VIBRATIONS OF A CROSS-PLY CERAMIC MATRIX COMPOSITE BEAM WITH MATRIX CRACKS IN LONGITUDINAL AND TRANSVERSE LAYERS

V. Birman[1] and L.W. Byrd[2]

[1]University of Missouri-Rolla, Engineering Education Center
8001 Natural Bridge Road, St. Louis, MO 63121, USA
[2]Air Force Research Laboratory, AFRL/VASM, Bldg. 65, 2790 D Street
Wright-Patterson Air Force Base, OH 45433-7531, USA

ABSTRACT

The paper illustrates the effect of matrix cracks in longitudinal and transverse layers of a cross-ply ceramic matrix beam (CMC) on its stiffness and vibration frequencies. Even if vibration amplitudes are small, the physical nonlinearity is introduced by the interfacial fiber-matrix friction in the vicinity of matrix cracks in the longitudinal layers. A closed-form solution for mechanical properties of a cross-ply beam with matrix cracks is developed in the paper. The frequency of free vibrations of a simply supported beam is derived as a function of the amplitude, accounting for the effect of matrix cracks. The conclusions that follow from the numerical analysis enable us to suggest a simple and accurate design formula for the fundamental frequency.

KEYWORDS

Ceramic matrix composites, matrix cracks, vibrations, physical nonlinearity, bimodular material.

INTRODUCTION

As follows from observations of a matrix crack development in cross-ply beams, cracks appear in transverse layers at a relatively low load [1]. These cracks, called tunneling cracks, are perpendicular to the load direction and parallel to the fibers of the corresponding layer. As the load increases, the crack density increases as well, until the tunneling cracks reach saturation. After saturation, the density of tunneling cracks remains constant under increasing load.

As the load continues to increase, cracks begin to develop in longitudinal layers where they are perpendicular to the fibers. The cracks approaching the fiber-matrix interface can break the fiber or pass it and continue their propagation in the matrix. The cracks propagating in the matrix of a longitudinal layer, without breaking the fibers, are called bridging cracks. Such cracks represent a typical mode of damage in CMC.

The density of cracks in transverse layers is often different than that in longitudinal layers. For example, Erdman and Weitsman [2] reported measurements of the matrix crack density in cross-ply SiC/CAS composites. Dependent on the lay-up, the saturation density in transverse layers varied between 20 and 40 cracks/inch. However, the saturation density in longitudinal layers was between 122 and 238 cracks/inch.

This illustrates that the analysis should not be conducted by assumption that the densities of cracks in transverse and longitudinal layers are identical.

ANALYSIS

The analysis utilizes the following assumptions.

1. During the first phase of the initial quasi-static loading cracks in transverse layers have reached saturation.
2. Continued loading of the beam resulted in cracking of longitudinal layers that occurred without further changes in the stiffness of transverse layers.
3. Cracks in longitudinal and transverse layers remain open during the tensile part of the cycle of motion. Under compression, the cracks in all layers are closed.
4. The mode shape of vibration of the beam is not affected by damage.
5. Energy dissipation due to damping, interfacial friction, thermoelastic effect, and closing and opening of the cracks is disregarded.

The solution for the stiffness of a cross-ply CMC beam with matrix cracks in longitudinal and transverse layers is based on the theories of Han and Hahn for a cross-ply material with cracks limited to transverse layers [3] and Pryce and Smith for a unidirectional material with bridging matrix cracks [4]. In particular, the stiffness of a cross-ply material where tunneling cracks in transverse layers reached saturation, while longitudinal layers are still intact is [3]:

$$E' = E_i[1 + (h_l/\xi s_T)(E_T/E_L)\tanh(\xi s_T/h_l)]^{-1} \tag{1}$$

where s_T is a crack spacing (saturation spacing, in this case), h_l is a layer thickness, E_L and E_T are the longitudinal and transverse moduli of elasticity of a layer, $E_i = (E_L + E_T)/2$ is the composite modulus of the intact material, and ξ is a shear lag parameter. Note that the modulus given by (1) is used for the analysis of vibration cycling, when the cracks already exist. Accordingly, residual thermal stresses do not affect this modulus.

Once the value of E' has been determined, the modulus of the transverse layers can be obtained as $E_T' = 2E' - E_L$. The modulus of transverse layers remains unaffected by the stress during cycling, as long as the cracks remain open, i.e. the material is subject to tension. Under compression, the cracks are closed and $E_T' = E_T$ corresponds to the modulus of the intact material.

The average modulus corresponding to reverse fatigue loading of a unidirectional CMC lamina with bridging matrix cracks and the stress varying from zero to a maximum value was given in [5], based on the model of Pryce and Smith [4]:

$$E_L' = \tau[\tau/E_L + (r/4s_L)(\Delta\sigma_L/E_f)(V_mE_m/V_fE_L)^2]^{-1} \tag{2}$$

In (2), τ is an interfacial shear stress, r is the fiber radius, s_L is the matrix crack spacing in longitudinal layers, $\Delta\sigma_L$ is the range of stresses applied to the longitudinal layer, E_f and E_m are the moduli of fibers and matrix, respectively, and V_f and V_m are the volume fractions of the fibers and matrix, respectively.

Residual thermal stresses do not affect the average modulus E_L', as can easily be shown using the solution [4]. The interfacial shear stress decreases with cycling due to wear, as was shown by Holmes and Cho [6]. Other factors, such as lubrication, strain rate, and temperature have also been shown to affect this stress. However, during small-amplitude steady state vibrations, the changes of the factors affecting the interfacial stress are slow. Accordingly, the value of the interfacial stress during one cycle may be assumed constant. Equation (2) was obtained by assumption of a partial slip along the fiber-matrix interface during cycling, as can be expected in the case of small-amplitude vibrations. The limits of applicability of this equation are specified in [4].

The range of stresses acting in the longitudinal layer can be determined keeping in mind that the average stiffness of transverse layers remains constant during the tensile part of the vibration cycle. Accordingly, the ratio between the stress ranges in adjacent longitudinal and transverse layers is

$$\Delta\sigma_L/\Delta\sigma_T = E_L'(\Delta\sigma_L)/E_T' \tag{3}$$

where E_T' is constant. At the same time, the range of the applied composite stress is

$$\Delta\sigma = (\Delta\sigma_L + \Delta\sigma_T)/2 \tag{4}$$

Solving (3) and (4) together with (2), it is possible to obtain the stress range $\Delta\sigma_L$ and the average modulus for the longitudinal layers E_L' as functions of the range of the applied composite stress $\Delta\sigma$. Then the average composite modulus of the material with matrix cracks in both longitudinal and transverse layers, $E = E(\Delta\sigma)$, is available as $E = (E_L' + E_T')/2$. This modulus remains constant during the tensile part of the cycle when the stress varies from zero to a maximum value. During the compressive part of the cycle the modulus $E_L' = E_L$, according to (2). Physically this is justified since the crack closing strain could be estimated as a ratio of the crack opening displacement to the crack spacing. This ratio is small and it may be assumed equal to zero.

Consider now a nonlinear relationship between the range of the applied stresses and the range of strains for the section of a cross-ply CMC beam with matrix cracks that is subject to tension during reversed bending. The permanent offset strain that remains in the material after removal of the load that caused cracking does not explicitly affect the average modulus of elasticity during the motion. However, the effect of preloading is incorporated in the solution via the matrix cracks spacing. As shown above, we can determine the modulus E as a nonlinear function of the applied stress range $\Delta\sigma$. Subsequently, it is possible to find the corresponding strain range $\Delta\varepsilon = \Delta\sigma/E(\Delta\sigma)$.

It is necessary to characterize the curve $E = E(\Delta\varepsilon)$ by an analytical expression that can be used in the frequency analysis. For example, as shown below, the following function was in an excellent agreement with numerical results:

$$E_a = E_c + a\Delta\varepsilon + b\Delta\varepsilon^n \tag{5}$$

where E_c is the modulus of the damaged material subjected to a negligible tensile strain that does not affect the modulus of longitudinal layers, a and b are constants that have to be determined and n is an integer. Note that E_c in (5) is independent of the crack density in longitudinal layers. This is because the modulus of elasticity of longitudinal layers with bridging cracks based on the model of Pryce and Smith [4] is affected by the crack density only in the presence of applied stress. Physically, this model is justified since opening a crack in a longitudinal layer requires a tensile force to overcome friction along the damaged section of the fiber-matrix interface.

The solution for small-amplitude vibrations of a simply supported cross-ply CMC beam with matrix cracks in longitudinal and transverse layers is obtained by the energy method, based on the assumption that the energy dissipation is negligible. Accordingly, in the case of harmonic vibrations, the squared frequency is obtained from

$$\omega^2 = U_{max}/T_{max} \tag{6}$$

where U and $\omega^2 T$ are the strain and kinetic energies, respectively.

The evaluation of the maximum value of the strain energy is complicated due to the fact that the modulus of elasticity varies throughout the depth and the length of the beam. As explained above, if the dynamic strains are compressive, the modulus of the material remains constant and equal to E_i. Therefore, the cross-ply beam material behaves in a manner that resembles a nonlinear material of “bimodular” type, though in the present problem the response is more complicated since the material exhibits physical nonlinearity during the tensile part of the cycle but retains constant stiffness under compression.

The location of the neutral curve of the beam can be determined from $z_n = B/A$ where A and B are extensional and coupling stiffnesses, respectively. For example, if the section, $z_n < z < h/2$, of a unit-width beam of thickness h is subject to tension and the crack densities in both the longitudinal and transverse layers are independent of the thickness z-coordinate, these stiffnesses become

$$\{A,B\} = \int_{z_n}^{h/2} [E_c + a\Delta\varepsilon(z) + b\Delta\varepsilon(z)^n]\{1,z\}dz + \int_{-h/2}^{z_n} E_i\{1,z\}dz \qquad (7)$$

Note that $\Delta\varepsilon$ at each location is equal to the maximum dynamic strain achieved during the cycle. Given the value of $\Delta\varepsilon_{max} = \Delta\varepsilon(z{=}h/2)$, it is possible to obtain the local dynamic strain range as

$$\Delta\varepsilon(z) = [(z-z_n)/(h/2 - z_n)]\Delta\varepsilon_{max} \qquad (8)$$

Therefore, it is possible to determine z_n, as long as $\Delta\varepsilon_{max}$ is prescribed.

Now the maximum strain energy can be evaluated as $U_{max} = U_t + U_c$ where two components in the right side correspond to the contribution of the parts of the beam cross section subject to tension and compression, respectively. In the following analysis, the cracks are assumed uniformly distributed along the beam. Then the neutral curve becomes the neutral axis, i.e. z_n is independent of the axial coordinate. The maximum kinetic energy of the beam is readily available since it is not affected by cracks.

The analysis is carried out assuming that the mode shape of vibrations is unaffected by damage (assumption 4). Then the squared nondimensional frequency is the following nonlinear function of the amplitude of free vibrations (W):

$$F^2 = f_0 + f_1W + f_nW^n \qquad (9)$$

where f_0, f_1 and f_n are constants dependent on geometry, material, and damage and F is the ratio of the natural frequency of the damaged structure to that of the intact beam.

The position of the neutral axis can be found evaluating the extensional and coupling stiffnesses in terms of z_n. A quadratic equation for $Z_n = z_n/h$ is

$$n_2Z_n^2 + n_1Z_n + n_0 = 0 \qquad (10)$$

where n_i (i = 0, 1, 2) are functions of a, b, E_c, E_i and the strain range $\Delta\varepsilon_{max}$ (these functions are omitted here for brevity).

NUMERICAL EXAMPLES AND DISCUSSION

The material considered in the following examples is SiC/CAS with the following properties [7]:
$E_f = 200$ GPa, $E_m = 97$ GPa, $r = 8$ μm, $\tau = 5$ MPa, $V_f = 0.35$. The layer thickness adopted in these examples is 125 μm. The longitudinal and transverse moduli of this material are $E_L = 133$ GPa and $E_T = 118$ GPa, respectively. The transverse shear modulus that is needed to calculate the shear lag parameter was taken equal to 45 GPa. It should be noted that a relatively low value of the interfacial shear stress is adopted here to reflect the experimentally observed fact that these stresses decrease during vibrations due to a gradual smoothening of the fiber-matrix interface [6].

The ranges of stresses in longitudinal and transverse layers differ significantly, as the range of the applied composite stress increases, as follows from Figure 1. A difference between these ranges increases if the matrix crack spacing in longitudinal layers becomes larger. This is predictable since if damage in the longitudinal layers is relatively small, they absorb a larger fraction of the applied load.

Relationships between the composite modulus and the modulus of longitudinal layers and the range of the composite strain are shown in Figure 2. As follows from these results and other cases not presented here due to the space limitation, a very accurate approximation of the relationship $E = E(\Delta\varepsilon)$ can be obtained from equation (5) using $n = 2$. The coefficients a and b in equation (5) are listed in Table 1 for various matrix crack spacings in the longitudinal layers. These coefficients appeared insensitive to the matrix crack spacing s_T. However, the modulus E_c varied dependent on this spacing. The nonlinear quadratic representation of the composite modulus-applied strain relationship is very accurate. Maximum deviations of equation (5) with $n = 2$ and the coefficients listed in Table 1 from the actual curves were under 1%. It is also evident from Table 1 that the quadratic term in (5) is negligible when the strain range is small. This is reflected in Figure 2 where the composite modulus is almost a linear function of the range of the applied strain.

The changes in the position of the neutral axis due to a large strain were very small, not exceeding 5%, even as the tensile strain on the beam surface $z = h/2$ reaches 1%. This results in an important conclusion that it is possible to assume that the position of the neutral axis is unaffected by the strain. Accordingly, this position can be calculated from a simplified version of (10) that is simplified even more due to the observation that the quadratic term has little effect on the value of Z_n. Therefore, given the values of E_c and E_i, one can determine the position of the neutral axis from

$$Z_n = -(1/4)(1 - E_n)/(1 + E_n) \qquad (11)$$

The analysis of (9) with $n = 2$ illustrates that the effect of the amplitude of motion on the frequency is negligible. Then the nondimensional fundamental frequency is available form the simplified version of (9):

$$F^2 = [(1 + 2Z_n)^3 + E_n(1 - 2Z_n)^3]/2 \qquad (12)$$

The master curves are shown in Fig. 3. These curves can be convenient for the analysis of small-amplitude vibrations of cross-ply CMC beams with matrix cracks in longitudinal and transverse layers.

ACKNOWLEDGEMENT

This research was supported by the Air Force Office of Scientific Research through the contract F49620-93-C-0063. The project managers were Drs. H. Thomas Hahn and Daniel J. Segalman.

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Table 1: Coefficients (a, b) in equation (5) obtained using n = 2

s_L (μm)	125	250	500
a, b (GPa)	-268.83, 940.07	-153.0, 428.45	-81.46, 151.67

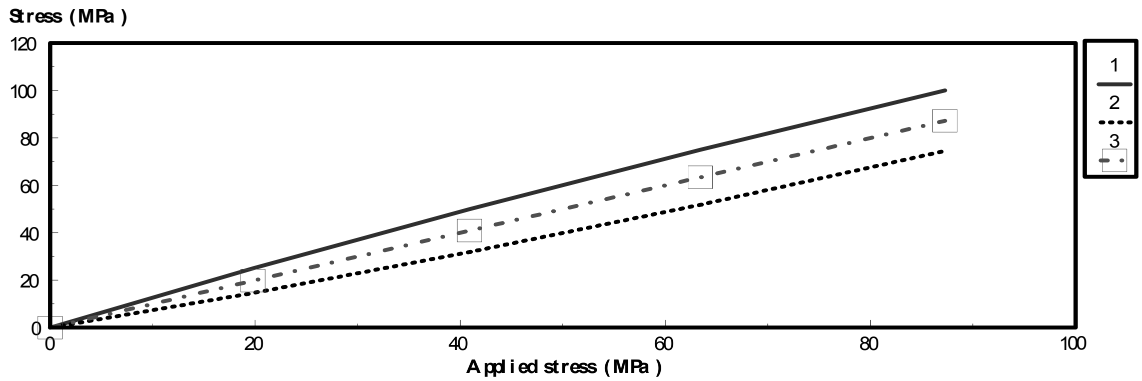

Figure 1: Relationships between the range of the applied composite stress (3) and the range of stress in longitudinal (1) and transverse layers (2) for the case where s_L = 0.125 mm and s_T = 0.250 mm.

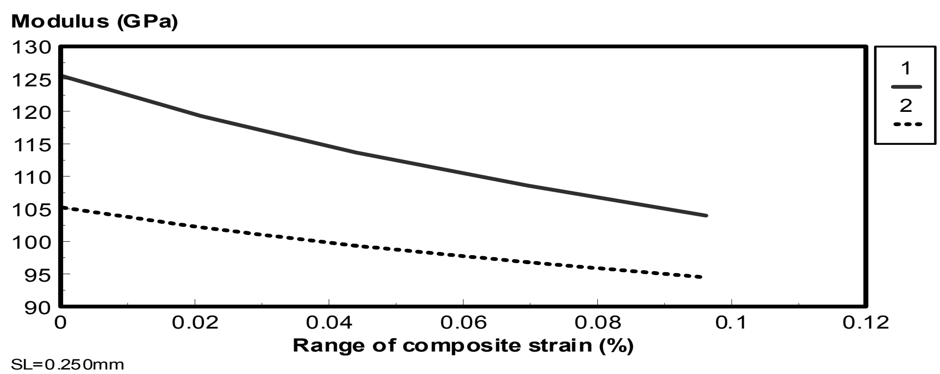

Figure 2: Relationships between the modulus of longitudinal layers (1) and the composite modulus (2) and the range of composite strain (s_L = 0.250 mm, s_T = 0.500 mm).

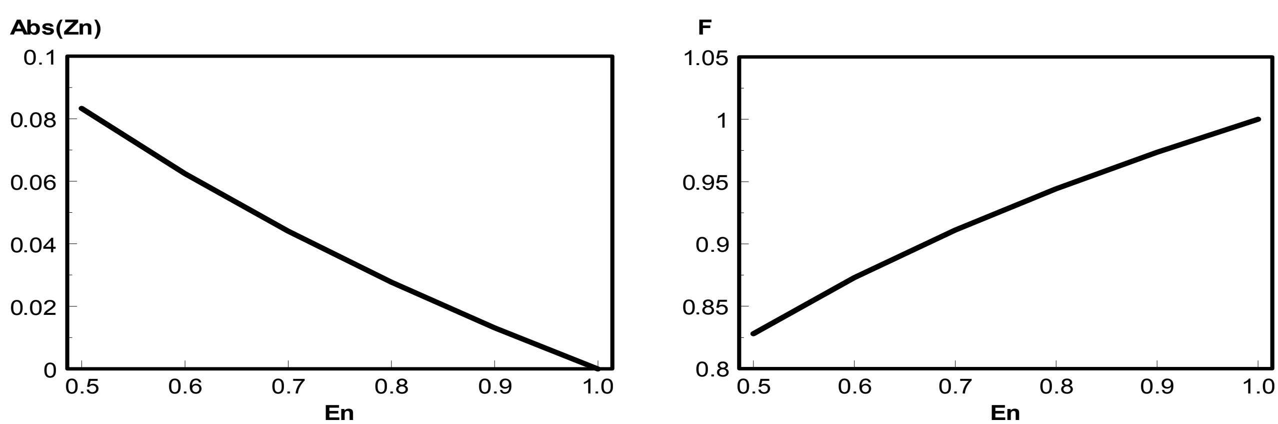

Figure 3: "Master curves" representing the nondimensional location of the neutral axis (left figure) and the nondimensional fundamental frequency, i.e. a ratio of the frequency of the beam with cracks to that of the intact beam, (right figure) as functions of the nondimensional stiffness of the beam ($E_n = E_c/E_i$).