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UNIVERSAL BRIDGING LAWS FOR CRACKS IN CREEPING MEDIA

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ABSTRACT

Some theoretical results for bridging traction/crack displacement laws for composite and monolithic materials in which cracks are bridged by rate-dependent ligaments are reviewed. Laws derived from experiments and laws proposed on theoretical grounds for different material systems and for different loading conditions often display similar functional characteristics. It is therefore proposed that engineering creep/fracture tests be analysed by simple, trial bridging laws of the most common functional form, with parameters to be determined by fitting test data such as crack length vs. time and load. A simple bridging law of this kind, with only a few unknown parameters to be evaluated, offers a reasonable balance between consistency with the underlying physical mechanisms and engineering practicality. An approach to engineering certification of damage tolerance for materials containing cracks bridged by rate-dependent ligaments is outlined.

KEYWORDS

Certification, crack bridging, rate dependence, creep, ceramic composite, polymer craze

INTRODUCTION

This paper is motivated primarily by the need to establish certification procedures for fibre-reinforced ceramic matrix composites for high temperature use in applications such as rocket nozzles, turbine engine combustors, and hot airframe components. In many such composites, failure mechanisms involving fibre creep are likely to form a significant part of the boundary of the allowable design space [1,2].

In notched or damaged components, one archetypal problem of interest is that illustrated in the schematic of Fig. 1, where a system of matrix cracks emanates from a stress concentrator. At high temperature, a single dominant crack tends to emerge from the system, the others arresting and remaining relatively short [1,3]. Considerable evidence shows that at least for some important composites the primary rate effect is fibre creep [1-5]. Crack acceleration of the dominant crack results from reduction of the shielding effect at its tip. The essential characteristic of the composite required to model crack growth and eventual failure is the relation between the stress, σ_0, in the fibres on the matrix crack plane and the relative displacement, u, of the fibres and the matrix on the same plane (the matrix crack opening displacement; or more generally the displacement discontinuity across a band of localised damage). If this function (which of course will also involve time and temperature) is known, bridged crack analysis will predict crack propagation.

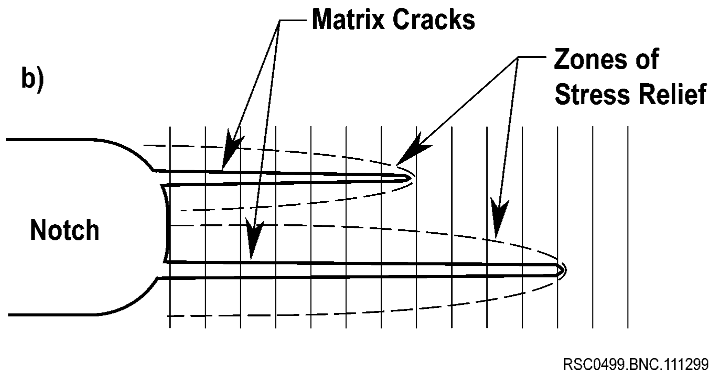

Figure 1: System of matrix cracks emanating from a stress concentrator, with one crack becoming dominant.

A simple model has led to an expression for the differential function $\dot{u}(\sigma_0)$ (the derivative of *u* with respect to time) [6]. The function bears close similarity to laws found for other material systems, especially polymers. This leads to the conjecture that a method of engineering certification for crack growth in ceramic composites and polymers in the presence of rate-dependent bridging that is based on standardisable tests fitted by a universal canonical traction law might be attainable. Such a method is needed to make the certification problem sufficiently straightforward.

TRACTION LAW FOR CREEP-RUPTURE PROBLEMS IN CERAMIC COMPOSITES

The physics of a creeping fiber being pulled out of an elastic matrix can be summarised by dividing the fibre into three zones in which different mechanisms operate (Fig. 2). Far from the matrix crack plane lies a so-called intact zone ($z > z_2$), where the fibre and matrix have equal strains in the axial direction, z. Displacements remain continuous across the interface in this zone and the loads are assumed to be low enough that the creep threshold of the fibres is not exceeded. Closer to the matrix crack lie two frictional zones, where the fibre has slipped relative to the matrix against the spatially varying friction stress, $\tau(z)$. In the further of these zones from the matrix crack plane (the "friction zone", $z_1 < z < z_2$), the axial stress in the fibres remains below a threshold for creep. Here any variation in the friction stress is caused by elastic contraction of the fibre in the radial direction due to Poisson's effect alone. In the nearer of the zones to the matrix crack plane (the "friction/creep" zone, $d < z < z_1$), the axial stress in the fibre exceeds the creep threshold and fibre shrinkage due to creep also modifies the friction stress. Nearest to the matrix crack a so-called debond zone may exist ($0 < z < d$), where the radial shrinkage of the fibre is so great that contact has been lost with the matrix and so there is no friction. One seeks the time history of the pullout displacement, u_0, for any history of the fibre stress at the fracture plane, σ_0, which is proportional to the bridging traction, p.

For fixed loads, a steady-state configuration exists, in which the friction/creep and friction zones propagate invariantly along the fibre away from the crack plane at a constant velocity, v. Simple analytical results can be obtained for general load histories in the limiting case that changes in stress are sufficiently slow that the solution is always close to the steady-state solution corresponding to the instantaneous value of the load, which will often be the case. In this quasi-steady state condition, the displacement rate for fibers obeying power-law creep with exponent m, i.e., $\dot{\varepsilon} \propto (\sigma - \sigma_{th})^m$, where σ_{th} is the creep threshold, can be written [6,7][1]

$$\dot{U}_0 = \frac{2\tau_0\lambda}{\gamma_2}\left\{\Sigma_0^m D + \frac{2}{m+1}\Sigma_0^{m+1}\right\} + \frac{2\tau_0}{\gamma_3 E}\frac{d}{d\bar{t}}\left[\Sigma_0 D\right] + \eta\frac{2\tau_0}{\gamma_3 E}\left[\frac{\sqrt{\pi}}{2}\frac{\Gamma\left(\frac{m+3}{m+1}\right)}{\Gamma\left(\frac{m+5}{2m+2}\right)}\right]2\Sigma_0\dot{\Sigma}_0 \tag{1}$$

where a dot denotes differentiation with respect to a normalised time variable, $\bar{t} = 2\gamma_2 t/\gamma_3\left(2\tau_0/\gamma_3\right)^{m-1}$; λ is a creep rate parameter; τ_0 is the pristine friction stress; E is the fibre modulus; d is the length of the debond zone (Fig. 2); U_0, D, and Σ_0 are normalised forms of u_0, d, and σ_0; γ_3 and η are elasticity/geometry parameters; and Γ is the Gamma function. The first term on the right hand side of Eq. (1) expresses the contribution of fibre creep to the displacement rate, with the term involving D arising from the debond zone and the term Σ_0^{m+1} from the friction/creep zone; the second term represents elastic stretching of the fibre in the debond zone; and the third term represents elastic stretching of the fibre in the friction/creep zone. The contributions involving D are history dependent, since

$$D = \int_0^{\bar{t}} V d\bar{t} \tag{2}$$

where V is the velocity (rate of self-similar translation) of the friction/creep zone, which is a function of σ_0 and therefore of the normalised time, $\bar{t}$. For the limits used to reduce the results of [6,7] to Eq. (1), one finds that

$$V = \frac{2}{m+1}\Sigma_0^{m+1} \quad . \tag{3}$$

In the approximation that bridging tractions can be represented by continuous, averaged tractions, p, acting smoothly over the crack faces rather than only at the location of bridging fibres, a traction law, $\dot{u}(p, \dot{p})$, follows from Eq. (1) by substituting $p = f\sigma_0$ where f is the fibre volume fraction. The differential form of Eq. (1) implies that bridged crack problems will have to be solved by integrating over the history of the applied load, while accounting for any crack growth [4,5].

Equation (1) has been derived under the assumptions that fibre/matrix friction is not uniform but is modified by creep shrinkage of the fibres; and that the debond and friction/creep zones evolve in a quasi-steady state condition, which is valid for moderate rates of change of the bridging stress. Begley, Evans, and McMeeking (BEM) derived a bridging law for fibres that undergo linear creep but remain coupled to the matrix by a constant and uniform friction stress, τ_0 [8]. Thus in BEM there is no debond zone (no fibre shrinkage) and the steady-state conditions of the model underlying Eq. (1) cannot be considered. The bridging law of BEM describes the transient response to the first period of loading. In its simplest form (omitting certain terms containing convolutions that are small), BEM reduces to

$$\dot{u}_0 = m_1\sigma(t)_0\left[\dot{\sigma}_0(t) + m_2\sigma_0(t)\right] \tag{4}$$

where m_1 depends on elastic constants and the fibre volume fraction and m_2 includes the creep rate constant, λ. The first term in brackets arises from the elastic straining of the fibre in the friction/creep zone. Because the friction stress is constant and uniform in BEM, their law reproduces the familiar quadratic relationship

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[1] From Eq. (37) of [6] and generalisations for non-linear creep in [7]. An error of sign is corrected and an integral term omitted from the former. The integral term corresponds to an elastic displacement rate arising from the friction zone, which is identically zero since the friction zone is bounded by the fixed stress condition, $\sigma = \sigma_{th}$, and is therefore invariant. The limit is taken here that Poisson's effect causes radial strains in the fibre that are small compared to its shrinkage due to creep.

between crack displacement and load, $u_0 \propto \sigma_0^2$, in the absence of creep. The last term of Eq. (1) refers to the same contribution and has the same form, even though the assumed stress distribution along the fibre is different (because of creep shrinkage). The second term in brackets in Eq. (4) arises from fibre creep in the BEM model. It corresponds to the term $\lambda \Sigma_0^{m+1}$ in Eq. (1), with equality of form in the case of linear creep (m = 1). Thus the contribution to the crack displacement of fibre creep within the friction/creep zone has the same functional form in both models.

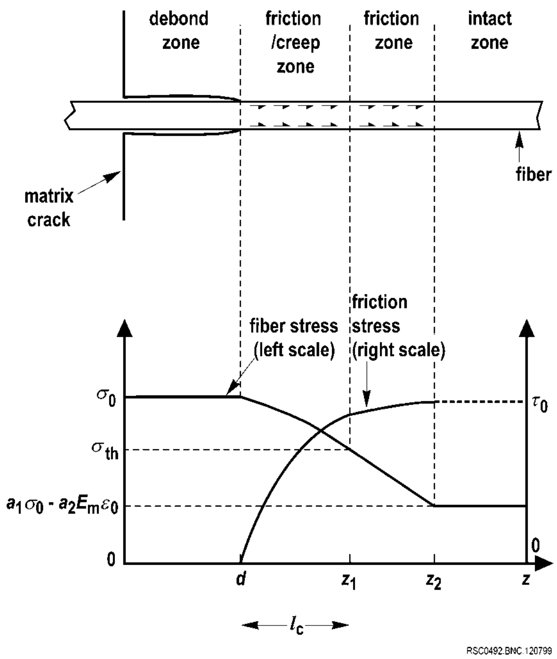

Figure 2: Four zones in a creeping fibre that bridges a matrix crack, with schematics of the axial stress and friction stress variations.

The similarity of the forms of BEM and the model of Eq. (1) when $m = 1$ is quite remarkable, since BEM is essentially a model of transient response, while the quasi-static approximation underlying Eq. (1) depends on the existence of a steady-state crack configuration. Furthermore, the stress distributions along the friction/creep zone in BEM and the model of Eq. (1) are not the same, even in the absence of Poisson's effect, being linear in BEM and nonlinear in the latter, due to non-uniformity of the friction stress. Yet the two models yield identical dependences on the applied load for both the elastic and creep contributions to the total displacement arising from the friction/creep zone. This interesting universality suggests that Eq. (1) might be a powerful basis for crack growth data reduction. The functional form of Eq. (1) is

$$\dot{U}_0 = c_1 \Sigma_0^{m} D + c_2 \Sigma_0^{m+1} + c_3 \frac{d}{dt}\left[\Sigma_0 D\right] + c_4 \Sigma_0 \dot{\Sigma}_0 \tag{5}$$

This law might be hoped to give good correlation with creep/rupture (crack growth) data for fixed temperature. The coefficients c_1, etc., will of course be different at different temperatures. While the micromechanical basis of Eq. (1) is a model of a single matrix crack, the same law, Eq. (5), is also proposed here for analysing damage bands consisting of multiple matrix cracks, or a non-planar matrix crack, or a crack system with complicated connectivity; and for non-unidirectional reinforcement, including wavy or misaligned fibres, such as in a textile composite, as long as a damage band can still be identified in each case.

RATE-DEPENDENT COHESIVE ZONES IN POLYMERS

The depiction of Fig. 2 for creeping fibres has a close analogy in the drawing and extension of fibrils in a craze zone in a polymer. Two processes dominate the extension of fibrils in a craze zone under normal loads: 1) creep or viscous extension of the fibrils, which is analogous to the creep extension of fibres in the debond zone of Fig. 2; and 2) drawing of new material from the bulk into the fibrils, which is analogous to the process of loss of contact between creep-shrinking fibres and the matrix in the friction/creep zone of Fig. 2. Micromechanical models of fibril drawing give rise to relations between the total displacement rate and the stress and stress-rate in the fibrils, which can be re-arranged to exhibit forms similar to Eq. (5) (e.g., [9,10]). Some quantitative distinctions are worth noting. First, because of the hardening mechanism in drawn polymers, creep extension of the fibrils once they have been drawn tends to be relatively small. Therefore, the first term in Eq. (5) is often omitted in craze models. Second, the stress dependence of the drawing rate in polymers is relatively strong. Thus where a power law relation is used between the stress (σ_0 in the current notation) and the displacement rate due to the drawing of new fibrils, the power is typically of order 10. Fibre creep tends to have milder stress dependence: the power for creeping fibres usually is bounded by $m \leq 4$.[2] Third, because of the dominance of the plastic displacement arising during the drawing process of newly drawn fibril, models of crazes have no analogue of the fourth term in Eq. (5) (elastic strain in the process zone). Thus in polymers, Eq. (5) might reduce to the second and third terms only, with a relatively high power expected in the second term.

FAILURE OF THE BRIDGING MECHANISM

At some point, the creeping fibres (or fibrils in a craze) will rupture, terminating the bridging traction. A plausible criterion for rupture of a creeping fibre is the attainment of either a critical creep strain or a critical stress. For a fibre exhibiting power law creep, $\dot{\varepsilon} \propto (\sigma - \sigma_{th})^m$, the rupture criterion will have the form

$$\int_0^t \left(\Sigma_0 - \Sigma_{th}\right)^m dt = \varepsilon_{crit} \tag{6a}$$

$$\sigma_0 = \sigma_{crit} \tag{6b}$$

for two material constants, ε_{crit} and σ_{crit}. In most cases of interest, Eq. (6a) is likely to be the first failure criterion to be attained: if the most highly-loaded fibres (those at a notch root) do not satisfy Eq. (6b) on first loading, then they never will, since the stress in them relaxes if the applied load is held constant [5].

AN APPROACH TO EMPIRICAL CREEP/RUPTURE LAWS AND CERTIFICATION

Certification of a part against creep-rupture by crack propagation requires a material calibration test that is simple and repeatable, from which any necessary material information can be deduced in a simple form without complicated computational analysis. For failure by crack propagation in the presence of long zones of bridging fibres, the critical material information is the rate dependent traction law, $\dot{u}(\dot{p}, p)$, or equivalently, $\dot{u}(\dot{\sigma}_0, \sigma_0)$, together with a bridging ligament failure criterion, such as Eq. (6).

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[2] The stress dependence of the rate of drawing of fibrils in crazes is so strong that it has also been modeled as an exponential function [11]. Whether the dependence is exponential or a power law with a power ~ 10, the effective result is that the fibrils will draw almost infinitely fast whenever the stress, σ_0, exceeds some threshold. A traction-displacement relation must result that is not far in its effect on crack growth from a Dugdale model of uniform stress in the cohesive zone.

An engineering certification procedure cannot depend on detailed knowledge of micromechanical mechanisms, such as those treated in deriving Eq. (1) as a problem in materials science. Therefore, Eqs. (5) and (6) are suggested as a parametric form of material data for the creep-rupture problem, with the parameters to be found empirically. Some speculations on the outcome of calibrating tests are as follows. 1) At low or moderate stresses, creep strain will dominate elastic strain and the last two terms of Eq. (5) will be negligible: the coefficients c_3 and c_4 will be difficult to determine. 2) At relatively short times, e.g., if the critical creep strain for rupture is relatively small, any debond zone will remain small and the first term of Eq. (5) will be negligible: the coefficient c_1 will be difficult to determine. At relatively long times, conversely, the first term will become the dominant creep term and the second term will be negligible, with c_2 difficult to determine. 3) The length of the bridging zone will depend on the notch size used in an experiment. With relatively small notches, a long zone of bridging fibres might evolve before the critical creep rupture strain is reached; with a long notch, the fibre at the notch root will be highly stressed and will reach the critical rupture strain while the bridging zone is relatively short. Thus tests for different notch sizes will yield information about different regimes.

CONCLUSIONS

A functional form, Eq. (5), has been suggested for a simple bridging traction law that could serve as the basis for empirical calibration of materials in which cracks are governed by rate-dependent bridging. The law has been derived with the first objective of establishing a certifying procedure for ceramic composites at fixed temperature. However, the law may also be useful for analysing time-dependent notch effects in polymers and the durability of bonded repairs. In different applications, some of the terms of the proposed law might be negligible and therefore could be omitted. The traction law will be complemented in general by a criterion for bridging ligament rupture, Eq. (6).

Acknowledgments

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