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STRESS SINGULARITIES WITHIN THE CELLULAR CORE OF A SANDWICH PLATE

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ABSTRACT

Structural sandwich panels with two-dimensional cellular core are subject to an increased delamination hazard compared to layered materials consisting solely of homogeneous materials. The increased delamination hazard is induced by an incompatibility in the modes of deformation of the cellular core and the facesheets which causes singular stress fields in the interface between the layers. A closed-form analytical and a numerical investigation show that the singularity is of the pure non-oscillatory power-law type. The order of the singularity as well as the associated stress intensity factor depend on the cell wall angle and on Poisson's ratio of the cell wall material.

KEYWORDS

Sandwich, cellular core, delamination, stress singularity, complex potential, stress intensity factor

INTRODUCTION

Sandwich plates and shells are widespread structural elements in lightweight construction with superior specific strength and stiffness characteristics. In essence, a sandwich consists of three layers, two homogeneous facesheets of high stiffness and strength and an intermediate low density core, see fig. 1. As typical core material often two-dimensional cellular structures are employed. Whereas the main task of the facesheets is to carry the inplane forces and bending moments the task of the core is to maintain the distance betweeen the facesheets and it is to carry the transverse forces.

For the proper function of a sandwich the integrity of the facesheet/core bonding is of essential importance. The problem of relatively large facesheet/core debondings or delaminations has already been considered by various authors as e.g. [1], [2], [3]. The case of a beginning delamination in sandwich plates with a hexagonal cellular core has been investigated by [4]. A detailed analysis of the mechanical situation along the adhesive bond of a hexagonal core without delamination has been performed by [5].

In comparison to the case of a homogeneous sandwich core in the case of a hexagonal cellular core

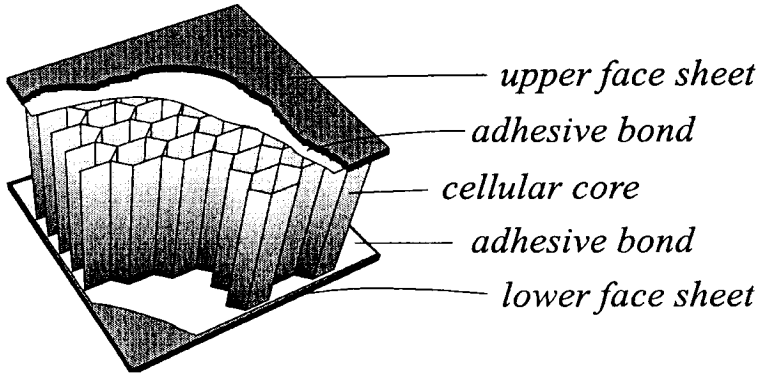

Figure 1: Sandwich construction principle

there is a somewhat increased delamination tendency which results from the incompatibility of the deformation modes of the cellular core and the facesheets respectively. Within the integrated sandwich compatibility is, however, enforced by the bond with the consequence of significant local stress concentrations in the vicinity of those interface locations where three cellular walls are joined to the facesheets. These stress concentrations have a singular character even in the absence of any delamination and thus are critical locations for the onset of delaminations.

The present work is concerned with this kind of singular stress concentration. By a closed-form asymptotic analysis it will be shown that the stress singularity is of a pure non-oscillatory power-law type. This is in good agreement with finite element analyses which in addition yield the according stress concentration factor.

CLOSED-FORM ANALYTICAL APPROACH

Fig. 2 shows the geometry of the hexagonal core of a sandwich plate to be considered. The cellular core is supposed to be doubly symmetric with two cell wall angles of the same size α. From manufacturing reasons the diagonal cell walls of fig. 2 are of simple thickness t whereas the horizontal cell wall has the thickness $2t$. For analysis reasons the coordinates x_1, x_2, r_I, φ_I, r_{II}, and φ_{II} shown in fig. 2 are introduced. As given loading a prescribed effective strain $\tilde{\varepsilon}_{11}$ is presumed.

For the closed-form analytical approach two simplifying assumptions are made. First, it is assumed that the facesheets due to their superior stiffness within the immediate vicinity force the cell walls to follow their deformation, in accordance with the underlying homogeneous effective strain $\tilde{\varepsilon}_{11}$. Second, the bending stiffness of the cell walls due to the small cell wall thickness is neglected. Then

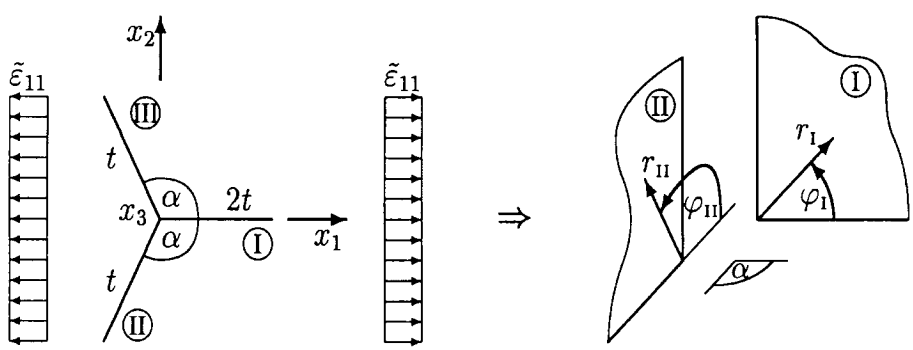

Figure 2: Cellular geometry and employed coordinates

the description of the mechanical field quantities within the cell walls can be accomplished by plane elasticity theory. For symmetry reasons it is sufficient to consider only two cell walls. With the local coordinates r and φ shown in fig. 2 the local stress and displacement fields can be represented through the formulas of Kolosov-Muskhelishvili [6]:

$$\begin{aligned} \sigma_{rr}+\sigma_{\varphi\varphi} &= 2\left(\Phi'(z)+\overline{\Phi'(z)}\right) \quad , \\ \sigma_{\varphi\varphi}-\sigma_{rr}+2i\sigma_{r\varphi} &= 2\left(z\Phi''(z)+\frac{z}{\overline{z}}\Psi'(z)\right) \quad , \\ 2G(u_r+iu_\varphi) &= \left(\kappa\Phi(z)-z\overline{\Phi'(z)}-\overline{\Psi(z)}\right)e^{-i\varphi} \quad , \\ & \text{with} \quad G=\frac{E}{2(1+\nu)} \quad , \quad \kappa=\frac{3-\nu}{1+\nu} \quad \text{(for plane stress conditions)}, \end{aligned} \tag{1}$$

where the quantities Φ and Ψ are complex potentials of the complex variable $z=e^{i\varphi}$, and the quantities E and ν are Young's modulus and Poisson's ratio, respectively.

In the present situation the following general representation is chosen for the complex potentials:

$$\begin{aligned} \Phi(z) &= Az^\lambda + Cz^\lambda \ln z \quad , \\ \Psi(z) &= Bz^\lambda + Dz^\lambda \ln z \quad , \end{aligned} \tag{2}$$

where the complex constants A, B, C, and D and the order $\lambda-1$ of the singularity are still undetermined. The stress and displacement components σ_{ij} and u_i have to satisfy the following boundary and compatibility conditions for an arbitrary radius r (for details see [8]):

$$\begin{aligned} u_r^I(\varphi=0) &= \tilde{\varepsilon}_{11}r \quad , \\ u_\varphi^I(\varphi=0) &= 0 \quad , \\ u_r^{II}(\varphi=\frac{\pi}{2}) &= u_r^I(\varphi=\frac{\pi}{2}) \quad , \\ u_\varphi^{II}(\varphi=\frac{\pi}{2}) &= (-\cos\alpha)u_\varphi^I(\varphi=\frac{\pi}{2}) \quad , \\ (-\cos\alpha)\sigma_{\varphi\varphi}^{II}(\varphi=\frac{\pi}{2}) &= \sigma_{\varphi\varphi}^I(\varphi=\frac{\pi}{2}) \quad , \\ \sigma_{r\varphi}^{II}(\varphi=\frac{\pi}{2}) &= \sigma_{r\varphi}^I(\varphi=\frac{\pi}{2}) \quad , \\ u_r^{II}(\varphi=\pi) &= \tilde{\varepsilon}_{11}(\cos^2\alpha)r \quad , \\ u_\varphi^{II}(\varphi=\pi) &= 0 \quad . \end{aligned} \tag{3}$$

As these conditions have to be fulfilled for all radii r the inhomogeneous set of equations can be transferred into a set of homogeneous equations by double differentiation. As the 8 equations (3) contain all two different kinds of functional dependences on r (with and without logarithmic dependence) they give in total 16 homogeneous equations for the 16 unknown real and imaginary parts of the complex constants A to D for the two considered cell walls I and II (see fig. 2). Conditions for the existence of non-trivial solutions for homogeneous systems of equations of the present kind have been given by [7]. These conditions comprise a zero-determinant of the coefficent matrix for the case of a pure power-law singularity. In addition the $(8-m)$th derivative of this determinant has to vanish, where m is the rank of the corresponding coefficient matrix. The determination of the eigenvalue λ from these conditions can be done numerically. When solving the system of equations no values λ with $\mathrm{Re}(\lambda)<0$ are permitted for energetic reasons. On the other hand also solutions λ with $\mathrm{Re}(\lambda)>1$ are excluded because these would belong to non-singular stress fields. By a correspondingly restricted numerical investigation it turns out that within the relevant range $0<\alpha\leq\pi$ and $0<\nu\leq 0.5$ there is no common solution with non-vanishing constants C and D. Thus, it can be concluded that $C=D=0$ and the stress singularity is of a pure power-law type. Considering the eigenvalues λ they are all found to be real, so that the singularity in any case is non-oscillatory. With the eigenvalue

λ for a given cell geometry with cell wall angle α and a given cell wall material with Poisson's ratio ν then the real and imaginary parts of the remaining complex constants A and B can be calculated from the corresponding homogeneous system of equations, so that finally the stress and displacement components are available in the form

$$\begin{aligned} \sigma_{ij}(r,\varphi) &= K^* r^{\lambda-1} f_{ij}(A,B,\varphi) \quad , \\ u_i(r,\varphi) &= \frac{1}{2G} K^* r^{\lambda} g_i(A,B,\varphi) \quad . \end{aligned} \tag{4}$$

with appropriately defined angular functions f_{ij} and g_i. The quantity K^* in (4) denotes a stress intensity factor that cannot be determined by an asymptotic analysis but results from the surrounding far field. Due to the different order of singularity it is to be noted that the stress intensity factor K^* cannot be compared directly with the well-known stress intensity factors K_I, K_{II}, and K_{III} of common fracture mechanics. The actual numerical exploitation of relations (4) reveals that in general all stress components σ_{rr}, $\sigma_{\varphi\varphi}$, and $\sigma_{r\varphi}$ are non-zero for $\varphi = 0$ and $\varphi = \pi$ which can be interpreted as the presence of a mixed mode stress singularity.

NUMERICAL APPROACH

In addition to the closed-form asymptotic analysis a numerical investigation has been performed by means of finite element method. To that end a detailed finite element model has been set up for a representative volume element of the given microstructure. The cell walls and the facesheets have been modelled by standard four-node shell elements in an enhanced-strain formulation with a mesh refinement in the region of the singularity. On the surfaces of the representative volume element periodic boundary conditions have been applied. The loading is given by prescribed displacements on the facesheets and along the outer cell wall edges of the representative volume element. The order $(\lambda - 1)$ of the singularity can be determined from the displacements u_1 of the nodal points along the cell wall junction. The exponent λ is obtained as the slope of u_1 as a function of the radial distance in a double-logarithmic representation.

Beyond this the stress intensity factor K^* can be determined from the stresses $\sigma_{\varphi\varphi}$ along the interface of cell wall I with the upper facesheet in the following way:

$$K^* = \lim_{r \to 0} \frac{\sigma_{\varphi\varphi}(r, \varphi = 0)}{r^{\lambda-1}}. \tag{5}$$

Within the numerical analysis this relation is used for the second to fifth element along the cell wall/facesheet interface. A convergence study gives stable results for a discretization of about 16000 finite elements for both the order of singularity and the stress intensity factor K^*. A comparison of the finite element results for the order of singularity shows very good agreement with the closed-form asymptotic analysis.

RESULTS

The presented closed-form asymptotic analysis reveals that the order $\lambda - 1$ of the considered singularity depends on both the cell wall angle α and Poisson's ratio ν of the cell wall material. The impact of α and ν on the exponent λ is shown in fig. 3, whereas fig. 4 shows the dependence of the stress intensity factor as it comes out from the performed finite element analyses. Obviously, the exponent $\lambda - 1$ vanishes for the limit case $\alpha \to \pi$ of a straight cell wall. On the other hand, the strongest singularity occurs in the case of a vanishing cell wall angle $\alpha = 0$. The impact of Poisson's ratio on the exponent λ is relatively weak, at least in the typical range $\nu = 0.2 \ldots 0.4$. For the stress intensity factors K^* smaller cell wall angles lead to significantly larger values, in particular when Poisson's ratio

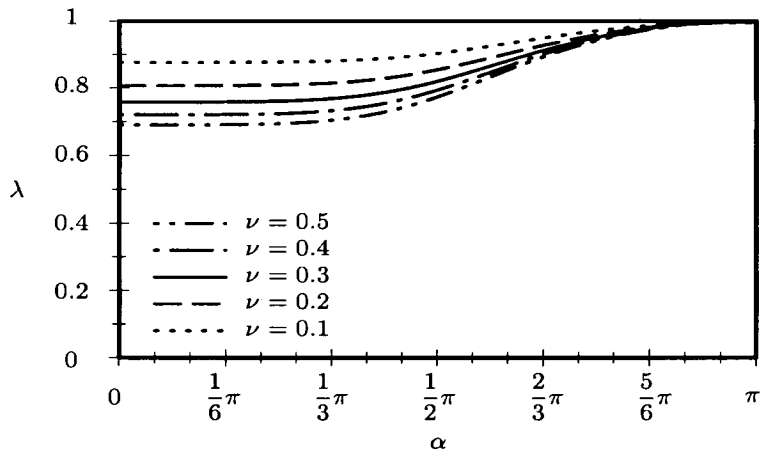

Figure 3: Order λ of the singularity as a function of α and ν

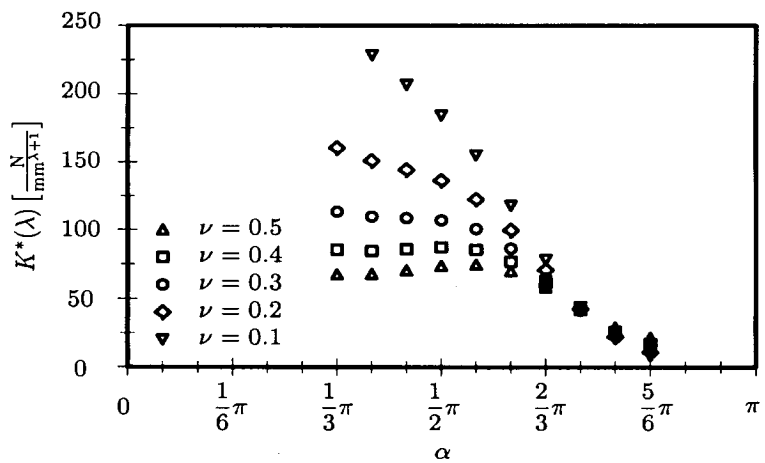

Figure 4: Stress intensity factor K^* as a function of α and ν

is small. Thus, with decreasing cell wall angle α the considered stress singularity increases in terms of the stress singularity exponent $\lambda - 1$ as well as in terms of the stress intensity factor K^*.

In addition to the variation of the cell wall geometry and Poisson's ratio of the cell wall material the influence of the stiffness of the facesheets has been investigated, too. In this regard the result is that there is no significant impact of the facesheet stiffness on the order of singularity $\lambda - 1$ and on the stress intensity factor K^* as long as Young's moduli E_f of the facesheets and E_c of the cell wall material and the thicknesses t_f of the facesheets and t_c of the cell walls are in the order of magnitude that is realistic for actual applications ($E_f \geq 50\text{GPa}$, $t_f \geq 1\text{mm}$, $E_c \approx 70\text{GPa}$, $t_c \leq 0.1\text{mm}$).

CONCLUSIONS

A corner singularity has been investigated that occurs at the junction of the cell walls of a hexagonal cellular core with the facesheets. It has been shown that this singularity is of a pure non-oscillatory power-law type with an order of singularity $\lambda - 1$ that depends on the cell wall angle α and Poisson's ratio ν of the cell wall material. These two quantities have also a significant impact on the resultant stress intensity factor K^*. The thickness and stiffness of the faceheets, on the other hand, have only a minor influence.

A fracture mechanical assessment of the delamination tendency is not directly possible on the basis of the available stress intensity factors as these correspond to varying orders of singularity. Some transfer of fracture mechanical concepts is however possible by means of a critical distance from the cell wall triple points. Beyond this, the calculated local stress fields can be employed as a basis for the analysis of hypothetical delamination cracks.

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