

SHEAR CAPABILITY OF REINFORCED CONCRETE BEAMS WITHOUT STIRRUPS PREDICTED USING A FRACTURE MECHANICAL APPROACH

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ABSTRACT

A host of experiment investigations on shear failure of reinforced concrete beams without stirrups show the fact that the shear failure of such beams is a very complex process and exhibits significant size effect. Therefore, how to acquire physically sound analytical model that enables to yield satisfactory results for all cases of the shear failure of the reinforced concrete beams is at all time the strong interest of many academic researchers and practical engineering designers.

As the result, in this work, a state-of-the-art review is presented on both the various empirical approaches that are used in the current design codes in the worldwide and some existing fracture mechanics-based approaches for predicting the shear bearing capability of reinforced concrete beams without stirrups, among which, more care is put to an analytical shear model proposed by Gasteble and May in terms of mode I fracture energy of concrete materials. Then, based on this shear failure formula, together with either mode II fracture toughness K_{IIc} or fracture energy G_{IIF} of concrete materials that are determined by our new developed analytical solution and preferable loading arrangements, a new analytical formula is developed. This formula accurately shows the size effect in shear fracture, the contributions of the shear span-depth ratio, the reinforcement ratio and the concrete quality to shear strength. Finally, an estimate of the shear bearing capability of reinforced concrete beams without stirrups from this new formula is evaluated and compared to that calculated by means of the Gasteble and May' model, CEB-FIP Model Code and the ACI 318-89 Code respectively. The good agreement between them further confirmed that fracture mechanics can be applied to the assessment of shear bearing capability of reinforced concrete beams without stirrups. It also provides a new knowledge to derive analytical formula for shear fracture problems in reinforced concrete members. However, the extensive fracture experiments for the measurement of pure mode II fracture parameters are expected to provide sufficient information for the efforts on applying fracture mechanics to shear fracture of reinforced concrete beams without stirrups.

1 INTRODUCTION

In last two decades, the experimental investigations focused on shear failure of reinforced concrete beams have been intensively performed by many researchers for capturing the failure mechanisms

[1-10]. As the consequence, various shear failure models were proposed to attempt acquiring adequate formulae to predict the shear strength of the reinforced concrete beams with good accuracy for designing engineers. However, the formulae that are used in various design codes are empirically yet [11-16] inasmuch as no physically sound analytical model that enables to yield satisfactory results for all cases of the shear failure of the reinforced concrete beams exists.

Recent years, some researchers have put their attentions on application of fracture mechanics to the shear failure of the reinforced concrete beams without stirrups for developing an analytical model with soundly physical fundament [7-10]. This is based on such a fact revealed in the intensively experimental investigations that the shear failure of a beam is triggered off by a series of fracture processes occurred in the beam. Therefore, the primary purpose of our study is to attempt to obtain a physically sound analytical model from the standpoint of fracture mechanics such that the shear bearing capability of reinforced concrete beams without stirrups is accurately predicted as well as its failure mechanism is well interpreted.

2 EMPIRICAL APPROACHES

According to substantially experimental observations carried out by many researchers, several models with somehow physical meaning, like the modified compression field theory (MCFT)[3] and truss models with concrete ties [5], were proposed; nevertheless, the corresponding calculation formulae are empirically constructed using regression analysis of a lot more experimental data. In fact, due to lack of the physically analytical model, various calculation formulae that are coded in the current design codes in the worldwide are empirically formed using regression procedures. Currently, ACI Subcommittee 445-F is making a new proposal [17] to present an empirical formula to predict the shear capability of the reinforced concrete beams without stirrups using a regression analysis of the experimental data based on the Evaluation Shear Database (ESDB) developed by the subcommittee. Herein, many empirical formulae in the current design codes in the worldwide such as the Canadian Standards Association Model Code (CSA Code) [14], the British design Code [12], the CEB-FIP Model Code [13], ACI 318-89 Code [11] are summarized.

3 FRACTURE APPROACHES

It was generally accepted that the shear failure of reinforced concrete beams is intimately associated with a series of fracture process and behaves marked size effect. Hitherto, some existing models with somehow physical meaning based on such intensive experiments on the shear failure of the reinforced concrete beams without stirrups only capture a little of interpretation for the shear mechanisms both in physics and mathematics. These studies mainly include Bazant's size effect formula [2][4][7], Gustafsson and Hillerborg' numerical stimulation using fictitious crack model [6], Jenq and Shah's diagonal shear crack analysis based on two-parameter fracture model [8], Karihaloo's modification [9] for Jenq and Shah's model and Gastebled and May [10]'s analytical model by a fracture energy approach. For the sake of convenience and clarity of deducing new

analytical formula in sequence part, in particular, the more detail presentation of Gasteble and May's shear fracture principle and corresponding calculation expressions is given.

4 A NEW ANALYTICAL FORMULA BASED ON GASTEBLED AND MAY THEORY

In terms of Gasteble and May' model and extensively experiment observations, the equation to predict the shear bearing capability of reinforced concrete beams without stirrups is anew obtained using fracture toughness K_{IIc} as follows

$$V_c = \frac{0.446}{\sqrt{H}} \sqrt{\frac{E_s}{E_c}} \left(\frac{H}{a_s} \right)^{1/3} \rho^{1/6} (1 - \sqrt{\rho})^{2/3} K_{IIc} bH$$

The corresponding comparison results are shown in Table 1.

Table 1 The shear bearing capability calculated using several formulae

Formulae used	ACI 318-89	CEB-FIP	Formula by Gasteble and May	Formula by the new developed
Shear stresses (MPa)	0.913	0.650	0.776	0.745

5 CONCLUSIONS

(1) The shear failure of reinforced beam without stirrups is a very complex fracture process and exhibits the noticed size effect. Hence, the lack of valid physically sound analytical model for the prediction of shear bearing capability of reinforced concrete beams without stirrups need more researches on application of fracture mechanics model to predict shear bearing capability of reinforced concrete members.

(2) The new analytical formula can accurately estimate the size effect in shear fracture, the contributions of the shear span-depth ratio, the reinforcement ratio and the concrete quality to shear strength and reasonably interpret the failure mechanism of reinforced concrete beams without stirrups.

(3) It is further verified that fracture mechanics can be utilized as a valuable knowledge to analyze the shear failure problems of reinforced concrete members without stirrups.

(4) It is rather important to perform more pure mode II fracture tests for various concrete materials to know both the mode II fracture toughness K_{IIc} and mode II fracture energy G_{IIIF} of concrete materials with different strength scales so as to better understand the shear failure process of reinforced concrete members.

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