ORAL REFERENCE: ICF100245OR

SOFTENING BEHAVIOUR OF PLAIN CONCRETE BEAMS

A. P. Fantilli and P. Vallini

Department of Structural and Geotechnical Engineering, Politecnico di Torino
Corso Duca degli Abruzzi 24, 10129 Torino, Italy

ABSTRACT

In this paper, a one-dimensional discrete cohesive model is put forward to study the softening behaviour of plain concrete beams. In particular, a block of beam bounded by two cracks is analysed, namely the "open crack" (with variable crack width) and the "closed crack" (i. e.with virtual crack width). By means of this model, both the bearing capacity and the post-peak response of concrete beams can be evaluated, taking into account the effects produced on the softening branch by fictitious crack models and by fracture energy as well. A good agreement between numerical and experimental results is obtained, with a fracture energy value higher than the conventional ones.

KEYWORDS

Plain concrete, fictitious crack model, bending beams, fracture energy.

INTRODUCTION

In many papers plain concrete beams have been the object of direct experimental researches. Oladapo [1], for example, inquired into the effects produced by different concrete mixtures on the mechanical behaviour of bending beams with the same dimension. Moreover, the author [1] describes the two stages of the gradual process of cracking in these beams (Fig. 1) as follows:

- The "Critical Stage" develops when the concrete tensile stress exceeds the strength f_{ct} (which is obtained by means of direct tensile tests), in the portion of the beam where the bending moment M is greater than the cracking moment M_{cr} . In this portion, a diffuse micro-cracking is present (Fig. 1a).
- The "Failure Stage" starts when the bending moment reaches the maximum value M_u (Fig. 1b) (where $M_{cr} / M_u = 0.7 \div 0.9$). In this stage, a single wide crack usually appears.

Bosco et al. [2], have investigated on the so-called "size effect" of three point bending elements, by changing the beam dimensions for the same concrete. The bending moment M-deflection η curves (Fig.1b) show an increase in beam brittleness for the greater dimension of the beams. The softening branch of the "Failure Stage" can be measured, even when a snap-back is present, if the loading process is controlled by a monotonically increasing function of the time, like the crack mouth opening displacement (CMOD) [2]. For this reason, Giuriani and Rosati [3] built a testing machine able to set the maximum value of crack width w in a concrete block in bending. In the literature, many theoretical nonlinear approaches for plain concrete beams have been proposed. As is well-known, for quasi-brittle materials like concrete, the dimension of fracture process zone is not small with respect to the beam's dimension. Therefore, nonlinear fracture mechanics (NFLM) models are needed for concrete elements in bending [4,5].

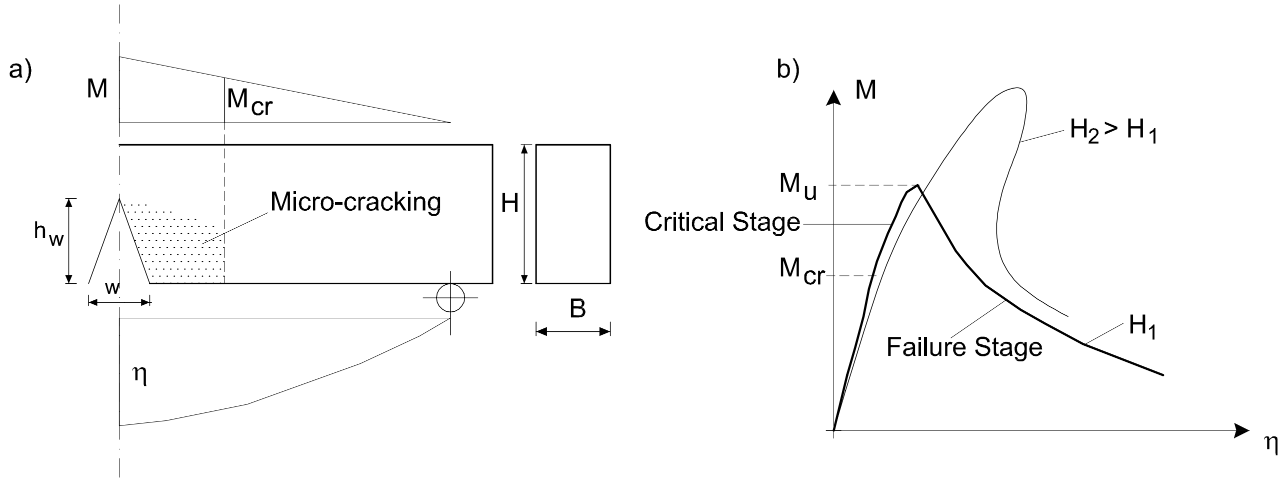

Figure 1: Plain concrete beam: a) three point bending test; b) possible M-η curves.

In particular, the Mode I cracking phenomenon can be reproduced by means of two nonlinear models: the fictitious crack model (or cohesive model) and the crack band model. With the fictitious crack model, introduced by Hillerborg et al. [6], the fracture process zone is represented through a crack line where the stress σ - crack width *w* relationship is applied. The simplest application of cohesive model for plain concrete beams, is made possible by means of the bidimensional Boundary Element Methods (BEM), i.e. the multidomain BEM [7] or dual BEM [8, 9]. In this case, all the body volume remains linear elastic and the nonlinearity is included in the crack line by the cohesive model. An alternative approach, is that of using bidimensional Finite Element Methods (FEM). With these methods, the cracking phenomenon is analysed in a "discrete" manner [6], changing the initial mesh (remeshing).
With the crack band model, the inelastic strains of the fracture process zone are smeared on a band of appropriate length, located around the crack. Several crack band models were developed to study the cracking phenomenon in concrete, not only for beams in bending. The so-called "smeared FEM", for example, are able to reproduce the mechanical behaviour of bidimensional structures, by considering concrete like a continuous body by means of suitable stresses-strains σ-ε law [4,5]. In other models, the authors exploited some simplifications to study plain concrete beams in a one-dimensional way, in particular they analysed a restricted portion of beam close to the main crack [10, 11]. It is also possible to reduce the observations to only cracked section [12], where the application of limit strain analysis can be useful to define the analytical formulation of the maximum bending moment M_u of the concrete beam [13].

PROPOSED MODEL

The bidimensional FEM approaches, discrete or smeared, are able to represent both stresses and strains also in the uncracked parts of a plain concrete beam far from the cracked section. Structural response of bending beams is usually calculated by a one-dimensional or sectional approach, thus the introduction of new one-dimensional models to study unreinforced concrete beams during the cracking phenomenon is justified. In the previous sectional [12, 13] or block [10, 11] models, hypothesis and simplifications were introduced about stress and strain distributions, so that is not possible to evaluate them in all the sections of the beam.
The proposed cohesive block model is able to define the structural response of cracked plain concrete beams, through the computation of stress and strain distributions in a wide portion of beam delimited by a crack. It can be considered an intermediate approach between the cohesive bidimensional FEM and the sectional models. The problem can be solved by means of a one-dimensional model, by introducing a suitable simplified strain profile for every cross-section of the beam. Cracking in plain concrete bending elements is characterized by one main crack [1, 2, 3], called "open crack", whose stress and strain profiles are depicted in Fig. 2a. The structural response of this section, with a plane crack, is evaluated with a trial and error procedure. In particular, for a given value of crack width *w* and depth h_w (Fig. 2a), the position of the neutral axis *x* can be computed by the static cross-sectional equilibrium condition. In this way, it is also possible to define the bending moment *M*, the slope of linear strain profile and the corresponding stresses.

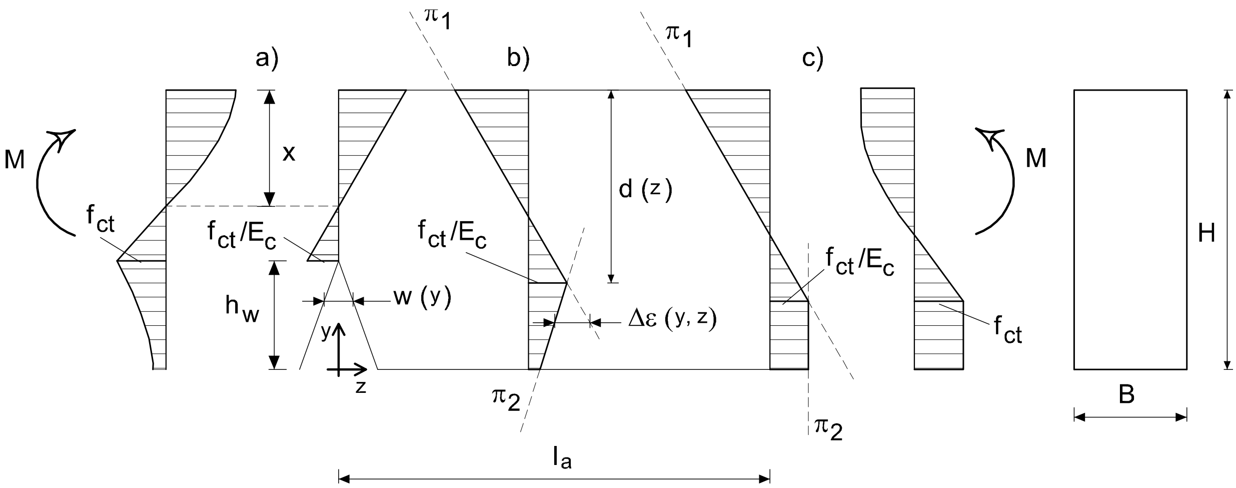

Figure 2: Observed concrete block: a) stresses and strains in the "open crack"; b) strain profile within l_a ; c) strains and stresses in the "closed crack".

In the "Critical Stage", when then bending moment is higher than the cracking one M_{cr} [1], in the concrete block there is a diffuse micro-cracking, which can be reproduced, according to cohesive model, as "virtual" cracks. In other words, besides the main crack, there are many cracked sections where the stresses could be computed by means of "closed crack" model. As shown in Fig. 2c, the concrete stress is constant and equal to the tensile strength f_{ct} within the crack length h_2 . Stress and strain in a closed crack section could be computed like in a continuous body, if the strain profile develops on two different planes π_1 and π_2 (Fig. 2c). The stress and strain analysis for a block delimited by the open cracked section and by the closed one (Fig. 2), allows to calculate the distance l_a between the two sections, which is an unknown of the problem. In a generic section within l_a , the strain profile lies on the planes π_1 and π_2 (Fig. 2b). In particular, both the slope of π_2 and the depth $d(z)$ of π_1 increase when shifting from the main crack (Fig. 2a) to the closed crack, where π_2 is vertical and $d(z)$ is maximum. At distance y from the bottom of the beam, the crack width $w(y)$ due to the strain difference $\Delta\varepsilon(y, z)$ measured on the two planes along l_a , can be computed through the following integration:

$$w(y) = \int_0^{l_a} \Delta\varepsilon(y, z) dz \qquad (1)$$

Based on this strain hypothesis, also supported by the experimental analysis [1, 3], it is possible to found an horizontal tangent in the function $d(z)$ near the tip of the main crack. In the case of constant bending moment M, $d(z)$ shows an horizontal tangent also near the tip of the virtual crack (Fig. 3a).
Broms [14] supposed that the maximum concrete tensile strain, in reinforced concrete elements in bending and tension, is localized inside a circle whose radius corresponds to the crack length and whose centre is on the reinforcement axis. Outside this circle, compressive or small tensile strains are present. By means of an analogy with this strain diffusion hypothesis, $d(z)$ could be considered as the zone where compressive or small

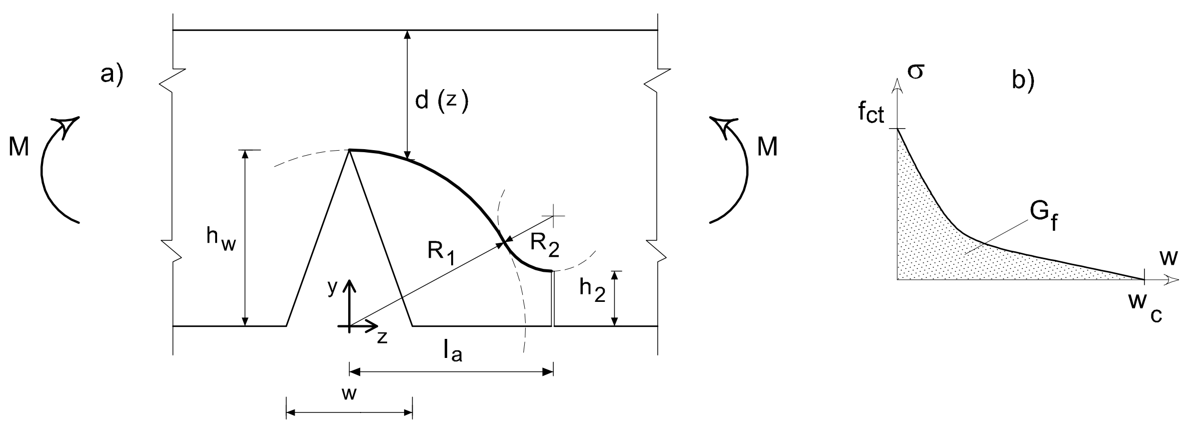

Figure 3: Observed concrete block: a) function $d(z)$ along the block; b) a cohesive model.

tensile strains are located, as shown in Fig. 3a by the thick line tangency at the tips of the open and closed cracks. Indeed, $d(z)$ is a function drawn on two tangency circles: the first circle has the centre on the bottom of the cracked section and its radius R_1 is the crack length h_w ; the radius R_2 of the second circle is the length of the virtual crack h_2 and the centre of this circle is distant $R_2 + h_2$ from the bottom of closed crack section. In the "Failure Stage", when $M < M_{cr}$, the virtual crack vanishes ($h_2 = 0$ in Fig. 3a) and it possible to observe only the presence of the main crack, with one circle in the open crack section [14]. Moreover, the distance between the open crack section and the closed one can be computed through the following equation:

$$l_a = \sqrt{(h_2 + h_w)^2 + 4(h_2)^2} \tag{2}$$

For the plain concrete beam of Fig. 1, whose constitutive laws for the uncracked concrete and the cohesive model (Fig. 3b) are known, the relationship M-η can now be obtained as the maximum width w of the open crack section increases, by means of the following trial and error procedure (Fig. 3a):

1. select a crack width w ;
2. assume a value of crack length h_w ;
3. compute, in the open crack section, the bending moment M ;
4. compute, in the closed crack section, the crack length h_2 ;
5. compute l_a according to equation (2) ;
6. compute $\Delta\varepsilon(y, z)$ within l_a ;
7. if the equation (1) is not verified, with a new trial value of h_w , go back to step 3.

For an imposed value of w in the main crack, this procedure yields the stresses and strains in a concrete block of length l_a , as in a bidimensional finite elements analysis. According to Bosco et al. [2], the loading process is controlled by w (a monotonically increase function), so that the possible positive slopes of the softening branch (in M - η curves), during the "Failure Stage", can be computed. Moreover, in this way, both the distance l_a between the cracks and the length h_w of the main crack increase monotonically, and in particular, at the end of "Failure Stage", they are both equal to the height H of the beam.

COMPARISON BETWEEN NUMERICAL AND EXPERIMENTAL RESULTS

Before testing the validity of the proposed model, it is necessary to define the two constitutive laws of the concrete. According to [4, 7, 12], a linear elastic stress strain relationship for the uncracked concrete is assumed (Fig. 4a). In this relationship, E_c represents the modulus of elasticity, f_c and f_{ct} are respectively the compressive and the tensile concrete strength. As is well-known, a cohesive crack model valid for all the concretes does not exist [4]. In fact, many σ-w curves were obtained for particular concrete with a limited number of experimental data. For the plain concrete beam in Fig. 1, the M-η response is strongly depending on the cohesive model adopted. As shown in [15], where a bilinear σ-w is used, the maximum bending moment M_u depends on the slope of the first part of the cohesive law, while the last part of σ-w influences the softening branch of the "Failure Stage". Moreover, if the fracture energy G_f , i. e. the area under the σ-w curve (Fig. 3b), were kept constant, both the "Critical Stage" and the "Failure Stage" of M-η should be different.

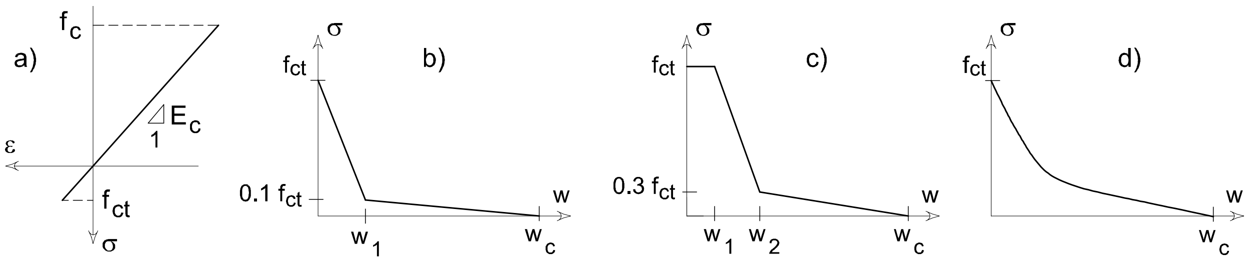

Figure 4: Constitutive laws: a) linear elastic law for uncracked concrete; b) cohesive bilinear model [16]; c) cohesive trilinear model [17]; d) cohesive exponential model [18].

Cohesive models adopted
In order to define the structural response of a plain concrete beam, in this paper the results obtained with three different fictitious models are compared. The bilinear model [16] (Fig. 4b), the trilinear model [17] (Fig. 4c) and the exponential model [18] (Fig. 4d) have been used. Usually, the practitioners do not know the σ-w relationship: they simply set the compressive strength f_c and the maximum aggregate size Φ_{max} . For these reasons, the adopted cohesive models, with the exception of the exponential model [18], depend on f_c and Φ_{max} .

Numerical analysis of the Giuriani's and Rosati's beam [3]
As a first approach, our numerical results and the experimental ones, obtained by Giuriani and Rosati [3], are compared. The Authors, concentrated in the cracked section of a concrete block (Fig. 5), where they imposed the maximum crack width w and measured both the crack depth h_w and the bending moment M. In particular, the crack growth were measured by means of geometric moiré. By introducing the cohesive laws reported in Fig. 4, the proposed model is able to evaluate, for the cracked section in Fig. 5, both the bending moment M - crack width w and the bending moment M -crack depth h_w curves. In Fig. 6, the numerical and experimental M-w and M-h_w diagrams are compared. In Fig. 6c, the proposed model calculated with the trilinear σ-w law,

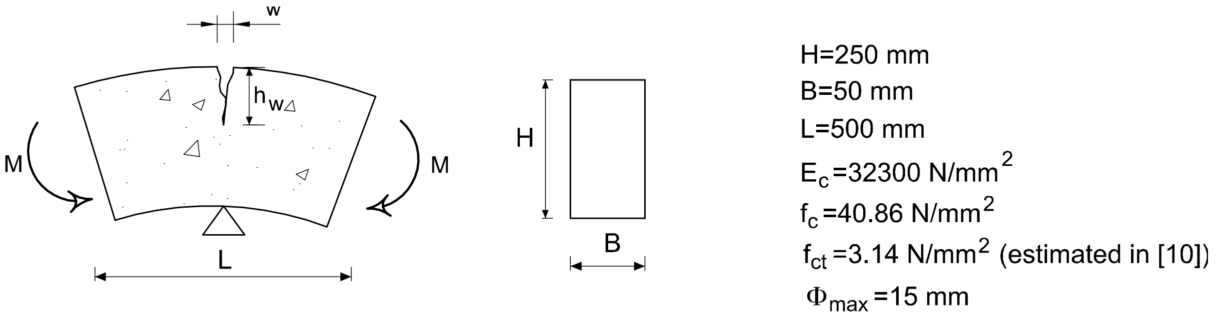

Figure 5: Giuriani's and Rosati's beam [3].

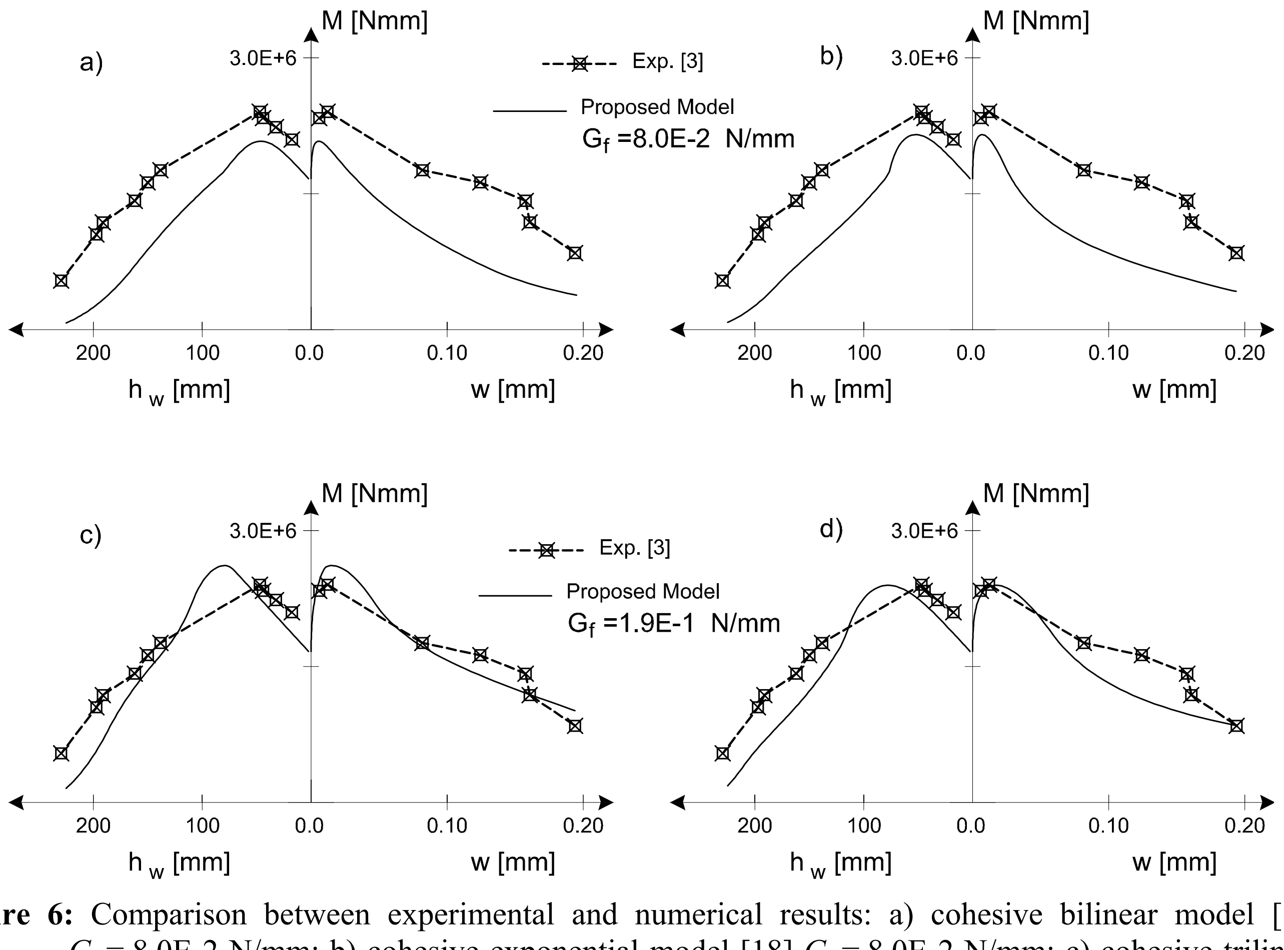

Figure 6: Comparison between experimental and numerical results: a) cohesive bilinear model [16] G_f= 8.0E-2 N/mm; b) cohesive exponential model [18] G_f= 8.0E-2 N/mm; c) cohesive trilinear model [17] G_f= 1.9E-1 N/mm; d) cohesive bilinear model [16] G_f= 1.9E-1 N/mm.

provides the best fitting of the experimental measurements. In this case, the fracture energy G_f = 1.9E-1 N/mm (which is function of f_c) is far greater than the value adopted for the bilinear and the exponential fictitious crack model (Fig. 6a and Fig. 6b, respectively), where G_f=8.0E-2 N/mm was estimated according to CEB Model Code [16]. As pointed out in [8] for the experimental results [2], if in the bidimensional BEM analysis G_f were higher than the conventional measured value, numerical results should be closer to the experimental ones. To confirm this observation, Fig. 6d shows a good agreement between the numerical and experimental results also for the bilinear model [16] with the higher G_f.

CONCLUSIONS

Due to limited number of comparisons with the experimental data, it is not possible, at the moment, to give general rules about plain concrete beams behaviour. Results obtained with the proposed model show a good agreement with the experimental ones [3], only when the fracture energy is higher than the conventional measured or computed values. In these cases, the proposed model represents an effective alternative to the classical bidimensional BEM and FEM analyses. Moreover, this model is able to define the better σ-w relationship to model the reinforced concrete beams, whose structural response depends both on the fracture mechanics of tensile concrete and on the bond between steel and concrete [19].

REFERENCES

1. Oladapo, I.O. (1964) *Magaz. Concr. Res.* 16, 103.
2. Bosco, C., Carpinteri, A. and Debernardi, P.G. (1990) *Eng. Fract. Mech.* 35, 665.
3. Giuriani, E. and Rosati, G. (1984) *Studi e Ricerche* 6, 151 (in Italian).
4. Bazant, Z.P. and Planas, J. (1998). *Fracture and Size Effect in Concrete and Other Quasibrittle Materials.* CRC Press, Boca Ranton.
5. Shah, S.P., Swartz, S.E. and Ouyang, C. (1995). *Fracture Mechanics of Concrete: Applications of Fracture Mechanics to Concrete, Rock and Other Quasibrittle Materials.* John Wiley & Sons, New York.
6. Hillerborg, A., Modeer, M. and Petersson, P.E. (1976), *Cem. Concr. Res.* 6, 773.
7. Maier, G., Cen, Z., Novati, G. and Tagliaferri, R. (1991). In: *Fracture Processes in Concrete, Rock and Ceramics*, pp. 561-570, van Mier, J.G.M., Rots, J.G. and Bakker, A. (Eds). E & FN Spon, London.
8. Aliabadi, M.H. and Saleh, A.L. (1999). In: *Computational Fracture Mechanics in Concrete Technology*, pp. 163-192, Carpinteri, A. and Aliabadi, M. (Eds). Wit Press, Boston.
9. Saleh, A.L. and Aliabadi, M.H. (1995) *Eng. Fract. Mech.* 51, 533.
10. Giuriani, E. and Rosati, G. (1987) *Studi e Ricerche* 9, 107.
11. Ulfkjaer, J.P., Krenk S. and Brinker, R. (1995) *ASCE Jour. Engng. Mech.* 121, 7.
12. Ananthan, H., Raghuprasad, B.K. and Sundara Raja Iyengar, K.T. (1990) *Eng. Fract. Mech.* 45, 195.
13. Aagren, P. and Harrison, N.L. (1986) *ACI Jour.* 83, 156.
14. Broms, B.B. (1965) *ACI Jour.* 62, 1237.
15. Alvaredo, A. M. and Torrent, R.J. (1987) *Mat. and Struct.* 20, 448.
16. Comité Euro-International du Beton (1993). *CEB-FIP Model Code 1990: Design Code.* Thomas Telford, London.
17. Liaw, B.M., Jeang, F.L., Du, J.J., Hawkins, N.M. and Kobayashi, A.S. (1990) *ASCE Jour. Engng. Mech.* 116, 429.
18. Hordijk, D.A. (1991). Doctoral Thesis, TU-Delft, Netherlands.
19. Fantilli, A.P., Ferretti, D., Iori, I. and Vallini P. (1998). In: *Minimum Reinforcement in Concrete members,* pp. 99-125, Carpinteri, A. (ed.). Elsevier, Amsterdam.