MODELING FATIGUE CRACK GROWTH IN CRYSTALLINE SOLIDS WITH DISCRETE DISLOCATION PLASTICITY

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ABSTRACT

Analyses of crack growth under cyclic loading conditions are discussed where plastic flow arises from the motion of large numbers of discrete dislocations and the fracture properties are embedded in a cohesive surface constitutive relation. The formulation is the same as used to analyse crack growth under monotonic loading conditions, differing only in the remote loading being a cyclic function of time. Fatigue, i.e. crack growth in cyclic loading at a driving force for which the crack would have arrested under monotonic loading, emerges in the simulations as a consequence of the evolution of internal stresses associated with the irreversibility of the dislocation motion. A fatigue threshold, Paris law behaviour, striations, the accelerated growth of short cracks and the scaling with material properties are outcomes of the calculations. Results for single crystals and polycrystals will be discussed.

1 INTRODUCTION

Fatigue crack growth occurs even when the driving force for crack growth is much smaller than what is needed for the same crack to grow under monotonic loading conditions. Dissipative mechanisms are key for fatigue. As a consequence, both the plastic flow mechanism and the process of material separation play important roles in determining the fatigue behavior.

Consider a cracked solid subject to loading corresponding to a stress intensity cycling between K_{min} and K_{max}. Fatigue crack growth occurs even when K_{max} is much smaller than the value of K needed for the same crack to grow under monotonic loading conditions. Typically, there is a threshold value of $\Delta K_{\mathrm{I}} = K_{\mathrm{max}} - K_{\mathrm{min}}$ below which cracks do not grow at a detectable rate. Above this threshold value, in the regime where the amount of crack growth per cycle, da/dN, is on the order of a few lattice spacings, there is a steep increase in da/dN with ΔK_{I}. For larger values of ΔK_{I}, the increase in da/dN becomes less steep and the Paris law regime [1] is entered where $da/dN \propto (\Delta K_{\mathrm{I}})^m$ (c.f. [2]).

Dislocations in crystalline solids play a dual role in the fracture process under monotonic loading, as noted by Cleveringa *et al.* [3]. On the one hand, plastic flow caused by the motion of dislocations delays crack initiation and increases the resistance to crack growth. On the other hand, it is the local stress concentrations associated with discrete dislocations in the vicinity of the crack tip that leads to stress levels of the magnitude of the cohesive strength, causing the crack to propagate. This dual role is key for fatigue in crystalline metals – the dissipation from dislocation motion provides the irreversibility, while the high stresses associated with the dislocation structures that form near the crack tip precipitate crack growth.

A series of analyses of crack growth in single crystals under cyclic loading conditions have been carried out in Deshpande *et al.* [4, 5, 6, 7]. Results for polycrstals are presented in Balint *et al.* [8]. Plastic flow arises from the motion of large numbers of discrete dislocations, which are treated as singularities in an isotropic elastic solid. The material model is independent of the presence of a crack. The fracture properties of the material are embedded in a cohesive surface constitutive relation [9] so that crack initiation and crack growth are driven by stress as well as deformation. A key aspect of the formulation is that the plastic stress-strain response and the evolution of the dislocation structure, as well as crack initiation and growth are outcomes of the solution of the boundary value problem.

Furthermore, the only distinction between an analysis of monotonic crack growth and fatigue crack growth is that in fatigue the remote loading is specified to be an oscillating function of time.

2 THEORY

A brief overview of the theoretical framework is presented; background and further descriptions are given in [4, 5, 6, 7] and references cited therein. Plane-strain conditions are assumed to hold. Initially, the crystal is assumed to be free of mobile dislocations, but to contain a random distribution of dislocation sources and point obstacles. The rules for dislocation nucleation and motion are based on those proposed in [10] and use the Peach-Koehler force as the driving force. The sources mimic Frank-Read sources and generate a dislocation dipole of opposite signed edge dislocations when the magnitude of the Peach-Koehler force exceeds a critical value for a specified period of time. The obstacles pin dislocations and release them once the Peach-Koehler force attains a specified obstacle strength. Annihilation of two dislocations with opposite Burger's vectors occurs when they approach each other within a critical annihilation distance. Dislocation motion is assumed to occur only by glide with no cross slip. The magnitude of the glide velocity of a dislocation is taken to be linearly related to the Peach-Koehler force. There is no special dislocation nucleation from the crack tip.

In [4, 5, 7], loading is prescribed in terms of displacements corresponding to the isotropic elastic mode I singular field remote from the crack tip, while in [6] remote uniaxial tension is imposed. There is a single cohesive surface [9] that lies in front of the initial crack. At each time step, an increment of the remote loading (the mode I stress intensity factor increment $\dot{K}_I \Delta t$ for small scale yielding) is prescribed. At the current instant, the stress and strain state of the body is known, and the Peach-Koehler forces on all dislocations can be calculated. On the basis of these forces the dislocation structure is updated, which involves the motion of dislocations, the generation of new dislocations, their mutual annihilation, their pinning at obstacles, and their exit into the open crack. After this, the new stress and strain state can be determined. The field quantities, i.e. the displacement u_i, the strain ε_{ij} and the stress σ_{ij} are determined using superposition [11].

Both reversible and irreversible cohesive traction-displacement relations are used. As the cohesive surface ahead of the crack separates, the magnitude of the traction increases, reaches a maximum and then approaches zero with increasing separation. In a vacuum, there is no oxidation of the newly formed surface and it is expected that this relation is followed in a reversible manner. When the newly formed surfaces oxidize, the cohesive relation will not be followed in a reversible manner. The effect of the formation of the oxide layer and the subsequent surface contact during unloading is modeled by specifying unloading from and reloading towards the monotonic cohesive law to occur according to a linear incremental unloading relation.

A reference stress intensity factor K_0 is introduced that provides a convenient normalization for the imposed stress intensity factor. It is defined in terms of the work of separation of the cohesive surface, ϕ_n, by

$$K_0 = \sqrt{\frac{E\phi_n}{1-\nu^2}}\,, \tag{1}$$

where E and ν are the Young's modulus and Poisson's ratio of the material. Crack growth in an elastic solid with the given cohesive properties takes place at $K_I/K_0 = 1$.

3 RESULTS

In the small scale yielding calculations in [4, 5, 7], the applied stress intensity is varied between K_{min} and K_{max} with a rather high loading rate to shorten the computation time. Fatigue threshold results from [4] are summarized in Fig. 1. Crack growth under cyclic loading occurs if and only if (i) the cyclic amplitude ΔK_I exceeds a critical value ΔK^*_{th}, and (ii) the maximum stress intensity K_{max} exceeds a critical value K^*_{max}. With a reversible cohesive consitutive relation, which models conditions in a vacuum, this can be rationalized as follows: For sufficiently low K_{max}, no dislocations are generated and the system is elastic. Therefore, since as discussed in [4] some irreversibility is required for fatigue to occur, with a reversible cohesive law, K_{max} must exceed some minimum K_{max} denoted by K^*_{max}. For $K_{\mathrm{max}} >> K^*_{\mathrm{max}}$, interactions within the now dense dislocation structure act to retard dislocation motion. Accordingly, a minimum

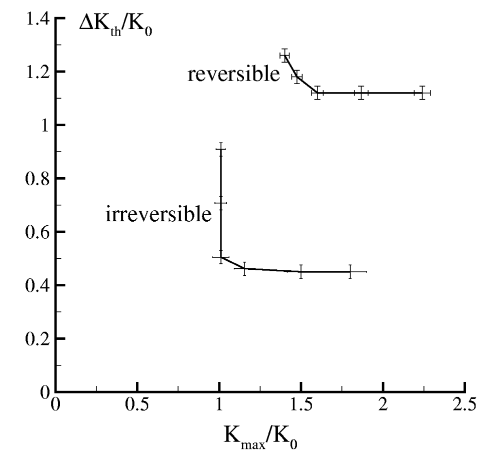

Figure 1: Discrete dislocation predictions for the variation of ΔK_{th} with load ratio, R [4].

cyclic stress intensity factor range ΔK_I is needed to induce dislocation motion during unloading and reloading. Thus, in this regime, a critical $\Delta K_{\mathrm{th}}^{*}$ is needed. For an irreversible cohesive relation, which models conditions in an oxidizing environment, contact plays an important role [4].

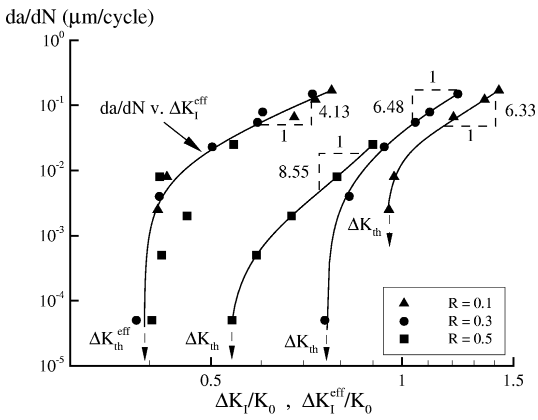

Figure 2: The cyclic crack growth rate da/dN versus $\Delta K_{\mathrm{I}}/K_0$ and $\Delta K_{\mathrm{I}}^{\mathrm{eff}}/K_0$ for an interface crack [5].

The form of the $\log(da/dN)$ versus $\log(\Delta K_{\mathrm{I}})$ curve seen experimentally, with a threshold and a Paris law regime, is captured in Fig. 2. The effective stress intensity range ΔK^{eff} responsible for crack growth is defined by $K_{\mathrm{max}} - K_{\mathrm{op}}$ where K_{op} is the stress intensity factor at which the crack faces first separate at the current location of the crack upon reloading. The effect of crack closure is more pronounced at the lower values of ΔK_{I} so that $\Delta K_{\mathrm{th}}^{\mathrm{eff}}$ is much less than ΔK_{th}.

The results of fatigue threshold calculations carried out in [6] for geometrically similar edge cracked specimens are shown in Fig. 3 which are identified by the edge crack length, a. For crack lengths less than 300 μm, the deviation from ΔK-governed fatigue increases with decreasing crack size, with the fatigue threshold for smaller cracks tending to be $\Delta\sigma$ rather than ΔK-governed. Thus, crack growth under cyclic loading conditions occurs even when K_{max} is less than K_0 which is the stress intensity at which the crack would grow in an elastic solid.

At least in the near-threshold and Paris law regimes, fatigue crack growth rates are relatively independent of the yield strength of the material but scale with the elastic modulus. This rather surprising observation has been borne out in experimental studies on a variety of metallic alloys. Results for $\Delta K_{\mathrm{th}}^{\mathrm{eff}}$ from [7] are shown in Fig. 4. Consistent with experimental data, the calculations show that $\Delta K_{\mathrm{th}}^{\mathrm{eff}}/E$ is rather independent of the normalized strength σ_{Y}/E

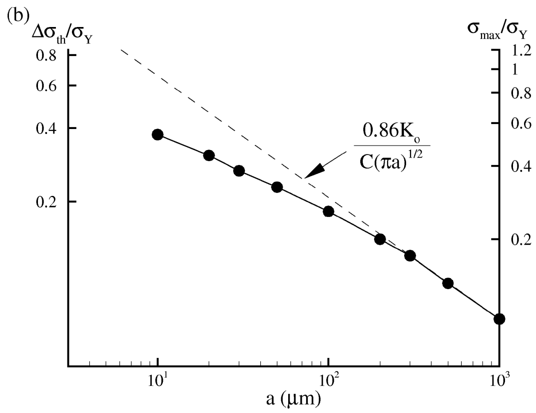

Figure 3: The fatigue threshold, $\Delta\sigma_{th}/\sigma_Y$, versus crack length *a* (corresponding values of σ_{max}/σ_Y are shown on the right axis) [6].

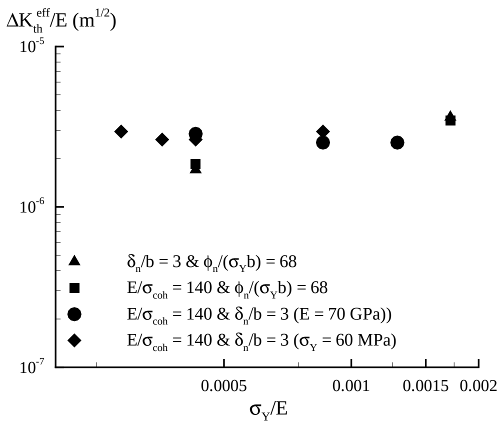

Figure 4: Discrete dislocation predictions showing that similar to experiments, ΔK_{th}^{eff} is relatively independent of the yield strength σ_Y and scales approximately linearly with Young's modulus, E [7].

over approximately a decade. gThe results in [7] show that the observed relative lack of dependence of the fatigue threshold in ductile metals on yield strength emerges from a cohesive fracture model with the stress concentration arising from near crack tip organized dislocation structures.

Results from Balint *et al.* [8] for polycrystals will also be discussed.

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