A 3D ISOTROPIC MESOMECHANICAL MODEL OF DEFORMATION AND CUMULATIVE DAMAGE OF CONCRETE

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ABSTRACT

Based on the general author’s mesomechanical concept (Kafka [1]), the processes of deformation under complex loading, of cumulative damage, of the resulting changes of elastic moduli and of volumetric changes are modeled and confronted with experimental data. Concrete is modeled as an isotropic heterogeneous medium consisting of elastic inclusions that are embedded in a plastically deforming and fracturing matrix. The configuration of the inclusions in the matrix is described by the so-called ‘structural parameters’ – free parameters that are to be determined from simple macroscopic experiments. After their determination they are used for the description of more complicated processes. The fracturing process of the matrix results in cumulative damage, in changes of elastic moduli, and in volumetric expansion.

In the current study, the model – created on the basis of the mentioned monograph – is an isotropic model, and its aim is description of very complicated loading conditions, in which the directions of principal stresses and strains do not change. The hypotheses of the model have been refined on the basis of confrontation with complex experimental data. These specifications we arrived at are: three kinds of currently proceeding deformation modes in different segments of the loading paths are differentiated: (i) concurrent elastic + plastic + damaging deformation, (ii) concurrent elastic + plastic deformation, (iii) only elastic deformation. Criteria for distinguishing between these three kinds of deformation modes have been newly formulated.

The merits of our model are demonstrated by confronting it with complex experimental data found from literature search. In the described experiments, cubes of concrete were loaded in three orthogonal directions by normal compressive loading in a very complex three-dimensional sequence, and the respective strain response has been measured. Comparison of the outcome of our model with these complex experimental data and with other generally known features of such kinds of processes corroborates the justness of our approach.

1 INTRODUCTION

Mechanical behavior of concrete represents a challenging problem due to its complexity. The respective modeling should cover elastic deformation, plastic deformation, quasihomogeneous fracturing, volumetric expansion and changes of elastic moduli. The respective models for concrete differ substantially from those for metals, as the effect of isotropic stress plays an important role and plastic deformation is accompanied by cumulative damage. On top of that, the assumption of normality of the strain increment to the yield surface, and the associated flow rule are not convenient enough. The phenomenon of cumulative damage must clearly be distinguished from plasticity, as it leads to significant changes of elastic moduli.

Phenomenological models can easily describe uniaxial processes, e.g. stress-strain diagrams in uniaxial compression. However, for more complicated loading paths, creation of proper models calls for a deeper understanding of the internal processes in the material. In our general mesomechanical concept (Kafka [1]), we presented models based on descriptions of internal structures of different materials, different processes in the structure, and internal strains and stresses, which were used as tensorial latent variables. The applications to metallic materials showed good agreement of the model prediction with experiments for plastic deformation of thin-walled tubes even under very complicated loading paths. For concrete, the application of the mesomechanical concept was limited in the monograph to the description of uniaxial compression. In a preceding author’s paper (Kafka [2]), a special transversely isotropic variant of our general concept has been created and applied to the description of deformation of concrete cylindrical specimens under relatively simple loading combinations – consisting of compression and twist. The agreement with experimental data of the deformation response calculated with the use of our model was good.

In the current study, a mesomechanical *isotropic* model – based again on our general concept – is derived and its merits investigated. The model predictions are compared with experimental data published by Stankowski [3]. In his study, cubes of concrete were loaded in three orthogonal directions by normal compressive loading in a very complex sequence, and the respective strain response has been measured. Comparison of the outcome of our model with these complex experimental data made it possible to test and refine the hypotheses on which our model is based.

2 MATHEMATICAL MODEL

Formulation of the model starts from our general mesomechanical concept (Kafka [1]). In this concept, description of the microscopic stress- and strains-fields is simplified by representing their distribution functions (functions of space and time) by products of functions of space and functions of time. Then it was possible – with the use of a variation procedure – to characterize the shape and continuity of the substructures by scalars called ‘structural parameters’. These structural parameters were defined as integral forms in the distribution functions and it follows from their definition that they are non-negative. The user of the model works only with the structural parameters as with free parameters. A successive decrease of continuity of one of the substructures is modeled by an increase of its structural parameters. If the values of these parameters go to infinity, the corresponding substructure turns into inclusions. If all the structural parameters have zero values, we receive the Voigt’s model (homogeneous strain), for all parameters infinite we receive the Reuss’ model (homogeneous stress).

Constitutive equation (in its differential form) *of the elastic inclusions* (labeled ‘e’) expressed in deviatoric and isotropic tensor components can be written as follows:

$$\mathrm{d}e_{ij}^{e} = m^{e}\,\mathrm{d}s_{ij}^{e}\,, \qquad \mathrm{d}e^{e} = r^{e}\,\mathrm{d}\,s^{e} \tag{1}$$

where e_{ij}^{e} / s_{ij}^{e} [$d_{ij} e^{e} / d_{ij} s^{e}$] are the deviatoric [isotropic] parts of the average strain/stress tensors in the e-material constituent, $m^{e} = (1+n^{e})/E^{e}$ and $r^{e} = (1-2n^{e})/E^{e}$ mean elastic compliances for deviatoric and isotropic parts, respectively, n^{e} meaning Poisson´s ratio and E^{e} Young´s modulus of the e-material.

Constitutive equation of the matrix (labeled ‘m’) takes into account its elastic response, its plastic deformation, and changes of its elastic moduli and of its volume due to the fracturing process:

$$\begin{aligned} \mathrm{d}e_{ij}^{m} &= m^{m}\,\mathrm{d}\,s_{ij}^{m} + s_{ij}^{m}\,\mathrm{d}m^{m} + s_{ij}^{m}\,\mathrm{d}\,l^{m}, \\ \mathrm{d}e^{m} &= r^{m}\,\mathrm{d}s^{m} + s^{m}\,\mathrm{d}r^{m} + \mathrm{d}w^{m} \end{aligned} \tag{2}$$

where stresses, strains and compliances have analogous meaning as in the above presented case of the elastic material, $\mathrm{d}\,l^{m}$ is the differential plastic multiplier, $\mathrm{d}w^{m}$ is differential increment of volumetric expansion of the matrix.

Furthermore, the static and kinematic relations in a composite read:

$$v^{e}\, s_{ij}^{e} + v^{m}\, s_{ij}^{m} = \bar{s}_{ij} \tag{3}$$

$$v^{e}\, e_{ij}^{e} + v^{m}\, e_{ij}^{m} = \bar{e}_{ij} \tag{4}$$

where $v^e, v^m (=1-v^e)$ are volume fractions of the material constituents, s^e_{ij}, s^m_{ij} [e^e_{ij}, e^m_{ij}] are average values of stress [strain] tensors in the respective material constituents, and $\bar{s}_{ij}$, $\bar{e}_{ij}$ are the respective macroscopic values.

According to our general concept (Kafka [2001], the following relations further complete our isotropic mesomechanical model:

$$\mathrm{d}e'^e_{ij} = \mathrm{d}e^e_{ij} - \mathrm{d}\bar{e}_{ij}\,, \qquad \mathrm{d}e'^e = \mathrm{d}e^e - \mathrm{d}\bar{e} \tag{5}$$

$$\mathrm{d}e'^m_{ij} = \mathrm{d}e^m_{ij} - \mathrm{d}\bar{e}_{ij}\,, \qquad \mathrm{d}e'^m = \mathrm{d}e^m - \mathrm{d}\bar{e} \tag{6}$$

$$\mathrm{d}e'^e_{ij} = m^e\,\mathrm{d}s'^e_{ij}\,, \qquad \mathrm{d}e'^e = r^e\,\mathrm{d}s'^e \tag{7}$$

$$\mathrm{d}e'^m_{ij} = m^m\,\mathrm{d}s'^m_{ij} + s'^m_{ij}\,\mathrm{d}m^m + s^m_{ij}\,\mathrm{d}l^m\,, \qquad \mathrm{d}e'^m = r^m\,\mathrm{d}s'^m + s'^m\,\mathrm{d}r^m \tag{8}$$

$$s^m_{ij} - s^e_{ij} + \frac{s'^m_{ij}}{h^m} = 0\,, \qquad s^m - s^e + \frac{s'^m}{h^m_o} = 0 \tag{9}$$

where $s'^m_{ij} + d_{ij}s'^m = s'^m_{ij}$, $s'^m = \frac{1}{3}s'^m_{ii}$; h^m, h^m_o are constant 'structural parameters' for the deviatoric and isotropic parts, respectively.

Equations (5) and (6) define strain variables with primes. The stress variables with primes are implicitly defined by equations (7) and (8). The above equations are modified forms of equations appearing in the monograph, where they have numbers II.2.3 to II.2.9. In the last named equation II.2.9 the values of h^e, h^e_o must be considered infinite, as the e-substructure is formed by inclusions in our case. Therefore, these structural parameters do not appear here explicitly.

Plastic deformation, quasihomogeneous fracturing (resulting in changes of elastic compliances) and volumetric expansion are interconnected processes, and therefore, it is assumed that the terms $\mathrm{d}m^m$, $\mathrm{d}r^m$, $\mathrm{d}w^m$ can approximately be expressed as follows:

$$\mathrm{d}m^m = k_m\,\mathrm{d}l^m \tag{10}$$

$$\mathrm{d}r^m = \frac{r^m}{m^m}\mathrm{d}m^m \tag{11}$$

$$\mathrm{d}w^m = k_w\,\mathrm{d}l^m \tag{12}$$

where k_m, k_w are material constants, k_m is dimensionless, k_w has dimension MPa. Comparisons with experiments led us to the conclusion that the elastic compliances (m^m, r^m) do not change in the segments of unloading ($\bar{s}_{ij}\,\mathrm{d}\bar{s}_{ij} < 0$), and therefore, in these segments the value of k_m is considered vanishing. On the other hand, plastic deformation can proceed in these unloading segments due to redistribution of internal stresses

From the above set of equations the following macroscopic constitutive equation can be arrived at in a straightforward way:

$$d\overline{e}_{ij} = \overline{m}\, d\overline{s}_{ij} + D_{ij}\, d l^{m} , \qquad d\overline{e} = \overline{r}\, d\overline{s} + V\, d l^{m} \tag{13}$$

where

$$\overline{m} = m^{m} \frac{v^{e} m^{e} + h^{m}(v^{e} m^{e} + v^{m} m^{m})}{m^{m} h^{m} + v^{e}(v^{e} m^{m} + v^{m} m^{e})} \tag{14}$$

$$D_{ij} = v^{m} \frac{h^{m}(m^{m} - m^{e})\overline{s}_{ij} + m^{e}(v^{e} + h^{m}) s_{ij}^{m}}{m^{m} h^{m} + v^{e}(v^{e} m^{m} + v^{m} m^{e})} (1 + k_{m}) \tag{15}$$

$$\overline{r} = r^{m} \frac{v^{e} r^{e} + h_{o}^{m}(v^{e} r^{e} + v^{m} r^{m})}{r^{m} h_{o}^{m} + v^{e}(v^{e} r^{m} + v^{m} r^{e})} \tag{16}$$

$$V = v^{m} \frac{[h_{o}^{m}(r^{m} - r^{e})\overline{s} + r^{e}(v^{e} + h_{o}^{m}) s^{m}](r^{m} / m^{m}) k_{m} + (v^{e} r^{e} + h_{o}^{m} r^{m}) k_{w}}{r^{m} h_{o}^{m} + v^{e}(v^{e} r^{m} + v^{m} r^{e})} \tag{17}$$

The evolution equations for the internal variables s_{ij}^{m} and s^{m} can easily be derived from the set of equations presented above. The incremental values $dm^{m}, d l^{m}, d r^{m}, d w^{m}$ bound by eqns (10), (11), (12) are derived by differentiation of the following yield condition:

$$m^{m} s_{ij}^{m} s_{ij}^{m} - 3K^{m} r^{m} (s^{m})^{2} \le C^{m} \tag{18}$$

assumed valid for $s^{m} \le 0$, i.e. for compressive isotropic stress in the matrix. Here K^{m} is a positive dimensionless material constant, C^{m} is a constant with dimension MPa.

This is a generalization of the Huber-Hencky-Mises (HHM) criterion (that – according to Hencky's interpretation – limits the value of the deviatoric elastic energy) in two points:

(i) The value of $s_{ij}^{m} s_{ij}^{m}$ increases if the absolute value of s^{m} increases.

(ii) The value of m^{m} is not a constant (as it is the case in the HHM criterion).

By differentiation of eqn (18) it results:

$$d\tilde{m}_{m} = 2 m_{m} \frac{-m_{m} s_{ijm}\, ds_{ijm} + 3K_{m} r_{m} s_{m}\, d s_{m}}{m_{m} s_{ijm} s_{ijm} - 3K_{m} r_{m} s_{m}^{2}} , \quad dm_{m} = (d\tilde{m}_{m} + | d\tilde{m}_{m} |)/2 \tag{19}$$

and with the use of eqns (10), (11), (12) it is straightforward to receive the expression for $d l^{m}$.

For a concrete application, only this differential form is sufficient to use. The determination of the values of C^{m} is not necessary, as in our model, we assume that the inelastic deformation ($d l^{m} > 0$) starts from the beginning of the loading process and dm^{m} and $d l^{m}$ are defined as vanishing (elastic strain increment) only in such cases, in which $d\tilde{m}^{m}$ resulting from eqn (19) is negative.

This means that with our model it is not necessary to specify explicitly the elastic regions. The above criteria distinguish between elastic and inelastic increments without doing it.

The following comparisons with experimental data show that these assumptions lead to a good approximation.

3 MODELING OF A VERY COMPLEX LOADING PROCESS

In what follows the experimentally found deformation response of cubical concrete specimens – published by Stankowski [3] – is used for demonstration of the merits of our model. The investigated material was normal strength concrete obtained by mixing cement, sand, gravel and water in the relations:
Cement : Sand : Gravel = 1.00 : 3.16 : 3.19
Cement : Water = 1.2

The investigated specimens were cubes (4 in.), loaded in a special multiaxial test apparatus. The triaxial compressive loading process was stress-controlled, proceeding in the extent of strain-hardening, ended before reaching the start of strain-softening. The strain response in three orthogonal directions was measured. The specimen was loaded by uniaxial stress increments in such a way, that alternately all three possible uniaxial and biaxial stress states were achieved. These tests were specifically aimed at verification of constitutive equations. The complicated loading path is shown in Fig.1. The correspondence between the experimental and our calculated theoretical response in the direction x_3 is demonstrated in Figs. 2. In the other two directions x_1 and x_2 the correspondence is similar. The parameters of our model – shown below – have been determined by a curve-fitting procedure:

$(\bar{E})_V$ = 30 GPa; $(\bar{n})_V$ = 0.18; E^e = 52 GPa; n^e = 0.18; v^e = 0.6665; h^m = 5 e-5; h_o^m = 4.8 e-5; k_m = 20; k_w = 0.3 MPa; K^m = 0.01.

Parameters $(\bar{E})_V$ and $(\bar{n})_V$ mean macroscopic Young's modulus and Poisson's ratio of the concrete specimens in the virgin undamaged state – determined from the respective experimental stress-strain diagrams. Parameters E^e and n^e mean Young's modulus and Poisson's ratio of the aggregate, the approximate values of which have been chosen from literature. Parameter v^e means volume fraction of the aggregate. It was determined as the sum of volume fractions of both the coarse aggregate and the sand. Parameters h^m and h_o^m are 'structural parameters' for the deviatoric and isotropic tensor parts, respectively.

The agreement of our theoretical diagrams with experimental data is far better than that received by Stankowski [3] with the use of a simple theory.

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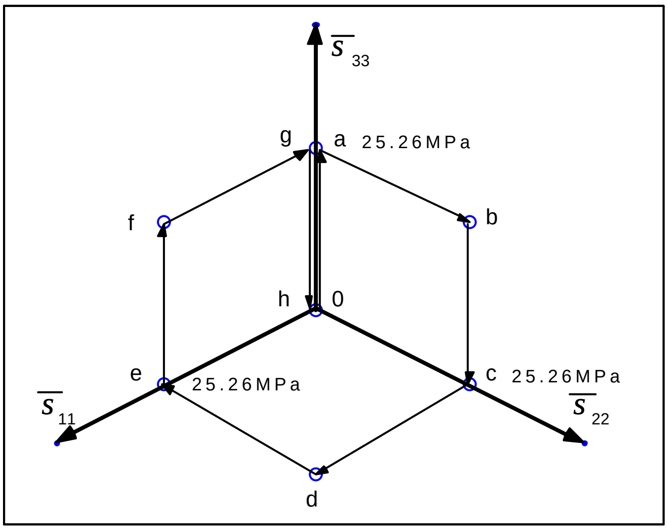

Figure 1: The complex loading path under investigation – a sequence of compressive loading processes in three orthogonal directions.

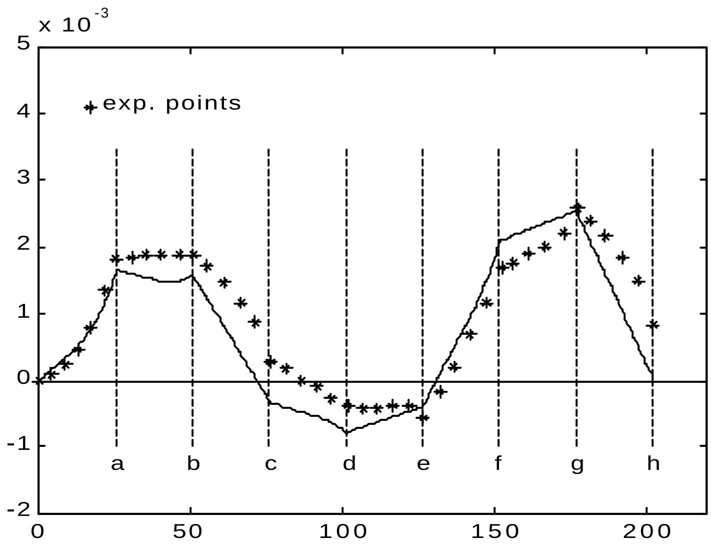

Figure 2: Comparison of the theoretical and experimental strain responses in th x_3 direction (the abscissa numbers correspond to the steps of absolute values of stress used in the calculation)