ON PRE-FRACTURE ZONE MODELLING FOR AN INTERFACE CRACK IN ANISOTROPIC BIMATERIAL

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ABSTRACT

Plane strain problem for an interface crack between two anisotropic semi-infinite spaces under the action of remote mixed mode loading is considered. Assuming that the interface layer is softer than the matrixes the pre-fracture zone is introduced at the right hand crack continuation. The normal displacement jump is admitted in this zone and the normal stress is assumed to be equal to some critical value which is constant for the whole zone. By use of presentations of mechanical fields via sectionally-holomorphic vector function the problem is reduced to the combined Dirichlet-Riemann boundary value problem with respect to the single function analytic in the whole plane except the region of the longed crack. An exact analytical solution of this problem is presented and the stresses at the pre-fracture zone continuation as well as the derivatives of the displacement jumps along the crack region are written in a clear analytical form. From the condition of the stress finiteness at the end of the pre-fracture zone the transcendental equation for the determination of this zone length is found. Besides the displacement jump in the initial crack tip is found as well. This jump can be considered as the main fracture parameter for the considered model. Numerical analysis has been performed for the combination of an isotropic material with an orthotropic one for different coefficients of their shear module. The pre-fracture zone lengths and the associated displacement jumps are presented for various coefficients of the remote normal stress and the critical stress of the interface layer.

1. INTRODUCTION

For homogeneous materials different fracture criteria and different ways of crack propagation prediction have been developed. Particularly the well known plastic strip model has been suggested in papers by Leonov and Panasuk [1] and Dugdale[2]. For a crack along an interface of two materials this model has not got a due attention in the literature because of the problem complexity and variety of materials used for composite constructions. In this direction the papers Kaminskiy et al [3] and Goldstein and Perelmuter [4] should be mentioned. In the first paper the Dugdale model has been generalized for a crack between two isotropic materials and in the paper [4] the bridge model at the interface crack tip has been considered. Besides, the plastic strip model has been developed in the paper by Sheveleva [5] concerning a crack between two anisotropic materials. In this paper an approximate analytical solution has been presented for a model accounting normal and transversal displacement jumps in the plastic (pre-fracture) zone.

In the present work the pre-fracture zone at the interface crack continuation between two anisotropic materials which takes into account the normal displacement jump is considered in an analytical manner.

2. THE BASIC RELATIONS

Let's consider a tunnel crack $c \le x \le a$, $y=0$ between two anisotropic semi-infinite spaces $y > 0$ and $y < 0$ with the elastic characteristics $C^{(1)}_{ijkl}$ and $C^{(2)}_{ijkl}$ (e.g. Lekhnitsky [6]) accordingly, which are loaded at infinity by uniform normal $\sigma_{yy} = \sigma$ and shear $\sigma_{xy} = \tau$ stresses. Besides, it is assumed that at infinity the normal stress $\sigma_{xx}^{(i)} = \sigma_{xi}^{\infty}$ is applied also which provide the satisfaction of conditions of continuity at the interface (e.g. Cherepanov [7]).

In the work by Herrmann and Loboda [8] for two bonded elastic anisotropic semi-infinite spaces with defects at the materials interface provided that stress-strain state does not depend on coordinate z, the following expressions for the displacement jumps and the stresses at the interface have been obtained

$$[\mathbf{u}'(x)] = \mathbf{W}^{+}(x) - \mathbf{W}^{-}(x)\,; \tag{1}$$

$$\mathbf{t}^{(1)}(x,0) = \mathbf{G}\mathbf{W}^{+}(x) - \overline{\mathbf{G}}\mathbf{W}^{-}(x)\,, \tag{2}$$

where $\mathbf{u}(x) = [u(x,0), v(x,0), w(x,0)]^T$, $\mathbf{t}^{(1)} = [\sigma_{12}^{(1)}, \sigma_{22}^{(1)}, \sigma_{32}^{(1)}]^T$, $\mathbf{G} = \mathbf{B}^{(1)}\mathbf{D}^{-1}$, $\mathbf{D} = \mathbf{A}^{(1)} - \overline{\mathbf{A}}^{(2)}\left(\overline{\mathbf{B}}^{(2)}\right)^{-1}\mathbf{B}^{(1)}$,

$\mathbf{W}(z)$ is the 3-components vector - function, analytical in the whole plane (x, y) except the areas of defects at the interface. The matrixes $\mathbf{A}^{(m)}, \mathbf{B}^{(m)}$ $(m = 1,2)$ are defined for every semi-infinite spaces as follows

$$\mathbf{A}^{(m)} = \left\lfloor a_{k\alpha}^{(m)} \right\rfloor,\ \mathbf{B}^{(m)} = \left\lfloor \chi_{i2\alpha}^{(m)} \right\rfloor,$$

where $\left\lfloor \chi_{ij\alpha}^{(m)} \right\rfloor = \left(C_{ijk1}^{(m)} + p_{\alpha}^{(m)} C_{ijk2}^{(m)}\right) a_{k\alpha}^{(m)}$ and $\mathrm{p}_{\alpha}^{(\mathrm{m})}$ and $\mathbf{a}_{\alpha}^{(m)}$ are the eigenvalue and the eigenvector vectors of the system

$$\left[\mathbf{Q}^{(m)} + p^{(m)}\left(\mathbf{R}^{(m)} + \mathbf{R}^{(m)T}\right) + \left(p^{(m)}\right)^2 \mathbf{T}^{(m)}\right]\mathbf{a}^{(m)} = 0\,.$$

Here $\mathbf{R}^{(m)} = \left\lfloor R_{ik}^{(m)} \right\rfloor$, $\mathbf{T}^{(m)} = \left\lfloor T_{ik}^{(m)} \right\rfloor$, and $Q_{ik}^{(m)} = C_{i1k1}^{(m)}$, $R_{ik}^{(m)} = C_{i1k2}^{(m)}$, $T_{ik}^{(m)} = C_{i2k2}^{(m)}$, $[f(x)] = f^{+}(x) - f^{-}(x)$ is the jump of a function $f(z)$ through the interface; $f^{\pm}(x) = \lim\limits_{y\to\pm 0} f(z)$, $z = x + iy$.

3. FORMULATION AND SOLUTION OF THE BOUNDARY VALUE PROBLEM

For the sake of consistency we assume that the materials are orthotropic with the main directions of orthotropy parallel to coordinate axes. In this case for the chosen external loading the plane strain state takes place and the associated part of the relations (1), (2) can be written as [8]

$$\sigma_{yy}^{(1)}(x,0) + im_j\sigma_{xy}^{(1)}(x,0) = t_j[F_j^{+}(x) + \gamma_j F_j^{-}(x)]\,; \tag{3}$$

$$[u'(x,0)] + iS_j[v'(x,0)] = F_j^{+}(x) - F_j^{-}(x)\,, \tag{4}$$

where $S_{1,2} = -m_{1,2}$, $\gamma_j = -\left(G_{21} - im_j G_{11}\right)/t_j$, $t_j = G_{21} + im_j G_{11}$,

$$F_j(z) = W_1(z) + iS_j W_2(z),\ i = \sqrt{-1}\,,\ j = 1,2\,,$$

and $F_j(z)|_{z\to\infty} = \tilde{\sigma}_j - i\tilde{\tau}_j$, $\tilde{\sigma}_j = \sigma/r_j$, $\tilde{\tau}_j = -m_j\tau/r_j$, $r_j = t_j(1+\gamma_j)$ (5)

We shall consider now, that the faces of the crack are traction free and at the right hand crack continuation $a < x < b$ the pre-fracture zone appear. In this zone the normal displacement jump is admitted, and the normal stress in this zone is equal to the critical stress σ_0 of the material interface. Then the boundary conditions along the material interface y=0 have the form

$$[u] = [v] = 0,\ \left\lfloor \sigma_{yy} \right\rfloor = \left\lfloor \sigma_{xy} \right\rfloor = 0 \quad \text{on } L = \{x \notin (c,b)\}\,; \tag{6}$$

$$\sigma_{xy}^{\pm} = \sigma_{yy}^{\pm} = 0 \quad \text{на} \;\; L_1 = \{x \in (c, a)\}; \tag{7}$$

$$\sigma_{yy}^{\pm} = \sigma_0, [u] = 0, [\sigma_{xy}] = 0 \;\; \text{on} \;\; L_2 = \{x \in (a, b)\}. \tag{8}$$

It is necessary to note, that the position of the point b while is not known, and the account only one pre-fracture zone near to the right tip of the crack is explained by the smallness of their lengths and insignificant influence of one zone on another. It is obvious that the functions $F_j(z)$ are analytic in the whole plane except the segment [c, b].

By introducing new functions $\Phi_j(z) = iF_j(z)$ the eqns (3), (4) will be rewritten as

$$i\sigma_{yy}^{(1)}(x,0) - m_j\sigma_{xy}^{(1)}(x,0) = t_j[\Phi_j^{+}(x) + \gamma_j\Phi_j^{-}(x)]; \tag{9}$$

$$i[u'(x,0)] - S_j[v'(x,0)] = \Phi_j^{+}(x) - \Phi_j^{-}(x). \tag{10}$$

Satisfying the conditions (7), (8) we arrive at the following combined Dirichlet-Riemann boundary value problem

$$\Phi_j^{+}(x) + \gamma_j\Phi_j^{-}(x) = 0 \;\; \text{at} \;\; L_1; \tag{11}$$

$$\mathrm{Im}\Phi_j^{\pm}(x) = \sigma_0 / r_j \quad \text{at} \;\; L_2; \tag{12}$$

$$\Phi_j(z)|_{z\to\infty} = i\tilde{\sigma}_j + \tilde{\tau}_j. \tag{13}$$

For a finding of all necessary factors it is enough to consider the problem (11) - (13) only for j=1. Therefore in the following analysis index j is omitted meaning, that the solution is under construction for j=1.

The solution of the problem (11), (12) can be presented as (e.g. Muskhelishvili [9] and Nahmein and Nuller [10])

$$\Phi(z) = P(z)X_1(z) + Q(z)X_2(z) + \Phi_0(z), \tag{14}$$

where $X_1(z) = ie^{i\varphi(z)} / \sqrt{(z-c)(z-b)}$, $X_2(z) = e^{i\varphi(z)} / \sqrt{(z-c)(z-a)}$,

$$\varphi(z) = 2\varepsilon \ln \frac{\sqrt{(b-a)(z-c)}}{\sqrt{l(z-a)} + \sqrt{(a-c)(z-b)}}, \quad \varepsilon = \frac{1}{2\pi}\ln\gamma;$$

$\mathrm{P}(z) = C_1 z + C_2$, $Q(z) = D_1 z + D_2$ are polynomials with real coefficients. The particular solution $\Phi_0(z)$ of the non-homogeneous problem can be written as $\Phi_0(z) = X_1(z)\Gamma(z)$, where $\Gamma(z)$ after some manipulations can be presented in the form

$$\Gamma(z) = \frac{\sigma_s}{\pi r}\left[\int_a^b \frac{\sqrt{(t-c)(b-t)}\, ch\varphi_0(t)}{(t-z)}dt - \sqrt{(z-a)(b-z)}\int_a^b \sqrt{\frac{t-c}{t-a}}\frac{sh\varphi_0(t)}{t-z}dt\right]. \tag{15}$$

From the condition at infinity (13) follow

$$C_1 = \tilde{\sigma}\cos\beta - \tilde{\tau}\sin\beta, \qquad D_1 = \tilde{\sigma}\sin\beta + \tilde{\tau}\cos\beta,$$

$$C_2 = -\frac{c+b}{2}C_1 - \beta_1 D_1, . \qquad D_2 = \beta_1 C_1 - \frac{c+a}{2}D_1 - R, \tag{16}$$

where $R = \frac{\sigma_0}{\pi r}\int_a^b \sqrt{\frac{t-c}{t-a}} sh\varphi_0(t)dt$, $\beta = \varepsilon \ln\frac{1-\sqrt{1-\lambda}}{1+\sqrt{1-\lambda}}$, $\beta_1 = \varepsilon\sqrt{(a-c)(b-c)}$, $\lambda = \frac{b-a}{b-c}$.

The obtained solution is valid for any position of the point *b*.

4. REAL PRE-FRACTURE ZONE LENGTH AND CRACK OPENING DISPLACEMENT

Taking into account that for $x \notin (c,b)$ the relation $\Phi^-(x) = \Phi^+(x)$ is valid from the equation (9) one gets

$$\sigma_{yy}^{(1)}(x,0) + im\sigma_{xy}^{(1)}(x,0) = -ir\Phi^+(x) = -ir\{[P(x)+\Gamma(x)]X_1(x) + Q(x)X_2(x)\}. \tag{17}$$

From the condition of the finiteness of stresses (17) for $x \to b+0$ the following equation is derived $P(b)+\Gamma(b)=0$. Transforming this equation with an account (5), (15) leads to

$$\cos\beta + mk\sin\beta - 2\varepsilon\sqrt{1-\lambda}(\sin\beta - mk\cos\beta) - \frac{\sigma_0}{\pi\sigma}M(\lambda) = 0, \tag{18}$$

where $k = \tau/\sigma$, $M(\lambda) = \frac{2}{b-c}\int_a^b \sqrt{\frac{t-c}{b-t}} ch\varphi_0(t)dt$, $M(\lambda) = \sqrt{\lambda}[4 + L(\lambda)]$,

$$L(\lambda) = \int_{-1}^{1}\left[\sqrt{\frac{2}{1-\tau} - \lambda}\; ch\psi_0(\tau) - \sqrt{\frac{2}{1-\tau}}\right]d\tau, \quad \psi_0(\tau) = 2\varepsilon\tan^{-1}\sqrt{(1-\lambda)\frac{1-\tau}{1+\tau}}.$$

The equation (18) is a transcendental equation with respect to the relative pre-fracture zone length λ. In a general case its solution can be found numerically. Further we define the displacement jump $[v(x,0)]$ on the interval (a, b). Taking into account, that in this interval

$$X_2^{\pm}(x) = \frac{e^{\pm\varphi_0(x)}}{\sqrt{(x-c)(x-a)}} \tag{19}$$

from the equations (10), (14) one gets

$$[v'(x,0)] = -\frac{2}{S\sqrt{x-c}}\left[\frac{P(x)+\Gamma(x)}{\sqrt{b-x}} ch\varphi_0(x) + \frac{Q(x)}{\sqrt{x-a}} sh\varphi_0(x)\right]. \tag{20}$$

Integrating of the last relation with an account $v(b,0)=0$ leads to

$$\delta = [v(a,0)] = -\int_a^b [v'(x,0)]dx. \tag{21}$$

The value of δ is the crack opening displacement in the initial crack tip and can be used as the main fracture parameter for the considered model.

5. NUMERICAL RESULTS AND DISCUSSION

The results of the computation of the values λ and $\tilde{\delta} = \delta C_{66}^{(1)}/[(a-c)\sigma]$ for $\tau = 0$, $C_{66}^{(1)}/\sigma_s = 2\cdot 10^4$ and different $k_0 = C_{66}^{(2)}/C_{66}^{(1)}$, σ/σ_s are presented in the Table 1. The upper material is an orthotropic plastic with $C_{11}^{(1)}/C_{66}^{(1)} = 18{,}1$, $C_{12}^{(1)}/C_{66}^{(1)} = 6{,}9$, $C_{22}^{(1)}/C_{66}^{(1)} = 16{,}2$ and the lower one is isotropic with $\nu^{(2)} = 0{,}3$ (the parameters $C_{11}^{(m)}$, $C_{12}^{(m)}$, $C_{22}^{(m)}$, $C_{66}^{(m)}$ correspond to the designations used in the reference by Lekhnitsky [6]).

Table 1. The values of the relative pre-fracture zone length and the correspondent crack opening displacement

k_0	σ / σ_0	$100\,\lambda_0$	$100\,\tilde{\delta}_2$
1.5	0.02	0.0224	0.447
	0.05	0.147	1.18
	0.1	0.604	2.44
	0.2	2.52	5.05
	1/3	7.45	8.62
4	0.02	0.0246	0.355
	0.05	0.154	0.887
	0.1	0.622	1.78
	0.2	2.55	3.58
	1/3	7.48	6.03
10	0.02	0.0241	0.253
	0.05	0.152	0.642
	0.1	0.617	1.30
	0.2	2.54	2.62
	1/3	7.48	4.43

It follows from the obtained results that similarly to the crack in a homogeneous material the values of λ_0 and δ are growing nonlinear with respect to the growing of parameter σ / σ_0.

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