.2W_c$. The concave point of case 4 is set at $W_{s3} = 0.3W_c$. For case 3 a

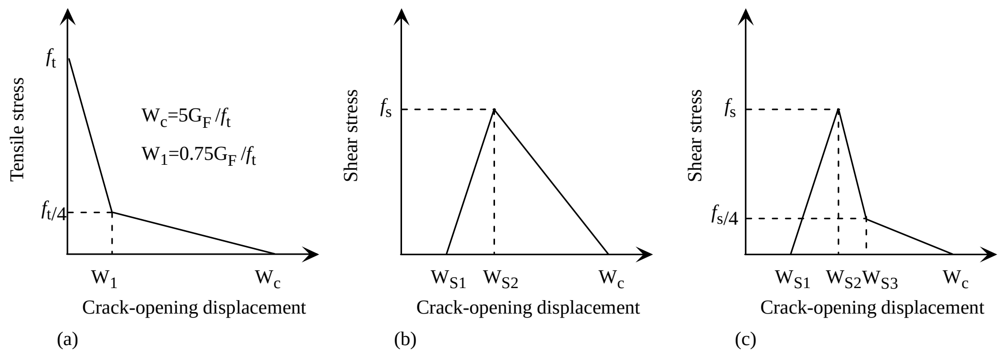

Figure 1: Assumed tension-softening and shear-transfer relations of concrete.

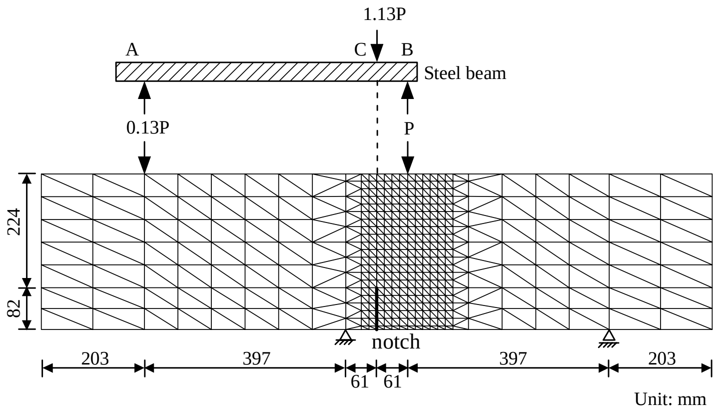

Figure 2: Finite element idealization for single-notched shear beam tests by Arrea and Ingraffea.

Table 1: Material properties of single-notched shear beam.

E (GPa)	ν	f_c (MPa)	f_t (MPa)	G_F (N/mm)
24.80	0.18	45.50	3.50	0.14

larger shear-lag is assumed with W_{s1} = 0.25W_c, and the shear strength is reached at W_{s2} = 0.5W_c. The material properties of the test specimen are summarized in Table 1.

Figures 4 and 5 show the experimental envelope and the numerical predictions of the load versus the CMSD curves, and of the crack trajectories, respectively. For comparison, the numerical results obtained under the mode-I condition are also illustrated. As seen from the load-CMSD curves of Fig. 4, the numerical analyses using the mode-I and mixed-mode conditions yield roughly the same peak loads (with variations of less than 3%) that agree reasonably well with the experimental results. The most obvious effect of the shear force on the global structural response is exhibited in the post-peak regions. With the bi-linear shear-COD relation of case 1, the global stiffness increases greatly in the post-peak regions compared with the observed structural response, and the material becomes relatively ductile. Apparently, lowering the shear strength in case 1 will reduce this trend, as shown by the response curve of case 2. Needless to say, reducing the shear strength to null the mode-I condition is then obtained, and the structural response becomes much more brittle. Imposing a larger shear-lag in case 3, a practically identical response curve of the mode-I condition is obtained up to a point beyond the peak load. As the shear transfer takes place in the post-peak region, the rapid decrease of the load is interrupted and the loading rebounds slightly as the structural deformation progresses further before the final failure of the beam. With a tri-linear shear-COD relation assumed in case 4, where shear transfer takes place mainly in a narrow band of the COD, the response curve in the post-peak region moves much closer to the experimental envelope.

Next, numerical results on the crack paths are examined. Figure 5 shows that the

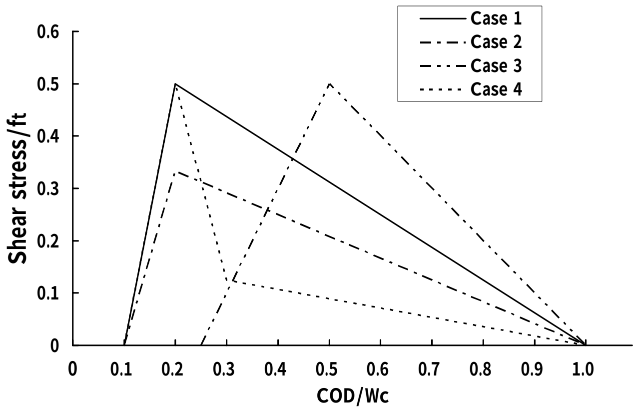

Figure 3: Four types of shear-COD relations.

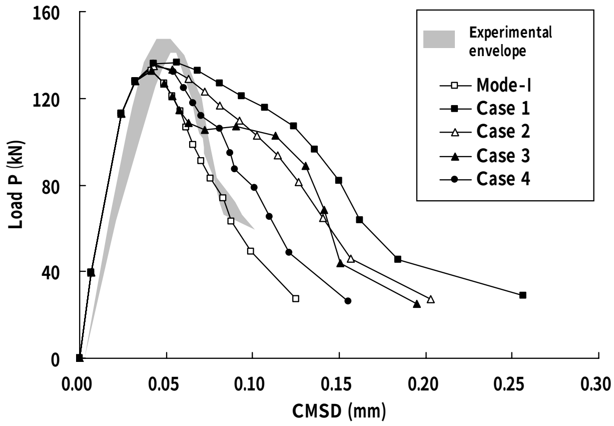

Figure 4: Experimental envelopes and numerical predictions of the load-CMSD curves under mode-I and mixed-mode conditions with four types of shear-COD relations.

crack trajectories obtained under the mode-I condition and the mixed-mode condition of case 1 are two opposite extremes, deviating from the experimental envelope as the crack penetrates deeper into the beam. While the mode-I path suggests a splitting failure, by further increasing the shear strength in case 1 a shear fracture may be approached. As seen, the crack paths in cases 2 and 4 basically fall into the experimental scatter band. The cracking behavior of case 3 is unique and serves to highlight the effect of shear. As the decrease of the load in the absence of shear is temporarily halted by the rise of the shear force at the crack surface as described above, the crack path diverges immediately from the mode-I path and reenters the experimental envelope.

4 CONCLUSIONS

A numerical study of the mixed-mode (mode-I and mode-II) fracture in concrete was presented. For crack propagation the maximum principal stress criterion was adopted,

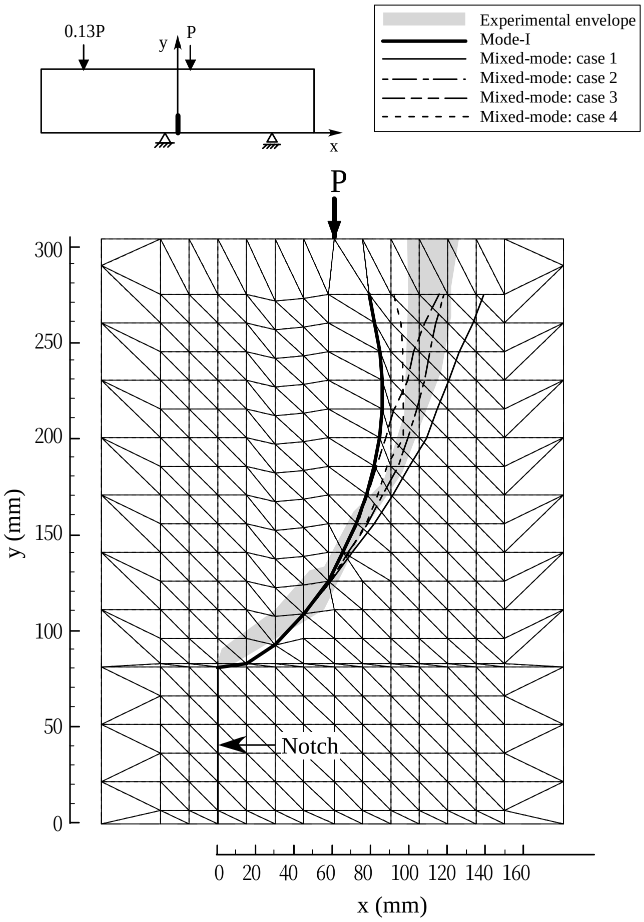

Figure 5: Experimental envelopes and numerical predictions of crack trajectories under mode-I and mixed-mode conditions with four types of shear-COD relations.

assuming that the mode-I condition is dominant at the tip of a mixed-mode crack. As a main feature of this study, normal and tangential tractions were applied directly to the crack surface, following specific tension-softening and shear-transfer laws. To verify the approach, the single-notched shear beam test by Arrea and Ingraffea was studied. Varying the shear-resistance characteristics on the crack surface, the numerical model predicted distinct transitions from the mode-I fracture to the mixed-mode fracture, which showed reasonable agreement with experimental observations. The main conclusions are as follows:
(1) The proposed shear-COD relation serves as a simple shear-transfer function for a mixed-mode fracture that can be easily implemented in a numerical model.
(2) Within the scope of the study, the influence of the shear force on the maximum load is found to be small and negligible. The main effect is shown in the post-peak structural response and the crack path.

REFERENCES

[1] Shi, Z. “Numerical analysis of mixed-mode fracture in concrete using extended fictitious crack model.” *J. Struct. Engrg.*, ASCE, 2004 (in press).

[2] Arrea, M. and Ingraffea, A. R. “Mixed-mode crack propagation in mortar and concrete.” *Report No. 81-13*. Dept. of Struct. Engrg., Cornel Univ., Ithaca, New York, 1982.

[3] Bazant, Z. P. and Gambarova, P. “Rough cracks in reinforced concrete.” *Journal of the Structural Division*, ASCE, 106(4), 819-842, 1980.