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STRAIN ENERGY DENSITY AS THE LINK BETWEEN GLOBAL AND LOCAL APPROACH TO FRACTURE

Hans-Jakob Schindler

Mat-Tec Ltd., Winterthur, Switzerland
(formerly EMPA, Duebendorf, Switzerland)

ABSTRACT

Fracture toughness is a material property only under certain restrictions concerning size, thickness, crack-length, loading mode, notch root radius, etc. To quantify such effects on the crack resistance the "local approach" is usually applied, which is very demanding in terms of numerical modelling and computational power. For practical engineering application as well as for teaching purposes, simpler analytical considerations are needed, which allow these effects to be predicted at least qualitatively. In the present paper simplistic mechanical models in conjunction with a suitable fracture criterion are outlined. A key element in the analysis is the local fracture criterion, which is required to link the global and local fracture behaviour. A suitable and promising parameter is the critical strain energy density. As pointed out in this paper, this parameter enables one to obtain relatively simple but surprisingly accurate formulas to predict effects of crack-tip constraints, finite notch root radius, and mixed mode loading on the fracture behaviour. Even problems of repeated loading like low-cycle fatigue of notched or cracked components can be dealt with.

KEY WORDS

Strain energy density, specific fracture energy, constraints, tearing, cleavage, notch, toughness.

INTRODUCTION

The concept of engineering fracture mechanics is essentially based on the hypothesis of autonomy of the fracture process zone, and a few parameters that are able to characterise the loading state of this zone, like the stress intensity factor in linear-elastic fracture mechanics or the J-integral in elastic-plastic fracture mechanics. The critical values of these parameters, called fracture toughness, characterise the resistance of the material against crack extension. Engineering fracture mechanics enables one to predict the behaviour of a crack without requiring a detailed analysis of the complex local fracture mechanisms. However, the autonomy of the fracture process zone is not guaranteed absolutely, since the fracture behaviour depends on the local constraints, which are affected by several system parameters, like component size, crack-length, geometry and loading case of the system. This means that fracture toughness is not a pure material property, but dependent on these parameters.

To quantify the effects of the above-mentioned parameters on the constraint conditions and the local fracture mechanisms, a detailed analysis of the crack-tip region is required, known as the "local approach". However, the corresponding non-linear 3D-FEM-analysis to compute the basic local parameters is quite demanding in terms of modelling, computational effort, hard- and software capacity, thus not well suited for engineering application. For such practical purposes as well as in teaching much simpler, rather analytical approaches are needed.

In this paper, semi-analytical ways to deal with the above mentioned local effects are pointed out, including constraints, notch root radius, mixed mode loading, and cyclic loading. A key element in such an analysis is a suitable failure criterion. It is shown how the critical strain energy density as suggested by Gillemot [1] can serve for this purpose. This material property seems to represent a linking parameter between local fracture and the global behaviour, and to control most aspects of ductile fracture. Therefore the first part of the paper deals with the definition and determination of this interesting physical quantity. Then simplistic local models to predict the behaviour of fracture toughness by simple closed-form formulas are outlined. Due to a lack of space the author restricts himself to discussing the key ideas and presenting the main results, rather than giving strict mathematical derivations. The objective of this paper is just to give an overview on the possibilities of these types of semi-analytical and semi-local approaches.

CRITICAL STRAIN ENERGY DENSITY

According to Gillemot's general failure criterion [1] an elastic-plastic material will fail in a ductile manner if the strain energy density U reaches a critical, material-dependent value U_f, i.e. if

$$U = \int_0^{\varepsilon_{ij}} \sigma_{ij} \cdot d\varepsilon_{ij} = U_f \tag{1}$$

U_f is a material property. Since (1) holds for any loading case, the simplest way to determine U_f is by a uniaxial tensile test, where it represents the area under the true-stress-true strain-curve up to fracture. (right hand side of Fig. 1). The average true strain during the entire test, including the necking phase, is obtained as $\varepsilon_t=\ln(A_0/A)$, where A_0 and A denote the initial cross section and the actual one, respectively. In the range of uniform strain (i.e. $0< \varepsilon_t < \ln(1+A_g)$) the true stress-strain-curve can be approximated as

$$\sigma_t = C \cdot \varepsilon^n \qquad \text{with} \qquad C = \frac{R_m \cdot (1+A_g)}{n^n}; \qquad n=\ln(1+A_g) \tag{2}$$

where A_g denotes the engineering strain at maximum load (i.e. standard uniform fracture strain). In the subsequent necking phase, the true stress-strain-diagram is often more or less straight, as shown in Fig. 1. With these assumptions, U_f is determined by

$$U_f = U_m + U_{nf} \quad \text{with:} \qquad U_m = \frac{R_m \cdot (1+A_g) \cdot n}{n+1} \qquad U_{nf} = \left[\frac{R_m \cdot (1-Z) + R_f}{2 \cdot (1-Z)}\right] \cdot \left[\ln\frac{1}{1-Z} - \ln(1+A_g)\right] \tag{3}$$

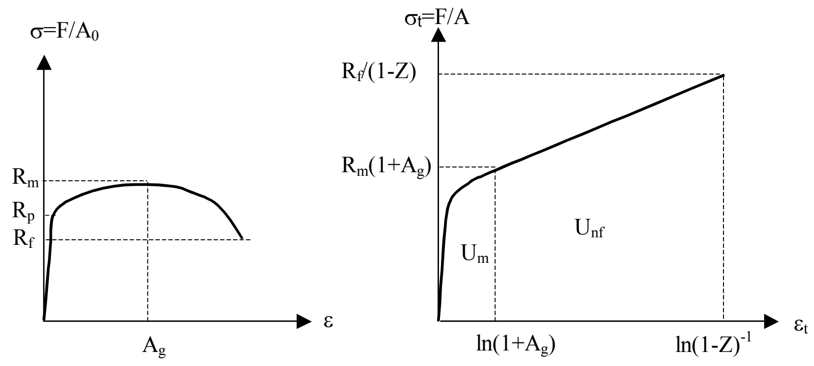

Fig. 1: Determination of the true stress-strain-curve (right) from the engineering one (left) and definition of the portions U_m and U_{nf} of the specific fracture energy $U_f = U_m + U_{nf}$

$Z=(A_0-A_f)/A_0$ denotes the standard reduction of area of a tensile test specimen. If R_f is not known, which often the case if the full stress-strain-curve is not available, the following approximate relation is useful:

$$U_{nf} = \frac{R_m}{2}\left[\frac{1+(1+A_g)\cdot(\ln(1-Z)-n)}{\ln(1-Z)-n}\right] \tag{4}$$

Eq. (4) follows analytically from the assumptions underlying (2) and (3) and the condition of continuity at the transition at $\varepsilon_t=\ln(1+A_g)$. If not even A_g is known, then we suggest the rough estimation

$$U_f \cong \frac{\sigma_f \cdot Z}{1-Z} \tag{5}$$

according to [3], where $\sigma_f=(R_p+R_m)/2$ denotes the commonly used flow stress.

LOCAL STRESSES AND STRAINS

It is well known from non-linear finite element calculations that the so-called HRR-field [3] breaks down in the vicinity of the crack tip. Fig. 2 shows schematically the actual distribution of the stress in y-direction. The peak stress, σ_{ymax}, can be expressed as

$$\sigma_{ymax}=\gamma\cdot R_p \tag{6}$$

where the factor γ depends primarily on the crack-tip constraints and the hardening behaviour of the material. It is about 3 for non-hardening elastic-plastic materials and standard constraint conditions, which means plane strain and saturated in-plane constraints. Actually, to determine γ accurately a 3D-FEM-analysis is required. However, according to previous findings of the author [4, 5], a rough estimation sufficient for practical purposes is possible by

$$\gamma \cong \left(2m + \frac{\upsilon \cdot T_{max}}{(1-2\upsilon)\cdot R_p}\right)\cdot\frac{\sigma_f}{R_p} \tag{7}$$

where ν is Poisson's ratio and m is the factor appearing in the basic relation

$$J=m\cdot R_p\cdot\delta \tag{8}$$

T_{max} is the "T-stress" (second term of Williams' expansion [6]) at maximum load of the system.

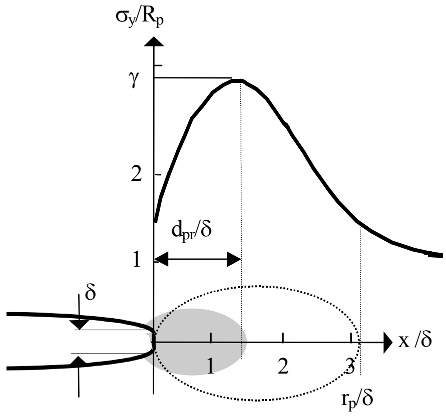

Fig. 2: Non-dimensional representation of the stress distribution in the vicinity of a crack-tip

The ductile fracture process by void growth and coalescence takes place in a relatively small area of width d_{pr} next to the crack-tip (shaded area in Fig. 2). The plastic strain in this "fracture process zone" in y-direction, ε_{yp}, can be assumed to be proportional to the crack-tip opening displacement δ, because the initial volume taking part in this process will not change much during the loading process. Thus,

$$\delta \propto \varepsilon_{yp} \tag{9}$$

Assuming that a certain portion of strain energy density, corresponding about to U_m, is already consumed in the preceding phase of general plastic straining according to the HRR-strain-field, Gillemot's criterion (1) applied to the process zone can be written as

$$\gamma \cdot R_p \cdot \ln(1+\varepsilon_{yp}) = U_{nf} \tag{10}$$

By (8) – (10) the local strain is related to the global crack load J.

EFFECT OF CRACK-TIP CONSTRAINTS ON FRACTURE TOUGHNESS

Ductile Tearing

The value of J near initiation of ductile tearing is size independent only if the corresponding standard size and geometry requirements [7] are met. The effect of reduced constraints on the J-value at crack-initiation, is experimentally well known [8], but difficult to model and predict analytically. However, from (8) – (10) a relation between the constraint-characterising parameters m and γ and the apparent fracture toughness J_{it} follows readily:

$$\frac{J_{it}}{m \cdot \left\{ \exp\left[\frac{U_{nf}}{R_p \cdot \gamma} \right] - 1 \right\}} = \text{const} \tag{11}$$

J_{it} represents a near initiation value of the J-R-curve, corresponding to the standard $J_{0.2/Bl}$ in the case of standard constraint conditions [7]. For a non-hardening material the latter correspond to about $m \approx 1.5$ and $\gamma \approx 3$, so $J_{it}(m \approx 1.5, \gamma \approx 3)$ equals $J_{0.2/Bl}$. By (11) the effect of reduced constraints (reflected by lower values of m and γ) on the fracture toughness can be determined. Predictions from (11) compare well with experimental results [4].

Cleavage Fracture

In some elastic-plastic materials an unstable cleavage fracture may be triggered at a certain value of J, J_c, which can be lower than the above considered J_{it}. J_c is known to be significantly constraint-dependent. In the following, this dependence is estimated. As discussed in [4] unstable cleavage require the following two criteria to be met:

i) The maximum stress in the vicinity of the crack tip must exceed the cleavage stress σ_c^*, i.e.

$$\sigma_{ymax} = \gamma \cdot R_p > \sigma_c^* \tag{12}$$

ii) The elastic energy $W_{el}^* = \int U_{el}\, dV$ stored in a critical Volume V* in the vicinity of the process zone must be sufficient to produce a cleavage fracture in the range $0 < x < d_{pr}$.

Criterion ii) means that the ratio W_{el}^*/d_{pr} has to exceed a certain critical value. Using the proportionalities $U_{el} \propto (\gamma \cdot R_p)^2$ and $V^* \propto \delta^2$ leads to $W_{el}^* \propto (\gamma \cdot R_p)^2 \cdot \delta^2$. With $d_{pr} \propto \delta$ one readily finds the proportionality

$$J_c \cdot \frac{\gamma^2}{m} = const \qquad \text{for } \gamma > \sigma_c^*/R_p \tag{13}$$

As discussed above, $J_c(m=1.5,\gamma=3)$ corresponds to the standard J_{uc} according to [7]. By (13) the effect of reduced constraints on J_{uc} can be determined. For $\gamma<\sigma_c^*/R_p$ no cleavage occurs. Predictions from (13) compare well with experimental results shown in [8].

NOTCH TOUGHNESS

Consider a sharp notch with a finite root radius ρ. If the end-points of the integration-path to calculate J are located on the parallel surfaces, J is path-independent and, thus, able to characterise the loading state of the notch. It can be calculated either on a remote path Γ_r (dashed line in Fig. 3) or a local path surrounding the notch root (dotted path-sections Γ_1 and Γ_4). In order to estimate the effect of the root radius on the critical J-integral, J_{inotch}, we calculate J for a notch that is assumed to be in its critical loading state, which means just before initiation of crack extension occurs. Under this condition, the local path to calculate J must not simply follow the notch surface (sections Γ_1 and Γ_4), because near y=0 there already are voids which act as discontinuities in the strain field, disturbing the path-independence of J. Therefore the integration path is chosen to surround the process zone by a "detour" denoted by Γ_2 and Γ_3 in Fig. 3. In case of sharp notches (i.e. ρ<<plastic zone width r_p) the stress-strain-fields in the vicinity of the x-axis is expected to be about the same as in the case of a crack, so the corresponding integration is expected to give about the value of the critical J of a crack, J_{icrack}. The parts Γ_1 and Γ_4 of the integration path deliver the contribution ΔJ_ρ resulting from the notch radius, which has the form $U_f \cdot \rho$ for dimensional reasons. Thus:

$$J_{inotch} = J_{icrack} + \Delta J_\rho = J_{icrack} + c \cdot U_f \cdot \rho \tag{14}$$

The adjustable factor c turned out to be about 0.7, for mild steel [2], high strength steel [9] and even brittle materials like ceramics.

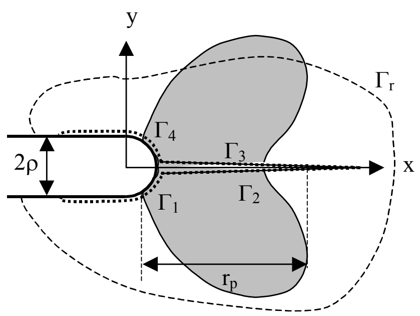

Fig. 3: Schematic representation of the root region of a sharp notch and the corresponding plastic zone

FRACTURE TOUGHNESS IN MIXED MODE

The maximum hoop-stress criterion as commonly used in LEFM for mixed-mode-loading is known to be not valid in the case of ductile fracture. Various experimental studies reveal that the critical stress-intensity factor in Mode II, K_{IIC}, is about 2 – 3 times higher than K_{Ic}. This behaviour can be explained, as shown below, by postulating for physical reasons an analogy between mode I and mode II and extending the relation (11) to mode II. In mode II there is essentially no constraint, so one can assume $\gamma=\gamma_{II}=1$ and $m=m_{II}=1$ (values of m for mode II are hard to find in the literature, eventually it lies in the range $0.6<m_{II}<1$). This leads to

$$\frac{J_{IIit}}{J_{it}} = \frac{m_{II} \cdot \exp\left(\frac{U_{nf}}{R_p}\right) - 1}{1.5 \cdot \left[\exp\left(\frac{U_{nf}}{3R_p}\right) - 1\right]} \tag{15}$$

where J_{IIit} denotes the critical J for pure antisymmetric loading (i.e. pure mode II). In case of mixed-mode loading, a physically plausible mode-interaction is a linear damage accumulation in terms of δ or J, thus

$$\frac{K_I^2}{K_{Ic}^2} + \frac{K_{II}^2}{K_{IIc}^2} < 1 \tag{16}$$

with

$$K_{IIc} = \sqrt{\frac{J_{IIit} \cdot (1 - \upsilon^2)}{E}} \tag{17}$$

This relation represents at least a qualitative explanation of experimental data [9].

DISCUSSION AND CONCLUSIONS

The purpose of the present paper was to show that even complex phenomena associated with local fracture, like constraint effects, influences of a mode-II-loading components, or a finite notch-tip radius, can be successfully and efficiently treated by simple analytical models. Unlike detailed numerical models, they result in easy to handle closed form formulas, which allow these effects to be discussed qualitatively, which is advantageous in practical applications as well as in teaching.

It shall be emphasised that the critical strain energy density can serve well for various purposes concerning fracture. Especially it is well suited to link global to local behaviour. It is the author's believe that the role and the possibilities of this parameter in engineering fracture mechanics is still not yet fully recognised. Another big field of its application, which was not discussed here due to the lack of space, is the behaviour of notched and cracked components under repeated elastic-plastic-straining, like low-cycle fatigue. As shown in [10], even high-cycle fatigue crack growth can be analytically treated based on a corresponding local fracture criterion.

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