ORAL/POSTER REFERENCE: ICF100839OR

STOCHASTIC ANALYSIS OF FATIGUE CRACK GROWTH

C. Proppe[1] and G.I. Schuëller[1]

[1] Institute of Engineering Mechanics, Leopold-Franzens University, A-6020 Innsbruck, Austria, EU

ABSTRACT

Due to cost and time limitations, experiments for fatigue crack growth are only feasible below certain cycle numbers. In this paper, it is investigated whether or not fatigue crack growth laws allow for an extrapolation of the statistics of the cycle number to reach a certain crack length from limited experimental data, and hence reduce costs.

KEYWORDS

fatigue crack growth laws, probabilistic fracture mechanics

INTRODUCTION

From experimental investigations [1, 2] fatigue crack growth appears as a process with random properties. These random properties seem to vary not only from specimen to specimen but also during crack growth, cf. Figure 1. A great number of stochastic models that account for the random behavior have been proposed. They are based either on suitable “randomized” empirical crack growth laws or on data fitting.

In this paper, the practical use of already proposed models is assessed. In engineering practice, there is often insufficient financial means as well as time available to carry out fatigue tests with a greater number of load cycles. As a consequence, there is a need to extrapolate the fatigue crack growth prediction by means of stochastic models from data that were obtained from a limited number of load cycles.

This paper investigates the requirements in complexity for a stochastic model of fatigue crack growth in order to approximately extrapolate the statistical characteristics of fatigue crack growth. For this, results from experimental investigations are compared to model predictions, where from all specimen only a part of the measured crack lengths were used to calibrate the model.

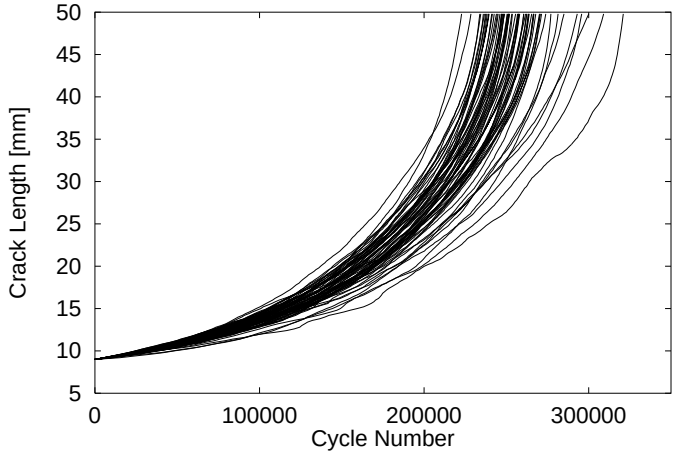

Figure 1: Fatigue crack growth for centre cracked specimen as measured by Virkler et al. [1].

FATIGUE CRACK GROWTH MODELS

Elementary fatigue crack growth laws assume a relationship between the stress state at the crack tip and the crack growth velocity. Several deterministic crack growth laws have been proposed for fitting the mean values of experimentally measured data. These laws have then been extended to account for the random phenomena inherent in crack growth. The main efforts are summarized in this section.

Deterministic Fatigue Crack Growth Laws

The extended crack growth law

$$\frac{\mathrm{d}\,a}{\mathrm{d}\,N} = C\frac{(1-R)^m \Delta K^n}{((1-R)K_c - \Delta K)^q}, \tag{1}$$

where a is the crack length, N the number of load cycles, ΔK the increment of the stress intensity, K_c the critical stress intensity factor and $R = \sigma_{max}/\sigma_{min}$ the stress ratio summarizes Paris' law ($m = q = 0$), Forman's law ($m = 0, q = 1$) and Walker's law ($q = 0$). Paris' law is a straight line in the $\ln(\mathrm{d}\,a/\mathrm{d}\,N)$-$\ln(\Delta K)$ representation. This is a good approximation for stable crack growth, but does not account for the different behavior of small cracks and in the range of instable crack growth. The typically S-shaped crack growth curve can only be approximated by a nonlinear relationship between the logarithm of the crack growth velocity and the logarithm of the stress intensity factor, respectively. For example, a cubic polynomial approximation would lead to

$$\ln(\frac{\mathrm{d}\,a}{\mathrm{d}\,N}) = C_0 + C_1 \ln(\Delta K) + C_2(\ln(\Delta K))^2 + C_3(\ln(\Delta K))^3, \tag{2}$$

where dependence on R can be accounted for by substituting Elber's [3] effective stress intensity factor $\Delta K_{eff} = (A + BR)\Delta K$, where A and B are constants that have to be determined from experiments, for ΔK. A similar approximation can be obtained from the hyperbolic sine law

$$\frac{\mathrm{d}\,a}{\mathrm{d}\,N} = 10^{(C_1 \sinh(C_2(\ln \Delta K + C_3)) + C_4)}. \tag{3}$$

Stochastic Crack Growth Models

Relatively simple stochastic crack growth models assume that the uncertainties in one or several parameters of the deterministic crack growth laws are modeled as random variables. These models, however, can not explain the variability of the crack growth rate during the crack growth process.

Models based on stochastic differential equations, in fact, are suited to account for this type of variability. E.g. Tsurui et al. [4, 5] and Tang and Spencer [6] proposed crack growth equations with a time-correlated stochastic process. A model with a jump process has been introduced in [7].

If the random variations of the crack growth process of a single specimen are attributed to material inhomogeneity, the correlation of the stochastic process should rather be attributed to the spatial dimension. Therefore, Ortiz and Kiremidjian [8] proposed a model of the form

$$\frac{\mathrm{d}\,a}{\mathrm{d}\,N} = f(\delta K)Z(a), \tag{4}$$

where $\log Z(a)$ is a Gaussian process whose correlation depends on the crack length.

Markov chain models [9] reflect the fact that the load process is often discretized into independent events and that an absorbing state - failure -can be introduced. Markov chain models can be directly fitted to experimental data. However, this makes predictions for other load conditions or geometrical configurations a difficult task. This problem can be circumvented by using a suitable stochastic crack growth model for the determination of the transition probabilities [10].

PARAMETER IDENTIFICATION

As has been pointed out by several authors [11, 12], parameter estimation should be carried out with the crack length as independent variable in order to avoid a systematic bias. Defining the approximation error as the sum of the squared differences between the experimentally and theoretically predicted cycle numbers at each measured crack length, an optimal set of parameters can be determined for each specimen. Assuming a distribution (e.g. log-normal for C and normal for n) for the random variables in the model, the distribution parameters as well as the correlation can be estimated from these optimal sets of parameters.

The properties of the stochastic process are then obtained from an analysis of the residuum. Considering all samples, the standard deviation of the difference between the cycle numbers obtained from the experiments and from the deterministic model with optimal sample parameters respectively can be computed either for each measured crack length separately or cumulated for all measured crack lengths. By assuming a Gaussian distribution with this standard deviation and zero mean for the error in the cycle number at each crack length, one obtains a stochastic process model with an non stationary and a stationary Gaussian white noise process, respectively.

SENSITIVITY ANALYSIS

The following arguments demonstrate that the sensitivity of the predicted cycle numbers with respect to small changes of the parameters in the crack growth law is very high. Thus, possible variations of these parameters have to be taken into consideration.

Consider the Paris law for an infinite plate under tension ($\Delta K = \Delta\sigma\sqrt{\pi a}$). The solution of the differential equation yields

$$N(a) = \frac{1}{C\pi^{n/2}(\Delta\sigma)^n}\left(\frac{a^{(1-\frac{n}{2})} - a_0^{(1-\frac{n}{2})}}{1-\frac{n}{2}}\right), \tag{5}$$

where a_0 is the initial crack length at $N = 0$. The derivatives of the number of load cycles with respect to the parameters C and n read:

$$\frac{\mathrm{d}\,N}{\mathrm{d}\,C} = -\frac{N(a)}{C} \tag{6}$$

and

$$\frac{\mathrm{d}\,N}{\mathrm{d}\,n} = -\frac{1}{2}N(a)\ln(\pi) - N(a)\ln(\Delta\sigma) + \frac{aN(a)}{2-n} - \frac{1}{2}\frac{\ln(a)a^{(1-\frac{n}{2})} - \ln(a_0)a_0^{(1-\frac{n}{2})}}{(1-\frac{n}{2})C\pi^{n/2}(\Delta\sigma)^n}. \tag{7}$$

From this, one can see that the number of cycles to reach a certain crack length is very sensitive to changes of the parameter C. Similar observations can be made for Forman's and Walker's law.

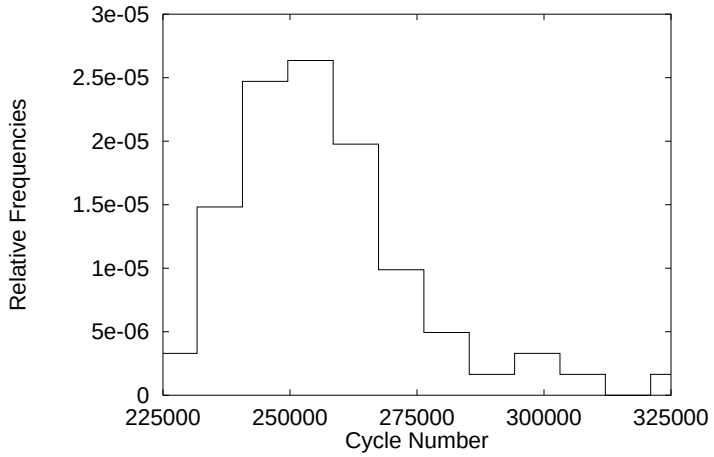

Figure 2: Histogram for the cycle numbers to reach a crack length of 49.8 mm obtained from the experiments carried out by Virkler et al. [1].

CYCLE NUMBER PREDICTION

The experimental results of Virkler et al. [1] have been used to study the prediction capabilities of Paris´ and Forman´s law with random parameters andwith an additional Gaussian white noise process which represents the randomness during crack growth. The target was the prediction of the cycle number to reach a crack length of 49.8 mm – the ultimate crack length measured by Virkler et al. From the experimental results, the mean cycle number to reach this crack length is 257164, with a standard deviation of 18310 (see Figure 1). This leads to a coefficient of variation of 7.12 %, which is relatively low, as the experiments have been carried out under well controlled conditions. The distribution is nearly log-normal with a skewness of 1.15. The histogram is given in Figure 2.

In the fatigue crack growth laws of Paris and Forman, the parameter C was assumed to be lognormally distributed and the correlation between $\ln(C)$ and m was determined by linear regression. The parameters were estimated from the full data set and from the first part of the cycle numbers, respectively. The relative errors, defined by

$$\frac{X_{theory} - X_{experiment}}{X_{experiment}}, \tag{8}$$

where X_{theory}, $X_{experiment}$ are the mean value, standard deviation, coefficient of variation (c.o.v.) or the skewness coefficient obtained from computations and experiments, resp., are summarized in Table 1 and 2. The theoretical results were estimated from 10000 generated sample curves.

One can see that at least 50% of the cycle numbers need to be considered in order to accurately predict the mean cycle number to reach the final crack length of 49.8 mm. However, the other statistical characteristics are not well represented, even by when all experimental data are being considered. Thus, the random variable model seems to be only suitable for mean value predictions.

Table 3 and 4 list the corresponding results for a stochastic process model with random variables and additional stationary Gaussian white noise. It can be seen that the error in the standard deviation is reduced by approximately 30%, but remains still relatively large. Results obtained with a non stationary process were nor much better. Also in this case, a limited amount of data (at least 50 % in order to accurately predict the mean value) is sufficient for the calibration of the model, i.e. its parameters.

TABLE 1:

PERCENTAGE RELATIVE ERROR FOR THE PREDICTION OF CYCLE NUMBERS TO REACH A CRACK LENGTH OF $a = 49.8\text{mm}$. PARIS´ LAW WITH RANDOM VARIABLES.

percentage of cycle numbers retained	mean	std. deviation	c.o.v.	skewness
100	-2	-88	-87	-63
50	-9	-62	-58	13
25	-15	12	32	18
10	-29	93	171	58

TABLE 2:

PERCENTAGE RELATIVE ERROR FOR THE PREDICTION OF CYCLE NUMBERS TO REACH A CRACK LENGTH OF $a = 49.8\text{mm}$. FORMAN´S LAW WITH RANDOM VARIABLES.

percentage of cycle numbers retained	mean	std. deviation	c.o.v.	skewness
100	-5	-84	-83	-60
50	-13	-74	-71	32
25	-19	-11	10	13
10	-32	62	137	40

TABLE 3:

PERCENTAGE RELATIVE ERROR FOR THE PREDICTION OF CYCLE NUMBERS TO REACH A CRACK LENGTH OF $a = 49.8\text{mm}$. PARIS´ LAW WITH RANDOM VARIABLES AND ADDITIONAL GAUSSIAN WHITE NOISE.

percentage of cycle numbers retained	mean	std. deviation	c.o.v.	skewness
100	-2	-66	-65	-94
50	-9	-45	-40	69
25	-15	22	45	-15
10	-29	97	177	38

TABLE 4:

PERCENTAGE RELATIVE ERROR FOR THE PREDICTION OF CYCLE NUMBERS TO REACH A CRACK LENGTH OF $a = 49.8\text{mm}$. FORMAN´S LAW WITH RANDOM VARIABLES AND ADDITIONAL GAUSSIAN WHITE NOISE.

percentage of cycle numbers retained	mean	std. deviation	c.o.v.	skewness
100	-5	-61	-59	-89
50	-13	-50	-43	81
25	-19	2	26	-32
10	-32	68	146	16

CONCLUSIONS

In this paper, the practical use of probabilistic fatigue crack growth models is assessed. By comparison with experimental data, the possibility to extrapolate the crack growth behavior is investigated for various models with different complexity.

It has been shown that random variable models may be used to extrapolate the mean cycle number to reach a certain crack length. However, these methods will fail to predict other statistical characteristics. Especially, they underestimate the variance. Hence, in order to account for these characteristics, more complex models have to be established.

Stochastic process models, where the residuum is explained by a Gaussian white noise process, lead to a better approximation of the second moments. However, the relative errors are still quite large.

Both random variable and stochastic process models respectively do not reproduce the crossing of measured crack growth curves from different specimen well. For the random variable models under consideration, crossing of crack growth curves is impossible, while for stochastic process models, the model predicts more crossings than are experimentally observed. This fact has been also noticed by other investigators, (see e.g. Bolotin [13], p. 213).

For reliability predictions with usually small probabilities, random variable models are much more efficient than stochastic process models, as variance reducing Monte Carlo simulation schemes can be easily applied.

REFERENCES

1. Virkler, D.A., Hillberry, B.M. and Goel, P.K. (1979). *J. Eng. Mat. Tech.* 101, pp. 148.
2. Ghonem, H. and Dore, S. (1987). *Eng. Fract. Mech.* 27, pp. 1.
3. Elber, W. (1971). *Damage Tolerance in Aircraft Structures*, pp. 230-242, ASTM STP 486.
4. Tsurui, A., Tanaka, H. and Tanaka, T. (1989). *Prob. Eng. Mech.* 4, pp. 120.
5. Tsurui, A., Nienstedt, J., Schuëller, G.I. and Tanaka, H. (1989). *Eng. Fract. Mech.* 34, pp. 153.
6. Tang, J. and Spencer Jr., B.F. (1989). *Eng. Fract. Mech.* 34, pp. 419.
7. Lin, Y.K., Wu, W.F. and Yang J.N. (1985). In: *Probabilistic Methods in Mechanics of Solids and Structures*, pp. 103-110, Eggwertz, S. and Lind, N.C. (Eds.), Springer, Berlin.
8. Ortiz, K. and Kiremidjian, A.S. (1986). *Eng. Fract. Mech.* 24, pp. 657.
9. Bogdanoff, J.L. and Kozin, F. (1985). *Probabilistic Models of Cumulative Damage*, Wiley, New York.
10. Oswald, G. F. and Schuëller, G.I. (1984). *Eng. Fract. Mech.*, 20, pp. 479.
11. Kozin, F. and Bogdanoff J.L. (1981). *Eng. Fract. Mech.* 14, 59.
12. Ostergaard, D.F. and Hillberry, B. M. (1983). In: *Probabilistic Methods for Design and Maintenance of Structures*, ASTM STP 798.
13. Bolotin, V.V. (1999). *Mechanics of Fatigue*, CRC Press, Boca Raton.