REFERENCE: ICF100665OR

STATISTICAL STUDY OF FRACTURE IN CONCRETE

Tai K. Wang[1], J.G.M. van Mier[2] and T. N. Bittencourt[1]

[1]Department of Structural Engineering and Foundation
Polytechnic School of University of Sao Paulo, Sao Paulo, Brazil
[2]Faculty of Civil Engineering and Geosciences
Delft University of Technology, Delft, the Netherlands

ABSTRACT: This paper describes the use of statistical distributions (Gaussian and Weibull distributions) for simulating the material heterogeneity of concrete in order to overcome the computational problems which limit the application of lattice-type fracture models in large-scale structures. Uniaxial tensile tests are simulated using a regular triangular lattice to compare with the numerical results obtained earlier by Van Vliet [3]. The numerical results obtained by this method proved to be useful to predict the structural behaviour of concrete. Since the statistical distributions omit the influence of real microstructure of concrete, the method appears to be less suitable for simulating correct crack patterns.

KEYWORDS: statistical distribution, numerical simulation, lattice model, concrete

INTRODUCTION

In the "Delft lattice model" concrete is schematised as a network of two noded Bernoulli beams [2,5,9]. Concrete is a heterogeneous material, and the most straightforward way to include effects from heterogeneity is through direct implementation of disorder. Several methods can be distinguished [1,5]. The most common way to include the material heterogeneity is to superimpose the mesh (lattice) on top of a computer generated or digital image of a real concrete structure [1,5]. For concrete three material phases are distinguished, namely aggregate, matrix and interfacial transition zone (bond). Different stiffness and strength are assigned to the beams that fall in each phase. By removing in each loading step the beam element with the highest stress over strength ratio, fracture is simulated. A simple fracture criterion is used as described in [2,5], which is based on the effective stress.

Note that lattice analyses are size dependent, although this deficiency is overcome to a large extent by including the condition that the individual beam length should be smaller than (at least) 1/3 of the minimum aggregate of the specimen. Then, it is necessary to use rather small beam lengths, which require considerable computational power. One of the solutions is to employ statistical distributions to introduce the material heterogeneity on coarser meshes without discretized aggregate structures. The strength of the beam elements is the most obvious choice as parameter in a statistical distribution. Several examples of the use of statistical distributions can be found in [5,6,7]. However, a systematic survey of the effect of a certain statistical distribution on fracture processes in concrete has never been carried out. In this paper Gaussian and Weibull distributions are employed to evaluate the effect of different scales of heterogeneity.

NUMERICAL SIMULATIONS

In order to allow for a comparison with the results obtained by Van Vliet [3], all the elastic computations were performed on a regular triangular lattice of 80 × 80 mm^2. Figure 1 shows the geometry, boundary and load conditions used by Van Vliet [3]. Firstly, five examples were run with the beam length l =1 mm and different aggregate structures were generated by computer with $2 \leq d \leq 16$ mm and $P_k = 0.75$ for these five examples respectively, where d is particle diameter and P_k the volume ratio of aggregates. The particles were assumed to be distributed according to a Fuller curve. The input parameters are: E_m=E_b=25 GPa, E_a=70 GPa (Young's modulus of matrix, bond and aggregate, respectively) and f_m=5MPa, f_b=1.25MPa, f_a=10MPa (tensile strength of matrix, bond and aggregate phases, respectively) where the beam height was given from the relation h/l with the global Poisson ratio $\nu = 0.2$ [10].

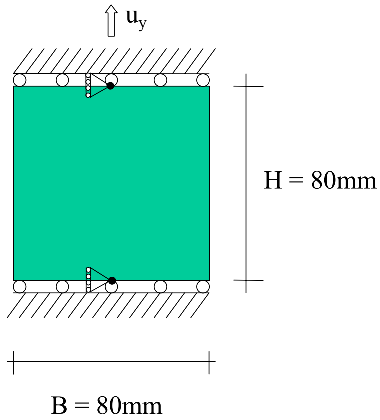


Figure 1. Geometry, boundary and load conditions used for numerical simulations.

Secondly, the same numerical simulations were performed using random number generators according to the Gaussian and the Weibull distribution, to assign different tensile strengths to the beams in a regular triangular lattice. Four different scales were investigated (i.e. beam lengths of 1, 2, 4 and 10 mm),which are compared with the five aggregate structures mentioned above with lattice length l = 1mm. A minimum and a maximum tensile strength was defined for the beam elements (f_{min} = 1.25 MPa and f_{max} = 10 MPa), which correspond to the tensile strength of the bond and the aggregate elements of the particle structures. When the generated strength is smaller than f_{min} or greater than f_{max} , a new value will be generated. For the Gaussian distribution, two input parameters were employed, namely the mean value (μ) and standard deviation (λ). The mean strength was taken equal to 6 MPa, whereas the standard deviations varied from 2 to 4 MPa. For the Weibull distribution, two parameters were employed, namely the scale parameter (δ) and the shape parameter (β). In this paper the scale parameter is assumed to be equal to 4 MPa and the shape parameter is varied from 1 to 3 MPa. The strength distribution of interface, matrix and aggregate elements in two different particle distributions was shown in figure 2 to compare with the strength distributions generated by Gaussian and Weibull distributions. More details about these two distributions can be found in Montgomery et al. [4].

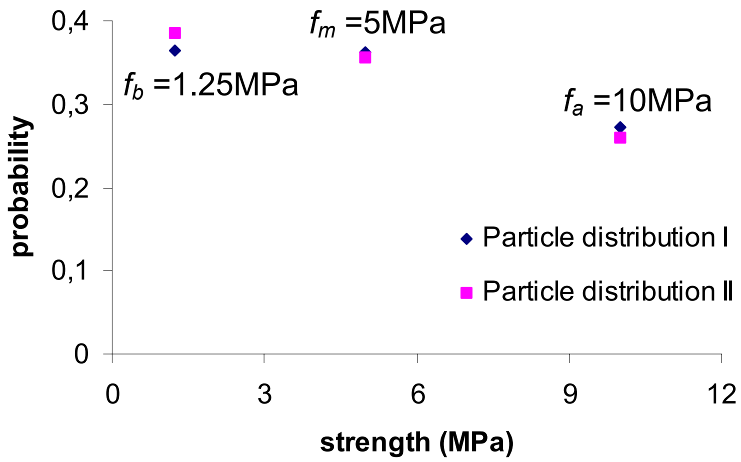
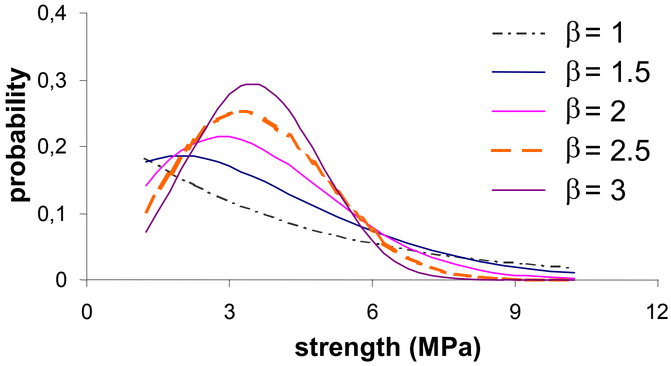
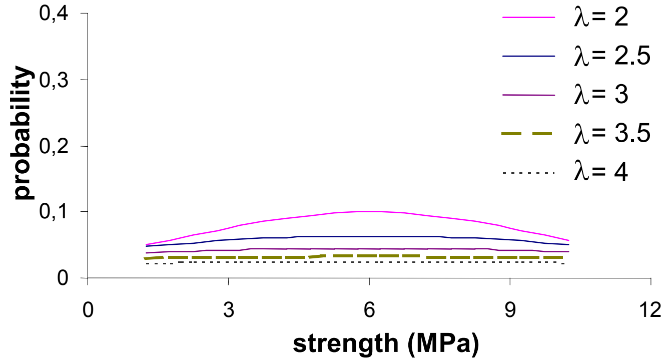


Figure 2a. Strength distribution of bond, matrix and aggregate phases in two particle distributions



(b)



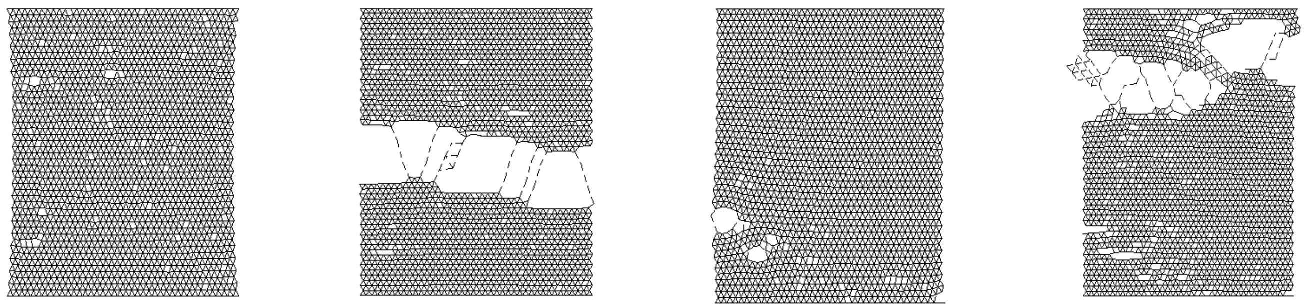
(c)

Figure 2b-c. Comparison of strength distribution: b) following different Gaussian distributions and c) following different Weibull distributions.

Figure 3 shows a comparison of crack patterns of an aggregate structure, of the Gaussian distribution structures with λ=2 MPa, *l*=2 and 4 mm and of the Weibull distribution with *β*=1 MPa, *l*=2 and 4 mm. The two crack stages in each analysis correspond to the situation at peak load and at the end of the analysis.

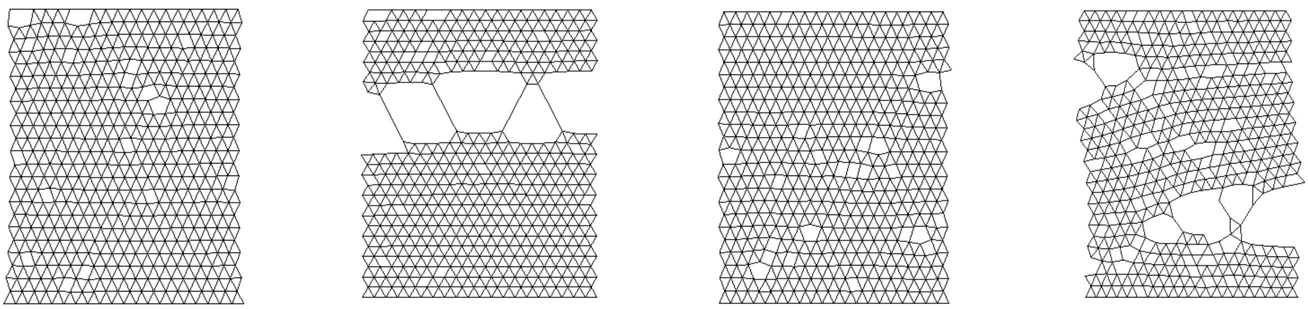
404 beams removed 1146 beams removed

a) crack propagation in a lattice with a computer generated particle distribution



63 beams removed 211 beams removed 50 beams removed 400 beams removed

b1) Gaussian distribution (*l*=2mm) c1) Weibull distribution (*l*=2mm)

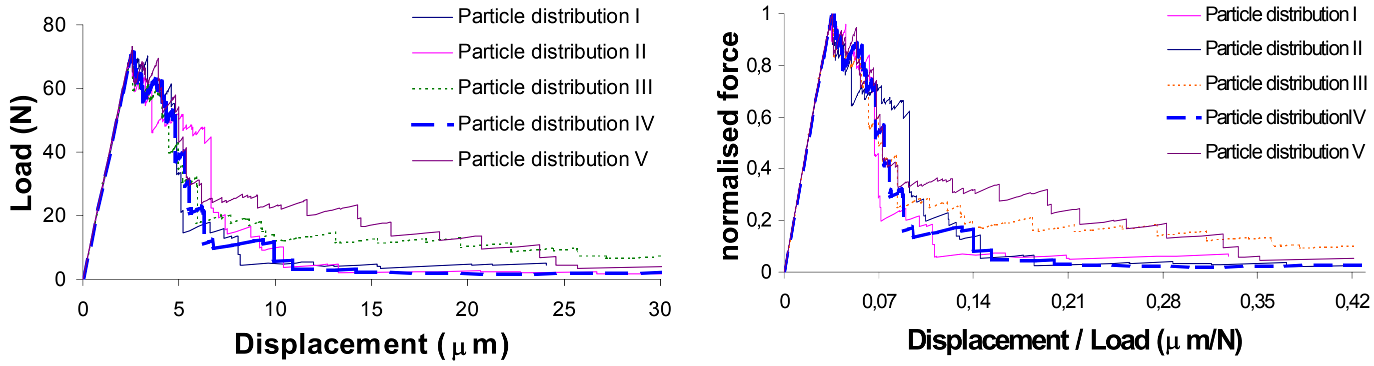


23 beams removed 75 beams removed 30 beams removed 145 beams removed

b2) Gaussian distribution (*l*=4mm) c2) Weibull distribution (*l*=4mm)

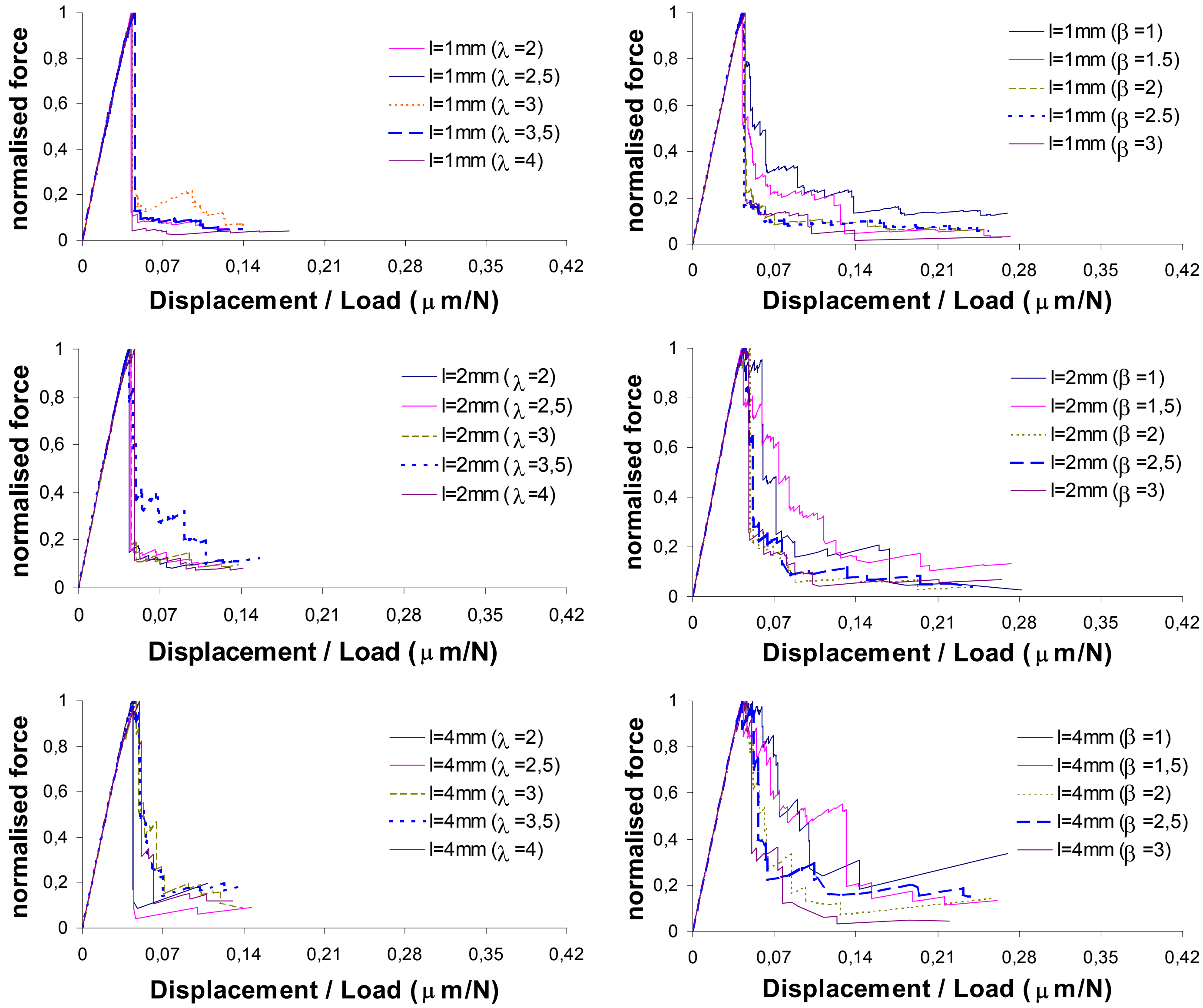
Figure 3. Comparison of crack evolution of (a) a computer generated particle distribution structure (b) Gaussian distribution structures and (c) Weibull distribution structures

The results of the numerical simulations with aggregate structures are shown in figure 4a1. In order to compare with the results obtained by statistical distributions, normalised load-displacement curves of aggregate structures and statistical distributions are presented in figure 4b and 4c.

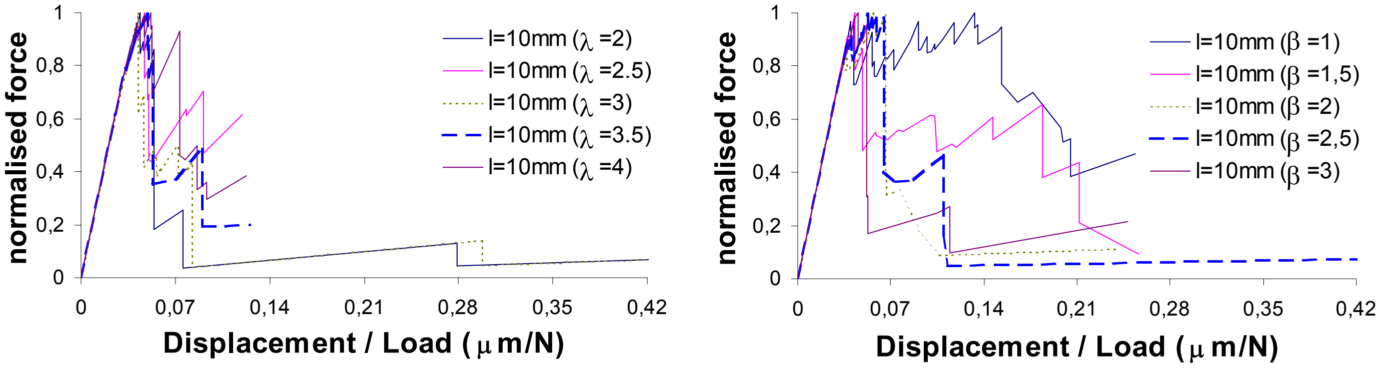


a1) load-displacement curves a2) normalised load-displacement curves

Figure 4a. Smoothened results of five different aggregate structures (l = 1mm)



b) Gaussian Distribution c)Weibull distribution



b) Gaussian Distribution c)Weibull distribution

Figure 4b-c. Comparison of normalised force–displacement: b) Gaussian distribution with λ varying from 2 to 4 and c) Weibull distribution with β varying from 1 to 3. Note that both the x-axis and y-axis have been normalised with respect to the maximum load in each analysis.

DISCUSSION

For the two statistical distributions, we can observe that, in a certain range of lattice beam sizes, the structural behaviour corresponds to the results obtained from the analyses with aggregate overlay. But when the mesh size increases, in particular to l = 10 mm, there is a clear discrepancy. It means that, to some extent, a rather coarse mesh (l = 2-4 mm) can be used with statistically distributed beam properties to analyse the behaviour of a large-scale structure instead of using a much more refined mesh with an underlying aggregate structure. With the increase of the beam length, the computational time per beam removal decreases significantly, whereas also less beams need to be removed until the end of the analysis in the coarser meshes. In this way the computational problems encountered in lattice-type fracture models for the analysis of large-scale structures can be dismissed. From figure 4a it can be observed that the different particle distributions affect mostly the softening parts of the load-displacement diagrams. Even for absolute force-displacement curves, quite small difference in maximum admissive tensile forces for different particle distributions exist (a maximum of 3.1% of difference was observed by numerical simulations). From the force-displacement curves of figure 4 it can be observed that the Weibull distribution corresponds better to the results obtained from aggregate structures. As shown in figure 2, the distribution of tensile strength in an aggregate structure in the lattice beams is more similar to a Weibull distribution than to a Gaussian distribution.

Comparing the crack patterns between regular lattices with and without underlying aggregate structures (see figure 3), it can be seen that cracking is initially more distributed when a statistical distribution of beam strength is used rather than a particle overlay. In the latter case, the positions of the particles have a big influence on the crack patterns. For the analyses with a particle distribution the weak elements are grouped in the interface zones around the aggregates. The variation of beam lengths also has a significant effect on the crack evolution: with increasing length much detail in the fracture patterns is lost. Fracture in a regular triangular lattice with the same strength assigned to all beam elements results in a subsequent removal of all the elements along a single row, see for example in Van Vliet [3]. The phenomenon does not occur when a particle distribution is imposed on the lattice, see figure 3a. Also from figure 3, it can be seen that subsequent removal of elements in the same row does not occur when a statistical distribution of beam strength due to the random distribution of weak elements is applied, which is especially true for the Weibull distribution. In the case of a Gaussian distribution the final fracture plane almost seems to follow a single row of elements (figure 3b1 and 3b2), but this should be expected in view of the relatively ‘flat’ strength distribution of Figure 2c.

Van Vliet [3] showed that the amount and location of weak elements decides how crack propagation proceeds, which explains why the use of statistical distribution without considering the microstructure of concrete leads to consequent incorrect crack patterns. This shows the limitation of this method where statistical information is used rather than a correct representation of material structure.

Experimental observations show that cracks in concrete are not continuous but show a lot of overlaps, mainly around the aggregate particles. Pieces of material bridge the two crack faces and as a consequence transfer of stress remains possible, see Van Mier [8]. Small overlaps (bridges) observed in experiments can not be captured in the simulations where the beam length is much larger than the size of the bridges. Thus, in order to obtain both correct crack patterns (and crack mechanisms) and load-displacement behaviour, the disorder implemented in the model has to be related to the real heterogeneity in the material. Unfortunately, however, this leads to an enormous increase of computational effort.

ACKNOWLEDGEMENTS: This research has been made possible through a grant from FAPESP-Brazil, which is gratefully acknowledged.

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