EVOLUTION OF THE EQUILIBRIUM STATE OF THE CURVILINEAR CRACKS WITH SURFACES INTERACTING WITH FRICTION DURING THE LOADING PROCESS

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ABSTRACT
The plane elasticity problem on evolution of an equilibrium state of a curvilinear crack (flattened cavity) was considered taking into account a possible contact of its surfaces with friction under the given trajectory of complex loading. A step by step method for sequential calculation of a stress-deformation state variation for a plane with a curvilinear crack or flattened cavity during the process of external loading was developed. Numerical method for searching for unknown boundaries of the zones of the crack surfaces opening and contact, slipping and sticking was developed. Normal and shear components of the displacement jumps of the crack surfaces are determined simultaneously by the numerical solving of the system of singular integral equations. Based on developed methods the solutions of the some problems on the equilibrium state evolution for the curvilinear crack and flattened cavities of the specific geometry under loading along different trajectories were obtained.

1 INTRODUCTION

Real structures and natural massifs often being in a complex stress state which is characterized by alternation of tension, compression and shear zones. As a consequence cracks occur in the zones where the compressive and shear stresses act along with the tensile ones. In this case to analyze the fracture process one needs to take into account a possibility of the crack surfaces contact. The boundaries of the crack opening and contact zones are unknown and need to be determined during the solution of the problem. If the crack surfaces interact with friction then one also needs to search for the boundaries of the slipping and sticking zones forming within the contact zone. Hence, the crack problem becomes nonlinear with unknown boundaries opposite to traditional problems with fixed boundaries separating the zones of different boundary conditions.

An analysis of elastic system with friction performed by Goldstein and Zhitnikov [1] shows that in case generally the stress-deformation state is not determined by the final values of the applied loads but is essentially dependent on the loading history. In particular, the sticking and slipping zones occur within the zone of crack surfaces contact in case of the Coulomb friction law. The slipping can stop on a part of the crack or on the whole crack during a quasistatic variation of the external loads in dependence on the loading parameter. As a result occurring two types of sticking zones is possible, with zero (Andreev at al. [2]) or nonzero (Goldstein and Zhitnikov [1, 3], Zhitnikov and Tulinov [4]) crack surfaces displacement jump. Hence, to study correctly the limit equilibrium of the cracks with contact zones one needs to develop the methods for solving the problems on equilibrium state evolution of cracks and flattened cavities at the loading process.

In the given paper we study the 2D-elasticity problem on evolution of a equilibrium state of a smooth curvilinear crack of an arbitrary shape at the loading process taking into account Coulomb friction in the contact zone of the crack surfaces. As an example the solutions of the model problems for the case of biaxial tension-compression along the different trajectories were obtained.

A study of 3D-problems on equilibrium plane cracks and flattened cavities was performed by Goldstein and Zhitnikov [1, 3] on the basis of an analysis of the solution asymptotics near the boundaries separating the zones of opening and contact, slipping and sticking of the crack surfaces. Some general properties of the solution were established, in particular, the conditions of the slipping start, transition to the sticking state (the conditions admit the convenient geometric interpretation), the uniqueness theorem in case of the given loading trajectory. The process of slipping was analyzed (Goldstein and Zhitnikov [4]) by a transition from the initial nonlinear problem to the incremental problem linear relative to an increment of the slipping angle. 2D-elasticity problem on evolution of the equilibrium state of a straight crack and kinked cracks were studied by Zhitnikov and Tulinov [4] and Zang and Gudmundson [5], respectively. The solution of the problem for an arbitrary curvilinear crack taking into account Coulomb friction in the zone of crack surfaces contact was obtained by Savruk [6] in case of occurring sticking zone with zero displacement jump only for the given static loads.

2 STATEMENT OF THE BOUNDARY VALUE PROBLEM

Let us consider the equilibrium of a linear elastic isotropic plane with a smooth curvilinear crack (flattened cavity) elongated along a line L (Figure 1). Denote by t the parameter referenced along the line L. Assume that the external loads are depending on the loading parameter θ (θ ≥ 0). Denote by $L_f(\theta)$ a part of the curve L where the crack opening occur, and by $L_c(\theta)$, $L_a(\theta)$ the zones of crack surfaces slipping and sticking, respectively. Let $v_n^o(t)$ crack opening at θ = 0 ($v_n^o(t) \equiv 0$ for the crack-cut). Assume that v_n^o is the single-valued function and its values are small as compared to the length of the curve L. Then the boundary conditions can be blown off on the line L:

$$\begin{array}{lll} N^{\pm}(t,\theta)=0\,, & T^{\pm}(t,\theta)=0 & t\in L_f(\theta)\,; \\ T^{\pm}(t,\theta)=\mu\upsilon_\tau(t,\theta)\,N^{\pm}(t,\theta)/\left|\upsilon_\tau(t,\theta)\right|, & v_n^+-v_n^-=-\,v_n^o(t) & t\in L_c(\theta)\,; \\ v_n^+-v_n^-=-\,v_n^o(t)\,, & v_\tau^+-v_\tau^-=v_\tau^f(t,\theta) & t\in L_a(\theta)\,; \end{array} \tag{1}$$

where $N^{\pm}$ and $T^{\pm}$ are the normal and shear components of stresses at the crack surfaces, $v_n^{\pm}$ and $v_\tau^{\pm}$ are the normal and shear components of the displacement vector, v_τ^f is fixed jump of the shear displacement component in the sticking zone, $\upsilon_\tau = d(v_\tau^+ - v_\tau^-)/d\theta$, μ is the friction coefficient. Note, that occurring of sticking zones with nonzero jump of the shear displacement component $v_\tau^f(t,\theta)$ is stipulated by variation of normal and shear stresses along the crack. As a result the slipping can stop.

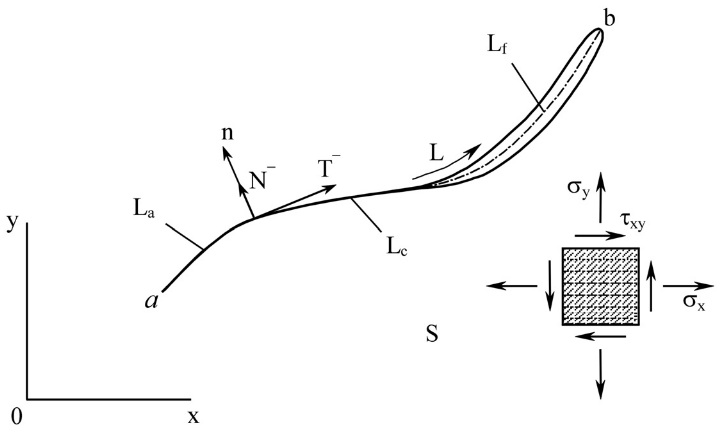

Figure 1: Elastic plane S weakened by curvilinear crack with opening L_f, slip L_c and stick L_a zones.

The following conditions need to be fulfilled in the different zones of the crack surfaces interaction

$$
\begin{aligned}
& \mathrm{v}_n^+ - \mathrm{v}_n^- \; > \; -\,\mathrm{v}_n^o(t)\,, && t \in \mathrm{L}_f(\theta)\,; \\
& \mathrm{N}^{\pm}(t,\theta) < 0, && t \in \mathrm{L}_c(\theta) \cup \mathrm{L}_a(\theta)\,; \\
& \mu \, \left| \mathrm{N}^{\pm}(t,\theta) \right| \; > \left| \mathrm{T}^{\pm}(t,\theta) \right|, && t \in \mathrm{L}_a(\theta)\,.
\end{aligned}
\qquad (2)
$$

These conditions enable to determine the current state of the crack surfaces at the given value of the loading parameter θ.

3 METHODS AND ALGORITHMS OF THE PROBLEM SOLVING

The methods of the Kolosov-Muskhelishvili [7] are used to obtain the formulae relating the stresses and the derivative of the displacement jump along the crack (see also Savruk [6]). By incorporating these relations we satisfy the boundary conditions (eqns (1)) and obtain the singular integral equations (SIE) which describe the equilibrium of the crack with contact zones in an elastic plane (see Andreev at al. [2]). The problem is solved in increments since the fixed jump of the shear displacement component in the sticking zone is unknown. Using the method of mechanical quadrature (Chawla and Ramacrishnan [8], Erdogan at al. [9]) the SIE are reduced to the system of the linear algebraic equations relative to the values of the unknown function (the derivative of the displacement jump) in the discrete set of the points. An iteration process [2] based on conditions (eqns (2)) is used to search for the unknown zones. The algorithm implies the following steps: 1. choosing an initial location of the unknown boundaries (e.g., substituting the stresses at the crack line in the continuum half plane in eqns (2)); 2. solving the SIE according to eqns (1) and using the chosen locations of the unknown boundaries; 3. computing the displacement jumps along the crack and stresses at the crack surfaces; 4. substitution of these values in eqns (2) and searching for the new locations of the unknown boundaries. Then the iteration process is repeated from step two. The iteration process is stopped if the locations of the unknown boundaries are no longer changed with a given accuracy. The numerical methods and algorithms are described in more detail in paper Andreev at al. [10].

4 NUMERICAL RESULTS

As an example of the methods application let us consider the problem on the crack-cut in the form of an elliptic arc under the bi-axial tension – compression and different loading trajectories $q = q(\theta) \geqslant 0$, $p = p(\theta) \leqslant 0$ (Figure 2).

The calculated values of the stress intensity factors (normalized by $q_{\max}\sqrt{l}$), relative sizes of the opening S_{fr} and sticking zones S_{st} (S_{cr} is the crack length), as well as the average value of the fixed jump of the shear displacements component in the sticking zone $<\mathrm{v}_\tau^f>$ (normalized by $l(1+\kappa)q_{\max}/G$, where G is the shear modulus, $\kappa = 3 - 4\nu$ for plane strain and $\kappa = (3-\nu)/(1+\nu)$ for plane stress, ν is Poisson ratio) are given in Table 1 for different loading trajectories and $\varepsilon = 2$, $\mu = 0.4$, $|p_{max} / q_{max}| = 5$. The appropriate values calculated for the same problem without the loading history are given in first column for comparison.

It is seen that the proportional loading (trajectory ODC) leads to the solution which coinsides with the solution obtained by nonincremental approach (i.e., without taking into account the loading history). The locations of the sticking, slipping and opening zones are not changed along trajectory OAC (OJEIC) on part OA (OJ), the displacements jump are only changed in the slipping and opening zones. This effect is related to a possibility to represent the stress intensity factors at the boundaries of the slipping and opening zones, slipping and sticking zones under one-parameter loading in the for $k_i^{\pm} = p(\theta) f(t_i^{\pm}, F_{geom})$ (i = 1, 2). Here, $t_i^{\pm}$ is the boundaries of the characteristic zones, F_{geom} is the function only depending on the crack geometry and orientation relative to the external load. The location of the characteristic zones boundaries is not changed since it is determined by the equation $f(t_i^{\pm}, F_{geom}) = 0$. Thus the jump of the shear displacement company in the sticking zone becomes equal to zero at attaining point A (J) (Figure 2).

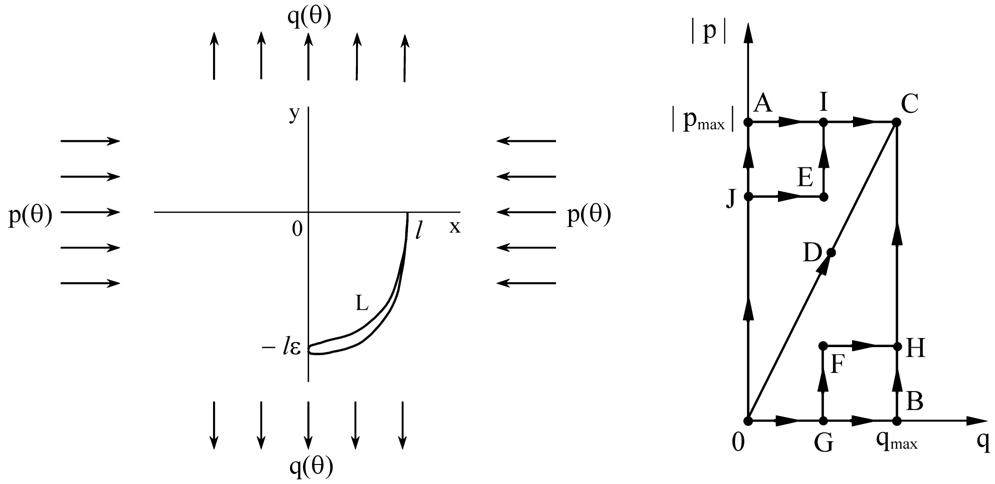

Figure 2: Crack in the form of an elliptic arc under the bi-axial tension – compression and different loading trajectories q = q (θ) ⩾ 0, p = p (θ)⩽ 0 (JE = EI = FH = GF).

Table 1: Calculated results for the crack – cut .

	Non	ODC	OAC	OJEIC	OGFHC	OBC
$k_1^- / q_{\max}\sqrt{l}$	1.0455	1.0455	1.0455	1.0455	1.0976	1.1320
$-k_2^- / q_{\max}\sqrt{l}$	1.0792	1.0792	1.0792	1.0792	1.1194	1.1445
$k_2^+ / q_{\max}\sqrt{l}$	0	0	0	0	0.1205	0.2411
S_{fr} / S_{cr}	0.1699	0.1699	0.1699	0.1699	0.1729	0.1759
S_{st} / S_{cr}	0.2724	0.2724	0.2724	0.2724	0.3848	0.3933
$\dfrac{< v_\tau^f >}{l(1+\kappa) q_{\max} / G}$	0	0	0	0	0.0586	0.1014

Part AC (JE) of trajectory OAC (OJEIC) represents the part of active loading since the decreasing of the sticking zone size occurs at this part of the trajectory. As a consequence the sticking zones with nonzero displacement jump do not occur under the loading along trajectory OAC. As a result the solution coincides with the nonincremental one. On the other hand the nonzero displacement jump occurs in the increasing sticking zone at loading along part EI of trajectory OJEIC. However, subsequent active loading (part IC) leads to decreasing of this jump and to the similar (nonincremental) solution. Hence, the obtained results enable one to conclude that the trajectories of monotone loading (without unloading) located in triangle OAC lead to the solutions coinciding with ones obtained without taking into account the loading trajectory.

A nonzero jump of the shear displacement component occurs under the loading along trajectories OGFHC and OBC. The stress intensity factor k_2^+ at the right crack tip is nonzero in the end trajectory point, while the loading region adjoins this tip. Hence, in the case crack growth can occur in spite of existence a sticking zero of crack surfaces. Note, that the transition from trajectory OAC (compression tension) to trajectory OBC (tension compression) leads to increasing of the stress intensity factors at the left crack tip (about 10%).

Hence, a class of loading trajectories which provide the same solution as obtained without taking into account the loading trajectory was separated. On the other hand the solutions for trajectories OGFHC and OBC, noticeably differ from the solutions obtained without taking into account loading trajectories. Hence, generally the numerical solution need to be performed by the suggested incremental method.

To study an influence of the initial crack opening on its limit equilibrium let us consider flattened cavities of the following two shapes: with smoothly joining surfaces ($\mathrm{v}_\mathrm{n}^\mathrm{o}(\xi) = \mathrm{v}_{\max}(1-\xi^2)^{3/2}$), and with elliptical opening near the tips ($\mathrm{v}_\mathrm{n}^\mathrm{o}(\xi) = \mathrm{v}_{\max}(1-\xi^2)^{1/2}$). Here, $\mathrm{v}_{\max}$ is the maximal cavity opening which need to be chosen such that the initial cavity opening will be small as compared with its length. The calculations were performed at $\mathrm{v}_{\max} = 0.1\,l$ (see Figure 2).

The results of solving the problems on flatten cavities are given in Tables 2 and 3. The cavities elongated along the elliptic arc were considered (Figure 2). Other parameters of calculations were the same as in the aforementioned examples.

Table 2: Calculated results for the flattened cavity with smoothly joining surfaces.

	ODC	OAC	OJEIC	OGFHC	OBC
$k_1^- / q_{\max}\sqrt{l}$	1.0615	1.0593	1.0593	1.1156	1.1479
$-k_2^- / q_{\max}\sqrt{l}$	1.1804	1.1786	1.1786	1.2211	1.2444
$k_2^+ / q_{\max}\sqrt{l}$	0	0	0	0.1202	0.2408
S_{fr} / S_{cr}	0.1759	0.1759	0.1759	0.1789	0.1820
S_{st} / S_{cr}	0.2724	0.2686	0.2686	0.3890	0.3933
$\dfrac{< \mathrm{v}_\tau^\mathrm{f} >}{l(1+\kappa)q_{\max}/G}$	0.0025	0.0009	0.0009	0.0663	0.1071

Table 3: Calculated results for flattened cavity with elliptical opening near the tips.

	ODC	OAC	OJEIC	OGFHC	OBC
$k_1^- / q_{\max}\sqrt{l}$	1.0568	1.0527	1.0527	1.1105	1.1427
$-k_2^- / q_{\max}\sqrt{l}$	1.2031	1.2003	1.2003	1.2417	1.2654
$k_2^+ / q_{\max}\sqrt{l}$	0.0075	0.0092	0.0092	0.1093	0.2288
S_{fr} / S_{cr}	0.1850	0.1850	0.1850	0.1881	0.1913
S_{st} / S_{cr}	0.2761	0.2686	0.2686	0.3890	0.3975
$\dfrac{< \mathrm{v}_\tau^\mathrm{f} >}{l(1+\kappa)q_{\max}/G}$	0.0073	0.0020	0.0020	0.0673	0.1088

In all cases, in particular under proportional loading (trajectory ODC, Tables 2 and 3) the sticking zones with nonzero jump of the shear displacement component occur. The trajectories OAC and OJEIC are equivalent, since the active loading along part IC leads to decreasing the sticking zone such that a difference in the fixed displacements jump in the end point of the trajectory caused by the difference between trajectories JAI and JEI disappears. Occurring a sticking zone near the right crack tip is characteristic for the cavities with smoothly joining surfaces. These zones occur at the initial part of the loading trajectory (with the exception of trajectories OGFHC and OBC). Occurring the sticking zone provides the equality of the stress intensity factor k_2^+ to zero. Note, that loading along trajectories OGFHC and OBC (tension-compression) is accompanied by essential increasing the sticking zone size and value of the shear displacement component within this zone. As a result the stress intensity factors are noticeably changed.

Hence, one can conclude that the stress-deformation state providing attaining the crack limit equilibrium is essentially influenced by the shape of the initial crack opening. Further, the incremental method enables to describe the evolution of the stress-deformation state near the flatten cavities since this method is convenient for computing the occurring and evolution the sticking zones with a nonzero jump of the shear displacement component.

The author acknowledges the financial support of President of Russia (grants MK-878.2003.01 and NS-1849.2003.01).

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