TIME DEPENDENT STRESS INTENSITY FACTORS IN A VISCOELASTIC STRIP

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ABSTRACT

The time-dependent stress fields around a moving crack tip in a viscoelastic strip is studied. Photoviscoelastic technique using an elliptically polarized white light is employed for the evaluation of the time-dependent stress distribution around a crack tip. The time-dependent stress intensity factor K_I^* which is extended for linearly viscoelastic materials is evaluated using a method based on the least-squares. The results show that the proposed critical stress intensity factor K_c^* for fast crack growth may be considered as a characteristic property of the material under monotonically increasing load.

KEYWORDS

Viscoelasticity, Crack growth, Stress intensity factor, Time- and temperature-dependent properties, Photoviscoelasticity

INTRODUCTION

Reflecting rapid expansion of the application of polymeric materials in various industrial fields, extensive investigations on crack threshold and subsequent growth in linearly viscoelastic materials have been reported from both theoretical and experimental viewpoints [1-8]. Various theories have been developed to describe crack initiation and propagation in such materials. However, the mechanism and the viscoelastic behavior in the fracture process are not yet clear since many difficulties, such as the detection of the initiation of crack growth or the constitutive behavior in the fracture process, still exist in this type of material.

Several optical methods, such as moiré interferometry or photoelasticity, have long been used in experimental mechanics to evaluate the deformation of solids under load and the mechanics of fracture [9]. These optical methods, which are non-contact and full-field measurement techniques, are very valuable in areas where other methods of analysis are not available or impractical. For the fundamental studies on the mechanics of viscoelastic materials, it has been pointed out that the photoviscoelasticity [10] is a very useful experimental method. Using photoviscoelastic technique, time-dependent stress state in the vicinity of crack tip can be calculated from the time-variation of fringe order and the principal direction of birefringence.

In this paper, the time-dependent stress fields around a crack tip at the instant of crack threshold and during subsequent growth in a viscoelastic material are discussed. The time-dependent stress state around a crack tip is successfully analyzed using photoviscoelastic technique with elliptically polarized white light taking a single shot image [11]. Then, the time-dependent stress intensity factor, which is extended for viscoelastic materials, is evaluated using a method based on the least-squares. The experimental results show that the proposed critical stress intensity factor K_c^* depends on not only the crack growth rate but also the temperature. As a result, the values of the critical stress intensity factor for fast crack growth may be considered as a characteristic property of the material under monotonically increasing load.

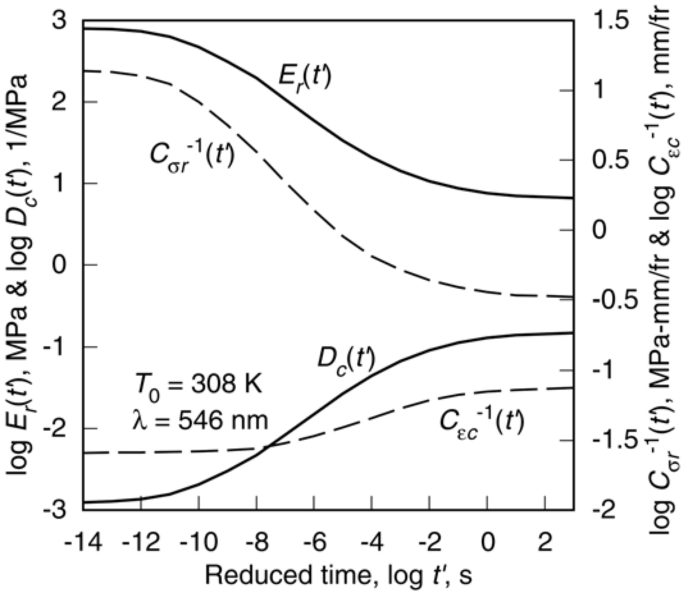

Figure 1: Master curves of the mechanical and opto-mechanical properties of the material

EXPERIMENT

Material Properties

The material used in this study is a soft epoxy resin (Epikote 871). This material is linearly viscoelastic, thermorheologically simple and birefringent. The glass transition temperature is measured as T_g = 258 K. Figure 1 shows the mechanical and opto-mechanical properties of the material. In this figure, the mechanical properties, i.e., the master curves of the relaxation modulus $E_r(t')$ and the creep compliance $D_c(t')$ as a function of the reduced time t' are shown by solid lines, and the inverse relaxation stress-birefringence coefficient $C_{\sigma r}^{-1}(t')$ and the inverse creep strain-birefringence coefficient $C_{\varepsilon c}^{-1}(t')$ are represented by broken lines. They are measured and determined through uniaxial tension tests under several constant strain rates at various temperatures, and constructed using the WLF time-temperature shift factor [12] at the reference temperature T_0 = 308 K. Then, each master curve is approximated by a Prony series by the collocation method to facilitate the calculation of the convolution integrals involved in the constitutive equations. As seen in Figure 1, the values of the coefficients vary remarkably between the rubbery and glassy states over a wide range of the reduced time on a logarithmic scale.

Experimental Procedure

The specimen adopted is a strip with w = 80mm in width, $2b$ = 40 mm in length and t = 0.5 mm in thickness, having an initial crack C_0 = 20 mm from the left edge as shown in Figure 2. The ratio of the initial crack length to the other dimensions of the specimen is chosen to satisfy approximately the condition of a semi-infinite crack [13]. A very sharp initial crack [6] is generated by extending a razor cut pre-crack of 8 mm in length to 20 mm under constant displacement rate loading at a temperature of 273 K, in order to realize good repeatability of crack growth under every test condition. The constant rate displacement loading is applied on the lower edge of the specimen normal to the crack surface under five different rates of the extension $V = 8.33 \times 10^{-3}$, 3.33×10^{-2}, 8.33×10^{-2}, 3.33×10^{-1} and 8.33×10^{-1} mm/s at the temperatures T = 263, 268 and 273 K. At these temperatures, the material shows marked viscoelastic behavior. However, the experiment under $V = 8.33 \times 10^{-1}$ mm at T = 263 K is excluded since the specimen exhibits brittle fracture under this condition.

Photoviscoelastic fringe patterns are observed through a polariscope arranged for an elliptically polarized white light photoviscoelasticity [11]. A tricolor type fluorescent lamp is used as a light source in order to reduce the attenuation of light intensities of photoviscoelastic fringe pattern. The variations of fringe pattern are recorded on color reversal films periodically. The photographic images of the propagating crack are later digitized into a computer using a film scanner.

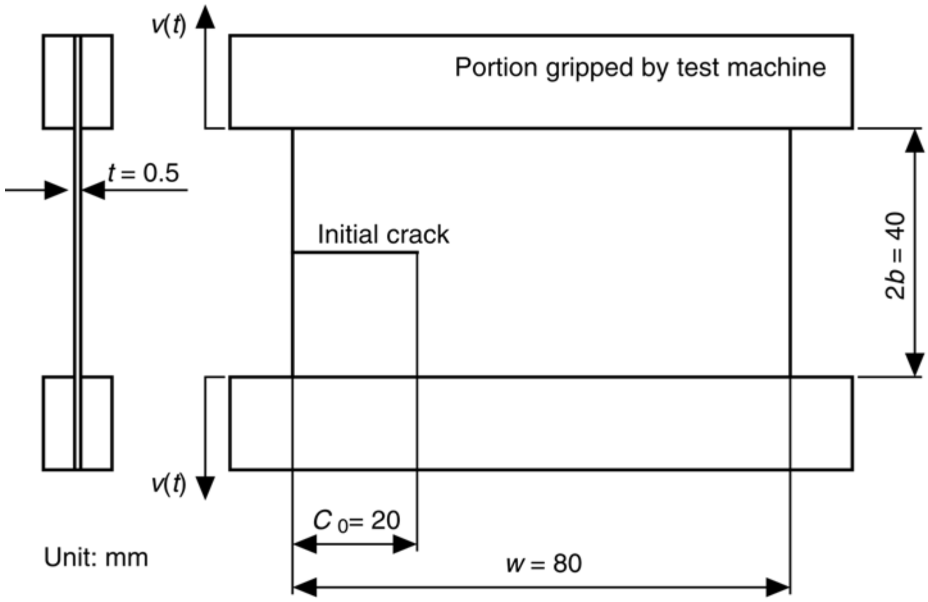

Figure 2: Specimen geometry

RESULTS AND DISCUSSION

Crack Growth Behavior

Figure 3 shows the crack extension curves, i.e., the relation between the increment of the crack growth $a(t)$ and the time t, measured from the careful observation of the photoviscoelastic image data. As reported by several researchers [2,3], the linear relation between the crack length increment and the time in a double logarithmic scale is observed. Thus, each crack growth curve is approximated by a power function as [3],

$$a(t) = \alpha' \, t^{\beta} \, , \tag{1}$$

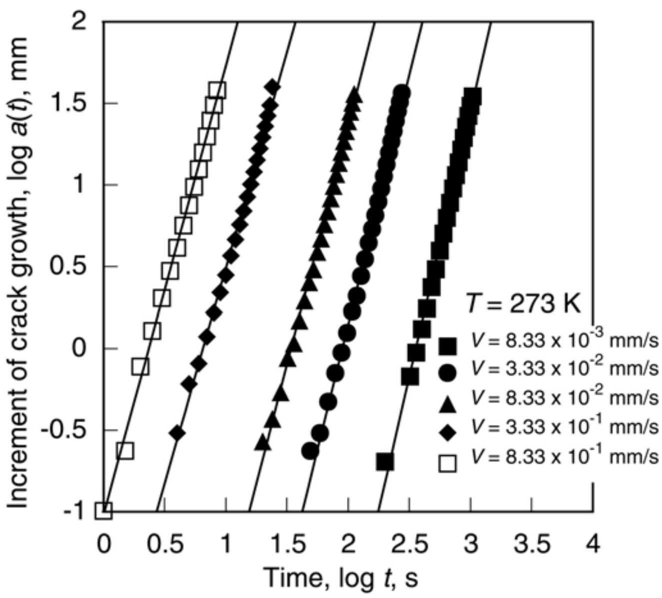

Figure 3: Examples of the crack extension curves (T = 273 K)

where α' and β' are constants. The crack growth curves approximated by the least-squares method are also shown in the same figure. Good correlations are observed. However, it is noteworthy to say that the crack threshold time, i.e., the critical condition for crack threshold cannot be determined from Eqn. (1). This fact is one of the unsolved difficulties in the discussion on viscoelastic fracture problem.

Fringe Pattern and Photoviscoelastic Analyses

Figure 4 shows examples of the photoviscoelastic fringe pattern around the crack tip at T = 273 K under the extension rate of $V = 8.33 \times 10^{-3}$ mm/s. Here, the originally colored images are expressed in gray levels. Typical mode I type fringe pattern varies with the movement of the crack tip. However, some of isochromatic fringes still remain at the initial crack tip and on the fracture surface behind the crack tip. This phenomenon indicates one of the features of the viscoelastic behavior.

The photoviscoelastic fringe patterns are analyzed using the method proposed by the authors previously [11]. Applying the procedure to a time-series of the photoviscoelastic images, the time-variation of fringe order and the principal direction of birefringence at selected points are determined. Then, the time variation and the

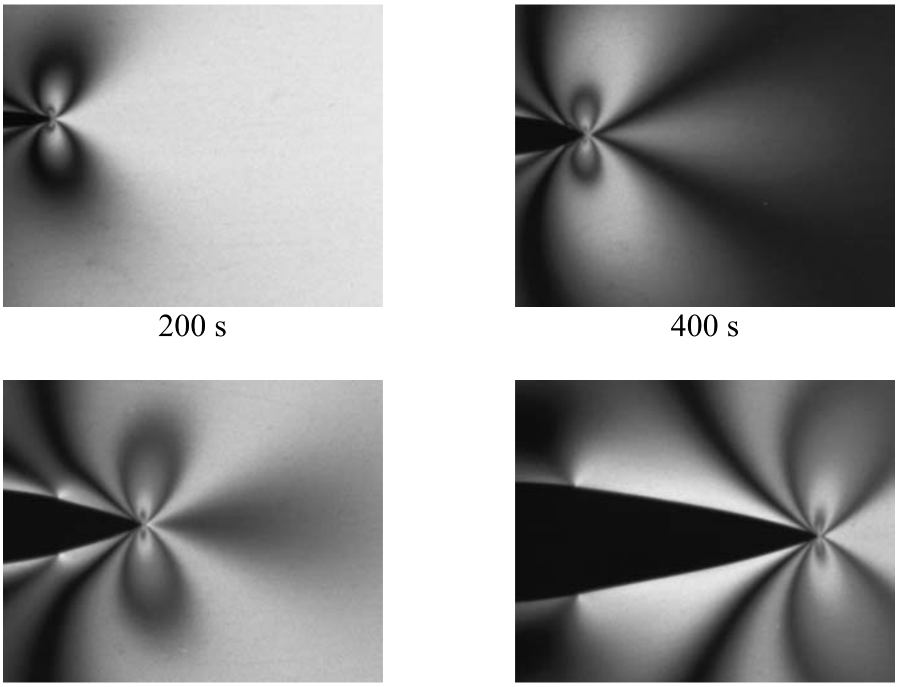

Figure 4: Examples of photoviscoelastic fringe pattern around a crack tip (T = 273 K, $V = 8.33 \times 10^{-3}$ mm/s)

distribution of the principal stress difference in the vicinity of the crack tip are calculated using the constitutive equations of photoviscoelasticity [10]. Unlike photoelastic case, the time-history of the variations of the fringe order and the principal direction of birefringence at each point have to be traced in photoviscoelastic case. In this study, to determine the fracture mechanics parameter of the moving crack at any instant, over 400 points on a specimen (19 points for an instant) are selected for tracing the history of the fringe order and the principal direction of birefringence.

Critical Stress Intensity Factor

The viscoelastic fracture studies reported up to date concerning the threshold condition of crack growth tend to search for a fracture mechanics parameter which could hopefully be independent of loading rate and temperature, such as the case of COD [2], M_{Ic} [4] or J_c' [5]. However, since discontinuous critical phenomena have not been observed in crack extension curves as shown in Figure 3, a new approach is developed in this study.

In viscoelastic crack problems, the expressions for stress fields, that is, the relations between stresses and the stress intensity factor, are not given unlike the elastic case. Thus, the conventional formula for extracting the stress intensity factor in photoelasticity does not provide an accurate estimation, except in the case of a stationary crack. In this study, however, since the expressions for stress fields in the vicinity of a running crack tip in viscoelastic materials cannot be obtained in general form, the value calculated by use of the conventional formula [9] is adopted as an apparent stress intensity factor K_I^* which is extended for viscoelastic materials. For the calculation of the viscoelastic stress intensity factor, the over deterministic algorithm based on the method of least-squares proposed by Sanford and Dally [14] is employed. The results of the viscoelastic stress intensity factor at the temperature of 273 K as a function of applied displacement $v(t)$ (= $Vt/2$) is shown in Figure 5. In this figure, curves represent theoretical values for a stationary crack obtained by the following equation.

$$K_I^*(t) = \frac{1}{\sqrt{(1-\nu^2)b}} \left\{ v(0)E_r(0) - \int_0^t E_r(t-\tau)\frac{dv(\tau)}{d\tau}d\tau \right\}, \qquad (2)$$

where b represents the half length of the specimen and ν is constant Poisson's ratio. Equation (2) can be obtained by invoking the correspondence principle, since the case of a stationary crack is a type of proportional loading problem.

From Figure 5 it is seen that in the early stage of the tests, the faster the loading rate is applied, the higher the values of K_I^* are observed. This observation is due to the viscoelastic response of the material. In this stage, the theoretical and experimental values show fairly good agreement. As displacement is increased the values of K_I^* initially increase monotonically, until the experimental values of K_I^* become constant, or sometimes decrease suddenly, at a particular value of the applied displacement in each curve. It may be considered that when the experimental value swerves away from the theoretical value, the actual effect of crack length increment on the optical and mechanical entities arises at this instant as an apparent crack threshold. However, the crack threshold time determined by the procedure mentioned above does not give a strict estimation. Judging from the crack extension curves shown in Figure 3, at the point at which the experimental value of K_I^* varies discontinuously, the crack has already grown about 0.9 ~ 1.8 mm, depending on the loading rate. From the viewpoint of the results, however, the stress fields in the vicinity of the crack tip change drastically at that moment. That is to say, the viscoelastic stress intensity factors K_I^* at the time can be evaluated as critical stress intensity factor K_c^* for fast crack growth.

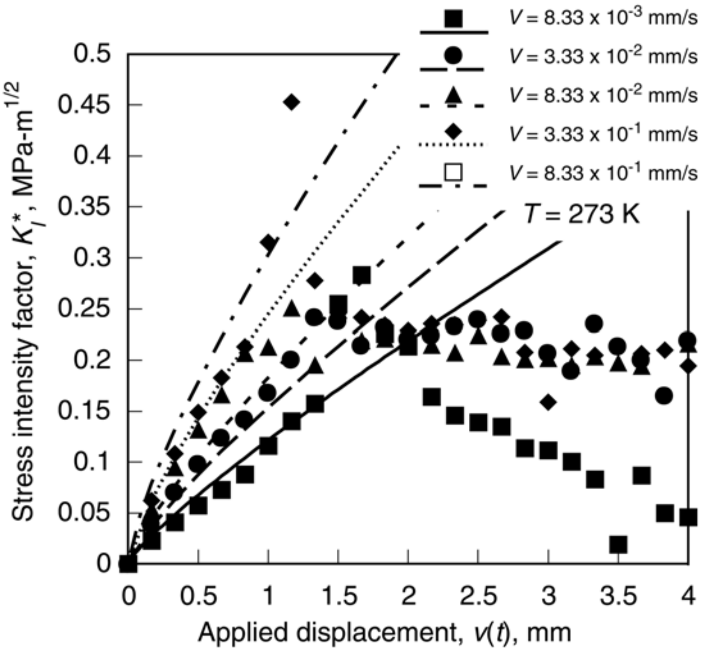

Figure 5: Viscoelastic stress intensity factor as a function of the applied displacement (T = 273 K)

Figure 6 shows a rough estimation of the critical stress intensity factor K_c^* as a function of the crack extension rate $da(t)/dt$, obtained from Figure 5. The crack extension rate is determined by differentiating the crack extension curves in Figure 3. The values of K_c^* increase with the increase of the crack extension rate. Also, the values of K_c^* depend on temperature. Thus, it is expected that a master curve of the critical stress intensity factor K_c^* can be constructed from the curves in Figure 6, in a manner similar to other material properties. Each curve in Figure 6 is shifted by WLF time-temperature shift factor. The results are shown in Figure 7. Here, the abscissa represents the logarithmic scale of the reduced crack growth rate $a'(t)$. As shown in this figure, the plots of the critical stress intensity factor can be approximated by a single curve. As a result, the proposed critical stress intensity factor for fast crack growth may be considered as a characteristic property of the material under monotonically increasing load.

CONCLUDING REMARKS

Using a new fringe pattern analysis method with an elliptically polarized white light and photoviscoelastic technique, an experimental approach to the viscoelastic fracture problem is discussed. The time-dependent fracture mechanics parameter K_c^*, which is extended for linearly viscoelastic materials under various loading rates and at several temperatures, is evaluated. The results are briefly summarized as follows.

1. The experimental values of the viscoelastic stress intensity factor K_I^* shows fairly good agreement with the theoretical values in the early stage of the tests, that is, for a stationary crack.
2. The discontinuous variation of the values of K_I^* is observed at a particular value of the applied displacement.
3. The values of the proposed critical stress intensity factor K_c^* for fast crack growth obtained from the discontinuous change of K_I^* depend on not only the crack extension rate but also the temperature.
4. The proposed critical stress intensity factor K_c^* may be considered as a characteristic property of the

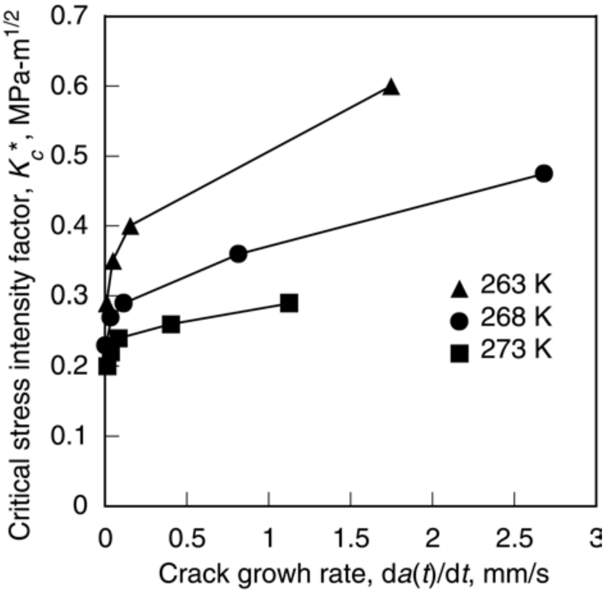

Figure 6: Critical stress intensity factor as a function of the crack growth rate

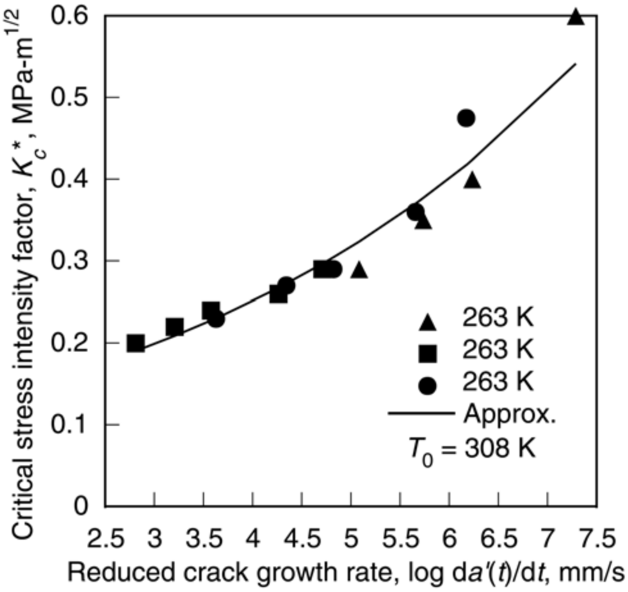

Figure 7: Master curve of the critical stress intensity factor

material under monotonically increasing load.

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