THE FORMULAE OF FATIGUE CRACK PROPAGATION IN POLYMETHYL METHACRYLATE

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Abstract: The formula for cyclic fatigue crack propagation (fcp) of PMMA derived from the static fracture model and modified to include the critical condition of fcp was deduced in this study. The formula can be used to represent the test results of cyclic fcp of four types of PMMA in near-threshold-, intermediate- and rapid crack propagation regions well. The correlation between the fcp rate and the stress intensity factor (ΔK), the fcp coefficient (B), the fcp threshold (ΔK_{th}) and the fracture toughness (K_{1c}) was revealed in the formula. The value of fcp coefficient can be calculated by Young's modulus as $B=15.9/E^2$. So the coefficient and the parameters involved in the formula all have definite physical meaning. Therefore, the above mentioned formulae could be thought as the almost perfect formulae for cyclic fcp of PMMA. The governing parameters of fcp of PMMA are, respectively, the effective stress intensity factor (ΔK-ΔK_{th}) in near-threshold region and intermediate region, and the difference between the fracture toughness and the maximum value of stress intensity factor (K_{1c}-K_{max}) in rapid crack propagation region. The fcp coefficient (B) and the ratio of fcp threshold to fracture toughness ($\Delta K_{th}/K_{1c}$) are the governing parameters in intermediate region. Comparing with those of the normal PMMA, the lower fcp rate of oriented PMMA mainly results from the higher values of fcp threshold, higher fracture toughness and the smaller ratio value of fcp threshold to fracture toughness.

Key words: PMMA, fatigue crack propagation, threshold, fracture toughness

1. Introduction

The explosion accidents due to crack in the aircraft cabin cover made of PMMA exist commonly after a certain period of service of PMMA. The results of cabin outfield employing show that the no crack life-span is always difficult to be predicted because of its large scatter. On the other hand, the safe application life is always composed of fatigue crack propagation (fcp) life, which is much longer than the crack initiation life in cabin sometimes. So, for economic and reliability reason, it is very important to accurately estimate the residual life of pieces having cracks and scientifically work out the schedule of checking period for cabin. Studies have shown that the life of fatigue crack propagation (fcp) has less scatter than that of fatigue crack initiation [1]. If a formulae can well describe the behavior of crack propagation of PMMA, the problem mentioned above can be solved. So the comprehensive and valid formulae is very important for estimating the residual life of pieces having cracks. The test results of the fatigue crack propagation rates of PMMA were fitted and expressed by using Paris' equation [2-4]. However, Paris' equation can not be used to fit the test results in near-threshold region and rapid region of the fatigue crack propagation, and the coefficient and the exponent in Paris' equation have no definite physical meaning [5]. In the present study, the test results show three regions of PMMA fatigue crack propagation and attempts are made to propose expression to fit those.

2. The materials and experiments

Four types PMMA are used in this study, i.e., two kinds of normal PMMA (type No: YB-3 and YB-4) and two kinds of oriented PMMA (type No: DYB-3 and DYB-4). The dimensions of the specimens are 400mm×100mm×10mm, and a center penetrate crack $2a_o$=10~15mm was prefabricated. The temperature of the test is at 23±2℃, and the sine wave load with 2.5Hz frequency and 0.1 stress ratio are employed in these fcp experiments. Figure 1 and Table 1 show the test results of fcp rate with test dots and the tensile properties respectively.

3. The basic formulae introduced to describe the fcp of PMMA

Even though the fcp rates of PMMA are mach higher than that of metallic materials, but when the test results were normalized by the parameter $\Delta K/E$(the ratio of stress intensity factor range to Young's modulus), the data of both kind of materials are scattered in almost a similar curve band [2]. On the other hand, striations were found in fatigue fracture surface of PMMA.[6] That is very similar to the ductile striation mechanism in fcp of metals. Thus, it may be also assumed that the crack propagation in PMMA occurs due to the fracture of the material elements located ahead of the crack tip, or the crack tip could not advance if the material elements ahead of the crack tip do not fracture under applied load. Based on this static fracture model for fcp in metals, the formulae introduced in this study was the expression derived by Zheng and Hirt as follows [5,8]:

$$\frac{da}{dN} = B(\Delta K - \Delta K_{th})^2 \tag{1}$$

Where ΔK_{th} is the fcp threshold defined as the value of ΔK below which no fcp will occur, and B is the fcp resistance coefficient depending on the fcp mechanism and the tensile properties,

$$B = 1/2\pi\sigma_{ff}^{\ 2} \tag{2}$$

Where σ_{ff} is the effective fracture stress of the material elements ahead of the crack tip. For metals, 0.1E may be taken as the estimated value of the effective fracture stress, and if the fatigue crack propagation by ductile striation mechanism we have:

$$B = 1/2\pi(0.1E)^2 = 15.9/E^2 \tag{3}$$

E is Young's modulus. Because of the similar striation propagation mechanism is of PMMA and metal, the basic formulae mentioned above was introduced to describe the fcp of PMMA:

$$\frac{da}{dN} = \frac{15.9(\Delta K - \Delta K_{th})^2}{E^2} \tag{4}$$

From the fcp test results shown in fig.1, it can be seen that the whole fcp test dots of PMMA consist of three regions, i.e., the near-threshold region, the intermediate region and rapid crack propagation region, which are similar to those of fcp curve of metals. As it has been pointed out in Refs. [5,8], eqn. (4) can be used only to describe the fcp in near-threshold region and intermediate region, see curves in figure 1. This is because eqn.(4) does not include the upper limit of fcp, i.e., the condition of critical propagation of fatigue crack when $K_{\max} = K_{Ic}$, where K_{1c} is the fracture toughness of materials; K_{max} is applied maximum stress intensity factor.

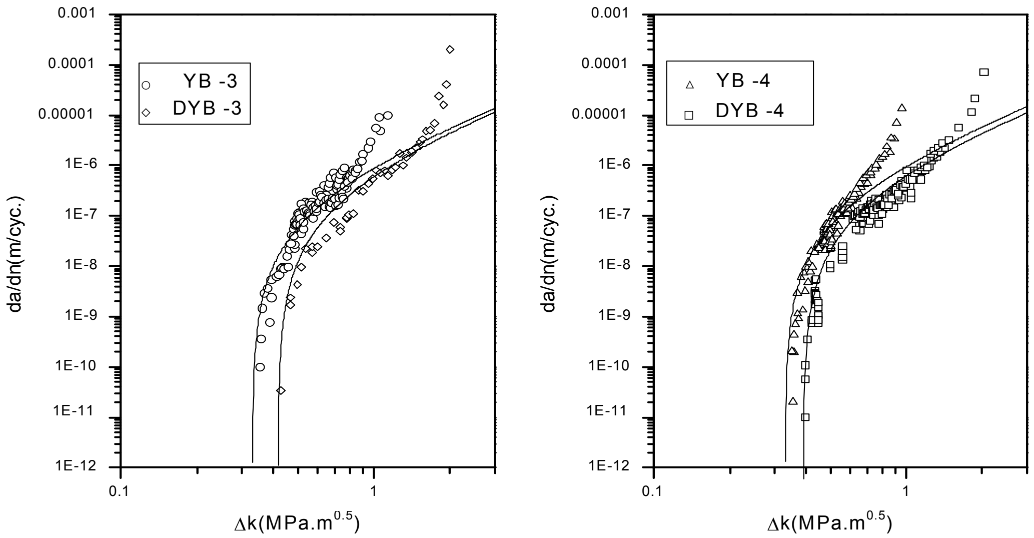

Figure 1 The Test Data of Four Types of PMMA and the Curves Obtained from Formula (4)

4. Modification to the formula

In order to enable eqn.(1) to describe the whole fcp curve of PMMA and to include the condition of critical crack propagation, eqn.(1) should be modified as follows [9]:

$$\frac{da}{dN} = B(\Delta K - \Delta K_{th})^2 f\left[\frac{K_{\max} - (K_{\max})_{th}}{K_{1c} - K_{\max}}, \frac{(K_{\max})_{th}}{K_{1c}}\right] \tag{5}$$

Where $(K_{max})_{th}$ is the fcp threshold defined as the value of K_{max} below which no fcp will occur.

Compared with the different $(K_{max})_{th}/K_{1c}$ values of metals (0~0.2), ceramics (0.4~0.8) and PMMA (0.2~0.4), the slope of fcp curve is closely related with the value of $(K_{max})_{th}/K_{1c}$. So the parameter $(K_{max})_{th}/K_{1c}$ should be located at the exponent place in the formula. Then we get eqn.(6):

$$\frac{da}{dN} = B(\Delta K - \Delta K_{th})^2 \left[\frac{(\Delta K - \Delta K_{th})/(1-R)}{K_{1c} - \Delta K/(1-R)}\right]^{\frac{\Delta K_{th}}{(1-R)K_{1c}}} \tag{6}$$

Where R is the stress ratio and the more details about this formulae can be found in Refs.(10). Figure 2 show the fcp curves drawn according to eqn.(6) and the test dots of test results. As it may be seen, eqn.(6) can well express the whole fcp curve of PMMA, including all three regions mentioned above, and it may be thought an almost perfect formula for fcp of PMMA.

5. Discussions

By analyzing the formula (6) and the slope of the curves in figure 2, It can be seen that:

1） the fcp rate of PMMA at near-threshold and intermediate regions are mainly controlled by the fcp coefficient (B) and the effective stress intensity factor (ΔK-ΔK_{th}). Because there is little difference of young's module (E) among different types of PMMA at room temperature, there is little difference of fcp coefficient too ($B=15.9/E^2$). So, the effective stress intensity factor (ΔK-ΔK_{th}) is the governing parameters of fcp in near-threshold region and intermediate region of PMMA.

2） The exponent ($\Delta K_{th}/K_{1c}$) of the formula modifies the shape of whole fcp curve. The higher of the value of ($\Delta K_{th}/K_{1c}$) the shorter of the intermediate region and the higher value of the curve slope. Comparing with normal PMMA, the lower fcp rate of oriented PMMA mainly results from the smaller ratio value of fcp threshold to fracture toughness ($\Delta K_{th}/K_{1c}$) as well as the higher values of fcp threshold and higher fracture toughness.

3） Every parameter in eqn.(6) has its certain physical meaning and the dimension relation between the both sides of the equation is the same. The material constants, ΔK_{th}, K_{1c} and B（or E） are the governing factors of fcp curve of PMMA, and if the value of these constants are known, the expressions of fcp curves of PMMA can be easily obtained without experiment.

Table 1 The Values of Young's Modulus, Fracture Toughness, Crack Propagation Threshold and Fcp Coefficient

Materials	E (GPa)	K_{1c} ($MPa.m^{0.5}$)	ΔK_{th} ($MPa.m^{0.5}$)	B (MPa^{-2})
YB-3	2.9	1.25	0.33	1.89E-6
DYB-3	3.1	2.23	0.42	1.65E-6
YB-4	2.8	1.10	0.33	2.03E-6
DYB-4	3.1	2.20	0.39	1.65E-6

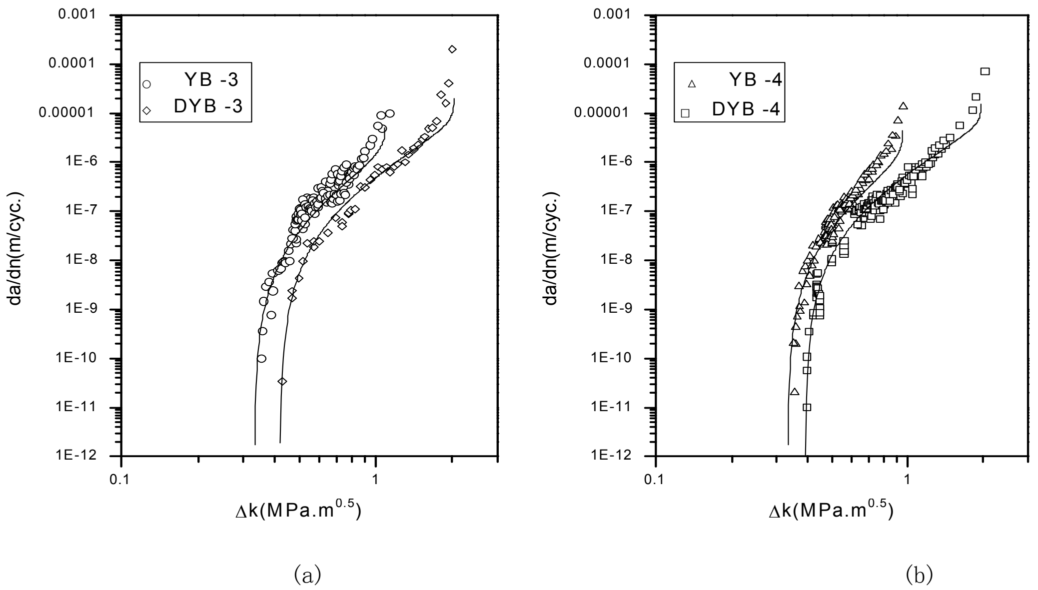

Figure 2 The Test Data of Four Types of PMMA and the Curves Obtained from Formula (6)

6. Conclusions

1） Eqn.(6) can be used to represent the test results of cyclic fcp of four types of PMMA in near-threshold-, intermediate- and rapid crack propagation regions well. This formula revealed the correlation between the fcp rate (da/dN) and the stress intensity factor (ΔK), the fcp coefficient (B), the fcp threshold (ΔK_{th}) and the fracture toughness (K_{1c}).

2） The value of fcp coefficient can be calculated by Young's modulus as $B=15.9/E^2$. So the coefficient and the parameters involved in the formula all have definite physical meaning. Therefore, the above mentioned formulae could be thought as the almost perfect formulae for cyclic fcp of PMMA.

3） The governing parameters of fcp of PMMA are, respectively, the effective stress intensity factor ($\Delta K-\Delta K_{th}$) in near-threshold region and intermediate region, and the difference between the fracture toughness and the maximum value of stress intensity factor (K_{1c}-K_{max}) in rapid crack propagation region. The fcp coefficient (B) and the ratio of fcp threshold to fracture toughness ($\Delta K_{th}/K_{1c}$) are the governing parameters in intermediate region.

4） Comparing with those parameters of the normal PMMA, the lower fcp rate of oriented PMMA mainly results from the higher values of fcp threshold, higher fracture toughness and the smaller ratio value of fcp threshold to fracture toughness.

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