METHOD FOR CHARACTERIZING THE FRACTURE SURFACE USING A TWO-DIMENSIONAL LOCAL HURST EXPONENT, AND ITS APPLICATION TO QUANTITATIVE EVALUATION OF STRETCHED ZONE WIDTH

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ABSTRACT

Fractal analysis has been widely used to characterize the fracture surface. It has been recognized that the local Hurst exponent, which is based on the concept of the self-affine fractal, is useful to detect the transition point of the fracture surface. In order to calculate the local Hurst exponent, a high-resolution profile is needed. The measurement of this profile, however, requires considerable time and effort, which thus makes it difficult to calculate all the profiles of the fracture surface and arrive at a detailed evaluation of the features of the fracture surface. In the present study, we propose a new method of calculating the two-dimensional local Hurst exponent, which can be used to evaluate the features of the fracture surface using the local Hurst exponent. To investigate the validity of the two-dimensional local Hurst exponent, our calculations were applied to grayscale images in which the stretched zone was observed and the width of the stretched zone (SZW_c) was measured. The SZW_c calculated by the two-dimensional local Hurst exponent and that detected by human observation were found to be in good agreement. We thus conclude that the two-dimensional local Hurst exponent is a useful means of detecting the transition point of the fracture surface.

1 INTRODUCTION

Fractography is an important field of study which investigates the causes of fracture accident. In recent years, numerical analysis has become essential in the estimation of stress from the fracture surface, since the results obtained by numerical analysis are objective and thus more reliable than results obtained by the conventional method, which depends on a human observer.

Fractal analysis has been widely used for the numerical analysis of the fracture surface.[1–3] Early studies made use of the fractal dimension (F_d), which was calculated from the image or topological data of the fracture surface. Mandelbrot,[2] for example, reported on the relationship between absorbed energy and the F_d.

In this paper, the two-dimensional local Hurst exponent, which is based on the concept of the self-affine fractal, is proposed to analyze the transition region using an image of the fracture surface. To investigate the validity of this parameter, it was applied to the estimation of the width of the stretched zone (SZW_c), which is related to a crack tip opening displacement (CTOD) and a critical J-integral (J_{Ic}).

2 METHOD FOR CALCULATING THE TWO-DIMENSIONAL LOCAL HURST EXPONENT

Unlike isotropic fractal geometry, self-affine fractal geometry includes the feature of self-similarity on condition that the scale is changed anisotropically. Specifically, assuming that $h(x, y)$ is a self-affine fractal, $h(x, y)$ is satisfied by the Equation (1) below..

$$h(x, y) \cong \lambda^{-H} h(\lambda x, \lambda y) \qquad (1)$$

Here, H is the Hurst exponent, which has a value from 0 to 1. Where $H = 1$, $h(x, y)$ is a flat plate. A fracture surface becomes more complicated, in which case H approaches 0. The method for calculating the two-dimensional local Hurst exponent is indicated in the following steps.

The following equation satisfies $h(x, y)$, which is a gray level image in the present paper.

$$h_{x_0,y_0}(x, y) = h(x + x_0, y + y_0) - h(x_0, y_0) \tag{2}$$

$$h_{x_0,y_0}(x, y) \cong \lambda^{-H} h_{x_0,y_0}(\lambda x, \lambda y) \tag{3}$$

In Eqn.(3), x_0, y_0 are the coordinates of pixel in the image.

The two-dimensional wavelet transform (WT), which is an effective method of investigating the local features and similarity of signal and image,[4] is applied to calculate the local Hurst exponent. The two-dimensional WT is defined by the following equation:

$$W(h(x, y), a, b_x, b_y) = \frac{1}{a} \int \int \overline{\psi \left(\frac{x - b_x}{a}, \frac{x - b_y}{a} \right)} h(x, y) dx dy \tag{4}$$

where ψ is the mother wavelet, and the overline indicates the complex conjugate. Moreover, (b_x, b_y) and a indicate the transition parameter and the scale parameter, respectively. Therefore, similarity and local features are respectively described by a and (b_x, b_y).

Next, we substitute Eqn.(3) for Eqn. (4) and the admissible condition of the mother wavelet:

$$W(h_{x_0,y_0}(x, y), \lambda a, \lambda b_x, \lambda b_y) = W(h(x, y), \lambda a, \lambda b_x + x_0, \lambda b_y + y_0) \tag{5}$$

Finally, we obtain the following:

$$W(h_{x_0,y_0}(x, y), a, b_x, b_y) = \lambda^{-H-1} W(h(x, y), \lambda a, \lambda b_x + x_0, \lambda b_y + y_0) \tag{6}$$

Therefore, the relationship between a and W at the neighborhood of (x_0, y_0) is expressed as the following equation:

$$W(h(x, y), a, x_0, y_0) \propto a^{H+1} \tag{7}$$

Specifically, the two-dimensional local Hurst exponent is obtained from the slope of the plot of a and W on the legalistic graph. Furthermore, the distribution of the two-dimensional local Hurst exponent is obtained by substituting the coordinate of pixel in the image for (x_0, y_0).

However, an almost linear relationship between a and W is not obtained. To solve this problem, Simonsen proposes the Averageed Wavelet Coefficient (AWC) method.[5] In the present paper, we expand on this method for two-dimensional WT as the following equation:

$$\overline{|W(h(x, y), a, x_0, y_0)|} = \frac{1}{w^2} \sum_{y=y_0-\frac{1}{2}w}^{y=y_0+\frac{1}{2}w} \sum_{x=x_0-\frac{1}{2}w}^{x=x_0+\frac{1}{2}w} |W(h(x, y), a, x, y)| \tag{8}$$

Finally, the two-dimensional local Hurst exponent is calculated from a linear relationship between $|W|$and a. A feature of an area whose width is w and whose center is (x_0, y_0) is contained in H, which is therefore expressed as $H(x_0, y_0)$.

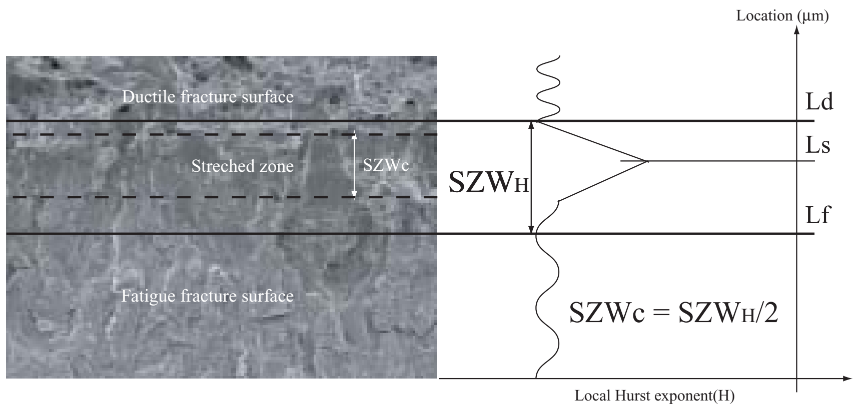

Figure 1: Illustration of SZW_c measurement using two-dimensional Hurst expoenents.

3 EXPERIMENTAL PROCEDURE

3.1 Method for detecting the stretched zone

In an elastic-plastic fracture toughness test, a stretched zone is observed on the fracture surface between a fatigue fracture surface and a ductile fracture surface. The stretched zone is a flat region compared to the fatigue and ductile fracture surfaces. Thus, the two-dimensional local Hurst exponent of a stretched zone is larger than that of the other regions (Figure (1)).

In order to detect the stretched zone by the two-dimensional local Hurst exponent, we first plot the relationship between the location, which is parallel to the crack propagation direction, and the two-dimensional local Hurst exponent, that is, the location where the two-dimensional local Hurst exponent is maximum and is set at L_s. Next, L_f and L_d are detected. L_f is set at the previous minima location from L_s and L_d is set at the next minima location from L_s. Here, L_f is considered to be the start location of the transition region from the fatigue fracture surface to the stretched zone, and L_d is considered to be the end location of the transition region from the stretched zone to the ductile fracture surface. Thus, the SZW_H in Figure (1) is considered to contain not only the stretched zone but also the fatigue and ductile fracture surfaces. For this reason, we define SZW_c as the following equation:

$$SZW_c = SZW_H/2 \tag{9}$$

However the SZW_c, which is important in stress estimation, depends on x_0 and is the averaged width in the image of the fracture surface. In the present paper, therefore, the averaged local Hurst exponent shown in Eqn.(10) is used.

$$\overline{H(y_0)} = \frac{1}{w_x} \sum H(x_0, y_0) \tag{10}$$

3.2 Test pieces and images of fracture surfaces

The test pieces for evaluating the SZW_c were CT test pieces with a chevron notch made from STPG370 (carbon steel). The elastic-plastic fracture toughness test was based on

ASTM E1820 and was performed by the unloading elastic compliance method at $200 \circ C$. Four pieces were tested, and 3 images were obtained from each test piece.

Scanning electron microscopy (SEM) was used to examine the fracture surface (ERA-4000; Elionix, Tokyo, Japan). Figure (2)-I shows the image of a fracture surface which has 256 gray levels (8bit/pixel). The magnification and resolution of the image are $\times 400$ and 800×600 pixels, respectively.

W used for the AWC method is 18 μm and the Daubechies (N=2) wavelet is used as the mother wavelet. The time for calculation was 90 minutes on Intel Xeon 2.2GHz.

4 RESULTS

Examples of an analyzed image and the distribution of the two-dimensional local Hurst exponent are shown in Figure (2). In Figure (2)-I, the white line indicates the stretched zone detected by human observation, Figure (2)-II shows the relationship between $H(y_0)$ and the location, and Figure (2)-III shows the distribution of the two-dimensional local Hurst exponent. Threshold (H_{th}) is 0.2. The white region is flatter than the black region. Figure (2)-IV also shows the distribution ($H_{th} = 0.25$).

Note that in Figure(2)-III, the white region is larger in the lower region than in the upper region, indicating that the fatigue fracture region is flatter than the ductile fracture region.

In addition, the white area in the ductile fracture region is related to the bottom of the dimple, which is large. For example, A in Figure (2)-IV is related to the large dimple observed the upper left of Figure (2)-I, while B in Figure (2)-IV is related to the set of small dimples. The two-dimensional local Hurst exponent is thus small.

Based on the data shown in Figure (2)-IV, the stretched zone is flatter than the fatigue and ductile regions. Furthermore, by setting the threshold appropriately, it is possible to detect the stretched zone shown in C (Figure (2)-IV). However, A,D in Figure (2)-IV indicates that there are regions in the fatigue and ductile regions which are flat, making these difficult regions in which to detect the stretched zone.

If we observe the fatigue and ductile regions, it becomes clear that the stretched zone is always flat in the x-direction while the other regions are not always flat in the same direction. Therefore, the SZW_c is evaluated by calculating the averaged local Hurst exponent shown in Eqn. (7).

Based on Figure (2)-II, it is obvious that the region detected by the proposed method is related to the region detected by human observation because the stretched zone is flatter than the other region and the two-dimensional local Hurst exponent is larger than the other area. Moreover, the maxima in the ductile fracture region is considered to be related to the large dimple observed in the upper left area of Figure (2)-I. Above all, the validity of the analytic model shown in Figure (1) is proved.

In the next, the SZW_c detected by the proposed method and the human observation at the each image from the test pieces are shown in Table.(1).

5 DISCUSSION

5.1 Evaluation of the stretched zone

In order for the stretched zone detected by the proposed method to be valid, the location of L_f and L_d must roughly agree with the stretched zone detected by human observation. Figure (2)-II shows that the boundary between the stretched zone and the ductile fracture region is in good agreement with the boundary detected by human observation. However, the boundary between the fatigue fracture region and the stretched zone does not show this agreement.

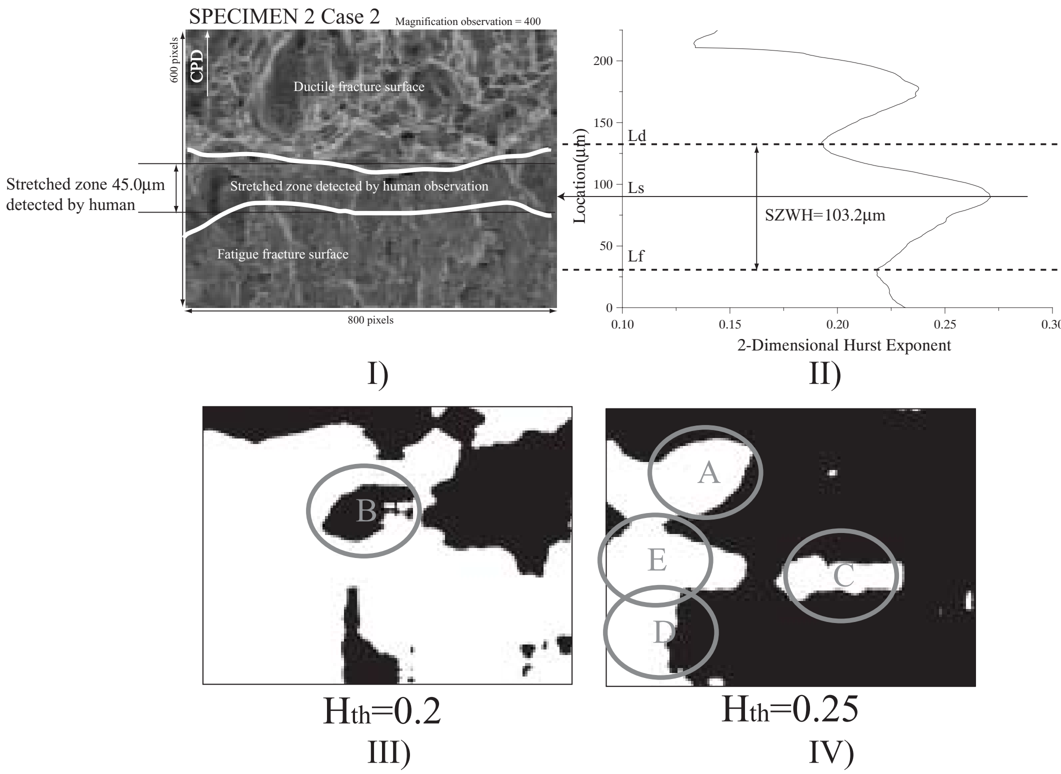

Figure 2: Distribution of two-dimensional local Hurst exponent and stretched zone detected by our proposed method. I) Gray scaled image of fracture surface. II) Relationship between two-dimensional Hurst exponent and the location. III) Distribution of two-dimensional local Hurst exponent($H_{th} = 0.2$). IV) Distribution of two-dimensional local Hurst exponent($H_{th} = 0.25$)

Table 1: Result of quantitative evaluation of $SZW_c(\mu m)$ by two-dimensional local Hurst exponent and by human observation (Italic).

		$SZW_c(\mu m)$	$Average(\mu m)$
	Case 1	45.9	
SPECIMEN 1	Case 2	41.1	43.4±2.00
J=134.5(kJ/m^2)	Case 3	43.1	*(38.4±3.0)*
	Case 1	35.4	
SPECIMEN 2	Case 2	51.6	47.3±8.51
J=92.0(kJ/m^2)	Case 3	54.9	*(43.4±1.8)*
	Case 1	35.3	
SPECIMEN 3	Case 2	28.7	33.2±3.19
J=81.2(kJ/m^2)	Case 3	35.6	*(39.3±7.7)*
	Case 1	47.6	
SPECIMEN 4	Case 2	38.4	42.0±4.05
J=79.1(kJ/m^2)	Case 3	39.8	*(40.9±1.9)*

Komai[6] conducted research on the depth of dimples, in which he found the depth to be a few μm ; the width of a dimple is approximately $10\mu m$. On the other hand, the height of striation reported by Furukawa is on the order of several hundred nm, and the width of striation is approximately $10nm$, because ΔK when the fatigue crack was propagated was $10Mpa\sqrt{m}$. Thus, the fatigue fracture surface is composed of smaller components than the ductile fracture surface, and the calculated boundary between the fatigue fracture surface and the stretched zone differs from that detected by human observation.

5.2 Evaluation of SZW_c

The difference between the SZW_c detected by the proposed method and that detected by human observation is almost $5\mu m$ as shown in Table (1), and it is nearly the with of the white line shown in Figure (2)-I. The SZW_c obtained by the proposed method thus shows good agreement with that obtained by human observation, proving that the two-dimensional local Hurst exponent is a useful and accurate method of detecting a stretched zone and measuring the SZW_c.

6 CONLUSIONS

In this paper, we propose a two-dimensional local Hurst exponent which can express the complexity of a fracture surface. To verify the applicability of this parameter, we applied it to the measurement of the SZW_c. Our results showed good agreement between the SZW_c detected by the two-dimensional local Hurst exponent and that detected by human observation. We thus conclude that the two-dimensional local Hurst exponent is an efficient method for measuring the SZW_c numerically.

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