NUMERICAL ANALYSIS OF SOME EXPERIMENTAL RESULTS RELATIVE TO SIZE EFFECTS ON THE FRACTURE PARAMETERS

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ABSTRACT

In order to study the compatibility of some theoretical models referring to the size effects on the fracture parameters relative to the experimental data, some experimental results [1], [2], referring to the tensile strength σ_N, critical strain w_u, and to the fracture energy G_F of some concrete and rock specimens, respectively, were studied by means of the procedures of the numerical analysis [3]. It was pointed out that for some concrete specimen (e.g. for some dry and wet samples [1]), the obtained experimental results were not homogeneous, some individual values of w_u having the character of rough errors.

Taking into account the particular interest shown by the fractal descriptions of the size effects on the fracture parameters, the experimental results of the classical work [4] were analyzed also. We have found that the experimental results of Fig.1 [4] correspond better to a multi-fractal [a spectral (size) distribution of limited fractals] than to a unique (ideal) fractal.

The compatibility of some basic theoretical models [5], [6] of the size effects on the fracture parameters relative to the existing experimental data was studied by means of a new processing procedure [7]. The obtained results seem to indicate that the fractal character of fracture surfaces and the (classical) elasticity theory implications on the size-effects represent cooperative processes, the most accurate descriptions implying (generally) contributions of both these factors, with properly chosen specific weights.

Some new similitude expressions of the fracture parameters in terms of the specimen size are also proposed.

1. INTRODUCTION

In 1984, Mandelbrot et al. [4] claimed that the fracture surfaces of metals are fractal (self-similar) over a wide range of sizes, introducing so the possibility to describe the size effects on some fracture parameters starting from considerations of the Fractal theory, and initiated 2 new experimental methods for the fractal dimension evaluation: the "Slit Island Analysis" (SIA) and the "Fracture Profile Analysis" (FPA), based on Fourier analysis. Despite of its large impact, the hypothesis of Mandelbrot et al. [4] was somewhat restricted by the following studies: a) the papers of Underwood [8], Pande [9], Lung [10] and Huang [11] affirmed that the fracture surfaces of metals can be **approximately** considered to possess a **certain** fractal character, b) Pande [9] concluded that the slit island analysis itself was imperfect in nature as a method for measuring the fractal dimension of fractured surfaces, c) Lung [10] found that the fractal dimension was largely affected by the measuring ruler employed and postulated the concept of inherent measuring ruler, d) Williford [12] tried to explain the obtained results in terms of multi-fractals, but this explanation seemed not to be satisfactory for some experimental results [13], [14], e) Huang et al. [11] pointed out that how to determine the fractal dimension of a fractured surface has always a problem of "argument", f) Delsanto et al. [15] pointed out the considerable difference between the values of the different effective fractal dimensions, etc.

Taking into account that: a) the work [4] did not mentioned the accuracy of the used experimental data, but: b) the Fractal theory presents in the last years even some "active" technical applications, e.g. the design and technical manufacturing of some super-capacitors [16], c) the correlation coefficients indicate only the degree of proximity of the confidence domains centers relative to the studied regression curves, d) for high accuracy of the experimental data, the theoretical relations are not more compatible with the experimental data, e) the existence of several experimental data referring to the size dependence of the fracture parameters [1], [2], etc, this work will accomplish a numerical analysis of some existing experimental data, as well as of the compatibility of the multi-fractal and similitude expressions of the fracture parameters relative to the analyzed experimental results.

2. ANALYSIS OF THE EXPERIMENTAL RESULTS OF FRACTURE PARAMETERS MEASUREMENTS [1], [2]

The study of the compatibility of some theoretical relations relative to the analyzed experimental data needs the previous elimination of the rough errors, as well as the evaluation of the square mean errors affecting the studied experimental results. In this aim, starting from the zero-order evaluation of the square mean error: $s(x)=\sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i-\tilde{x}_N\right)^2}$, we evaluated the reduced errors $z_i=\frac{x_i-\tilde{x}_N}{s(x_i)}$ and we used the Chouvenet's criterion [3], eliminating the individual values x_i whose absolute values of the reduced errors were larger than the Chouvenet's threshold: $z_{thr}=\arg\varphi\left(\frac{2N-1}{4N}\right)$, where $\arg\varphi$ is the inverse function (argument) of the errors integral: $\varphi(z)=\frac{1}{\sqrt{2\pi}}\int_0^z \exp\left(-\frac{\zeta^2}{2}\right)d\zeta$.

The use of this procedure to the study of the numerical values of some fracture parameters [1], [2], pointed out that the individual values δ_u = 7.7 μ*m* and δ_u = 9.7 μm, respectively of the critical strain corresponding to some specimens of size b = 20 cm of dry and wet concrete [1], respectively, are roughly erroneous. After the elimination of these individual values, the mean values $\tilde{x}_{Nc}$ and the square mean errors $s_c(x)$ were recalculated for the compatible individual values.

The obtained results concerning the relative standard errors corresponding to the main fracture parameters for specimens of different natures (materials) and sizes were synthesized in Table 1.

Table 1. Standard errors corresponding to some existing experimental data concerning the main fracture parameters of some concrete and rock specimens, respectively

Reference	Material	Specimen size (cm)	σ_N (MPa)	w_u (μm)	G_F (*N/m*)
[2]	Concrete	2.5 … 20.0	3.9 … 9.75 %	-	10.9 … 47.6%
[1]	Dry Concrete	5.0 … 160.0	4.0 … 16.2 %	6.7 … 42.1 %	7.2 … 14.3 %
[1]	Wet Concrete	5.0 … 40.0	2.5 … 11.6 %	3.7 … 25.0 %	9.8 … 20.9 %
[1]	Red Felser Sandstone	5.0 … 160.0	3.1 … 35.4 %	2.8 … 16.3 %	3.3 … 25.5 %

One finds the presence of somewhat large errors corresponding to the fracture parameters of the concrete and rocks specimens (for some additional details, see work [7]).

3. ANALYSIS OF THE EXPERIMENTAL DATA USED TO STUDY THE FRACTAL CHARACTER OF THE FRACTURE SURFACES OF METALS [4]

Using the well-known least-squares method corresponding to a linear correlation (the "regression line" method), we have found that: a) the correlation coefficient corresponding to the regression line *logA=f(logP)* from Fig.1 of work [4] is: $r \approx 0.979$, b) the relative square mean error corresponding to all 48 experimental points indicated in Fig.1 is: $\varepsilon_{48}(\log A_{\exp} - \log A_{regr.line}) \approx 7.43$ %, while the relative square mean error for the 6 extreme (first 3 and last 3) points of Fig.1 [4] is:
$\varepsilon_{extr.6}(\log A_{\exp} - \log A_{regr.line}) \approx 11.69\%$, c) the fractal dimensional increment corresponding (according to the text of work [4]) to the difference $i_F = D' - 1$ (where D' is the slope of the regression line *logA=f(logP)* from Fig.1 [4]) is: $i_F \approx 0.596$, while the fractal dimensional increment indicated by the caption of Fig.1 [4] is: $i_F(1) \approx 1.28$.

According to the text of the work [4], the fractal dimension of the fracture surface will be: $D = D'+1 \approx$ ≈ 2.596, which represents a value somewhat unusual, in disagreement with the values indicated in the captions of the Figs. 1, 2, as well as in Fig. 3 [4]. According to the better interpretation of the experimental data obtained by means of the slit island procedure, offered by work [16] (pp.64-65), the cross-section of area A of the fractured material is not fractal, therefore this area is proportional to the square of the slit island average radius: $A \propto R^2$, while the perimeter P of the slit-island is really fractal (of dimension: $D - 1$, where D is the dimension of the fracture surface), therefore: $P \propto R^{D-1}$ and: $A \propto P^{\frac{2}{D-1}}$. It results that the slope of the *logA=f(logP)* plot from Fig.1 [4] is: $\frac{2}{D-1} = s \approx 1.596$ therefore: $D = 1 + \frac{2}{s} \approx 2.253$. This value agrees well with the values indicated in the captions of Figs.1 and 2, in Fig.3 [4], as well as with the results obtained in other similar works (e.g. [17]).

Taking into account that all 6 extreme (first 3 and last 3) points of Fig.1 [4] are located under the regression line, we assumed that a nonlinear *logA = f(logP)* expression could agree better with the experimental data reported by this figure. In order to check this hypothesis, we used the well-known gradient method procedures [18], [19] to determine the parameters of the correlation:
$\log A = c_2 \cdot (\log P)^2 + c_1 \cdot \log P + c_o$, which ensure the best fit of the experimental data from Fig.1 [4].
The accomplished calculations led to the following results: a) the values of the parabolic correlation parameters which ensure the best fit are: $c_2 \approx -0.1769$, $c_1 \approx +2.4024$, $c_o \approx -1.5595$, b) the relative square mean errors corresponding to the parabolic correlation are considerably less than those corresponding to the regression line, both for all 48 experimental data considered by work [4] and for the 6 extreme (lowest 3 and largest 3 perimeter values) data:
$\varepsilon_{parab.48}(\log A_{\exp} - \log A_{calc.}) \approx 6.823\%$, $\varepsilon_{parab.,extr.6}(\log A_{\exp} - \log A_{calc.}) \approx 8.025\%$,

c) the fractal dimensional increments (calculated by means of the expression: $i_F = D' - 1$, where D' are the corresponding slopes of the parabolic correlation) for the smallest and largest perimeters, resp. corresponding to Fig.1 [4] are: $i_{F,smallest\ perimeter} \approx 1.0485$ and: $i_{F,l\arg est\ perimeter} \approx 0.0744$, therefore the ratio of the extreme values of the fractal dimension increment is approximately 14. Using the interpretation from work [16], the extreme values of the fractal dimension of the fracture surface corresponding to Fig.1 [4] will be: $D_{\min} = 1 + \frac{2}{2.0485} \approx 1.9763$, $D_{\max} = 1 + \frac{2}{1.0744} \approx 2.8615$.

One finds so that the explanation given by Williford [12], in terms of multi-fractals, of the experimental data concerning the fracture surfaces is considerably more realistic than the initial Mandelbrot's hypothesis. We have to underline that this explanation (multi-fractals) is supported also by the results obtained by A. Carpinteri [20], [21] for concrete samples, especially.

We consider that – at least in the case of the studied work [4] – multi-fractality represents a superposition of several fractals, each one with a specific (relatively narrow) field of self-similarity. Taking into account that the calculated fractal dimensions corresponding to the different slit-islands of the studied fracture surface are in the interval $D_S \in (1.97, 2.87)$, we used Fig.1 of work [4] in order to evaluate a "spectral (size)" distribution of fractals (slit-islands) – components of the ensemble. The obtained results are indicated by Table 2.

Table 2. The "spectral (size)" distribution of the fractals involved by Fig.1 [4]

Interval of Fractal Dimensions	(1.97, 2.07]	(2.07, 2.17]	(2.17, 2.27]	(2.27, 2.37]	(2.37, 2.47]
Approximate Interval of slit-islands Perimeter, μm	(10 … … 32.13)	(32.13 … … 90.86)	(90.86 … … 218.17)	(218.7 … … 460.89)	(460.89 … … 879.62)
Number of representative points in Fig.1	**6**	**15**	**10**	**7**	**6**
Percentage of representative points (fractals) for $\Delta D_S = 0.1$	**12.5%**	**31.25%**	**20.83%**	**14.58%**	**12.5%**

One finds also that the small values of the fractal dimension correspond to slit-islands of relatively small dimensions (perimeters of the magnitude order of μm), which corresponds to fracture surfaces not too curly, and even involving some surface breaks (which could explain eventually the seldom values little less than 2 of the fractal dimension corresponding to some small parts of the fracture surfaces).

Table 2. The "spectral (size)" distribution of the fractals involved by Fig.1 [4] (following)

Interval of Fractal Dimensions	(2.47, 2.57]	(2.57, 2.67]	(2.67, 2.77]	(2.77, 2.87]
Approximate Interval of slit-islands Perimeter, μm	(879.62 … … 1545.95)	(1545.95 … … 2539.78)	(2539.78 … … 3944.53)	(3944.53 … … 5845.15)
Number of representative points in Fig.1	**2**	**1**	**0**	**1**
Percentage of representative points (fractals) for $\Delta D_S = 0.1$	**4.17%**	**2.08%**	**1.04%**	

4. STUDY OF THE COMPATIBILITY OF SOME THEORETICAL MODELS OF THE SIZE EFFECTS ON THE FRACTURE PARAMETERS, RELATIVE TO THE EXPERIMENTAL DATA

An algorithm intended to the evaluation of the coordinates x_{ti}, y_{ti} of the tangency point of a confidence ellipse: $\left(\frac{x-x_i}{s(x_i)}\right)^2+\left(\frac{y-y_i}{s(y_i)}\right)^2-2r_i\left(\frac{x-x_i}{s(x_i)}\right)\left(\frac{y-y_i}{s(y_i)}\right)=2\left(1-r_i^2\right)\ln L$ to the theoretical relation plot $Y=f(X)$ was elaborated by us in frame of work [22]. The error risk at rejection of compatibility of studied theoretical relation $Y=f(X)$ relative to the "local" data referring to the "state" i is estimated in following as:

$$q_i=\exp\left\{-\frac{1}{2\left(1-r_i^2\right)}\left[\left(\frac{x_{ti}-x_i}{s(x_i)}\right)^2+\left(\frac{y_{ti}-y_i}{s(x_i)}\right)^2-2r_i\left(\frac{x_{ti}-x_i}{s(x_i)}\right)\left(\frac{y_{ti}-y_i}{s(y_i)}\right)\right]\right\}.$$

The detailed analysis accomplished in frame of the work [7] of the compatibility of the theoretical models [5], [6] of the size effects on the fracture parameters pointed out that each models fits better certain experimental data. We completed also the set of existing relations with 3 new similitude relations which ensure an description accuracy of the same magnitude order: 2 of the Bazant's type and one of the Carpinteri's type: $G_F=G_{Fo}\sqrt{1+\frac{b}{l_{GB}}}$, $w_f=w_{fo}\left(1+\frac{b}{l_{wB}}\right)$, $w_f=\frac{w_{f\infty}}{1+l_{wC}/b}$.

The accomplished study pointed out that the fractal character of the fracture parameters and the (classical) elasticity theory seem to describe cooperative processes to the size effects on the fracture parameters, the most accurate description implying (generally) contributions of both these factors, e.g.:

$$\sigma_N(b)=\frac{\sigma_{0B}}{\sqrt{1+b/l_{\sigma B}}}+\sigma_{\infty C}\sqrt{1+\frac{l_{\sigma C}}{b}}\ .$$

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