SIMULATION OF CRACK PROPAGATION IN HETEROGENEOUS BRITTLE SOLIDS: SCALING PARAMETERS

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ABSTRACT

Crack propagation in real (quasi)brittle materials possesses signs of stochasticity; a tortuous character of fracture surfaces, multiple cracking and crack branching observed in experiments are a vivid confirmation of it. Traditional approaches of fracture mechanics represent cracks as geometrically smooth objects with straight (or curved) crack fronts, thus usually neglecting morphology of real cracks. An introduction of a direct account for stochastic features of the failure process can lead to a more adequate description of real fracture processes. Numerical schemes accounting for a spatial scatter in material properties are proposed for a description of crack propagation. These schemes unify approaches of fracture mechanics with ideas of continuum damage mechanics (CDM). CDM describes a macroscopic manifestation of various failure processes developing at lower length scales. Such unification being one of the possibilities for a micro-macro transition in a study of failure processes allows an analysis of non-uniform crack propagation and of multiple cracking. A necessity to describe a crack with its length changing along its front presupposes an utilisation of local stress-intensity factors. A description of complicated morphology of cracks or of a random set of cracks in a case of multiple cracking necessitates an introduction of additional quantitative parameters characterising fracture processes in heterogeneous brittle materials. This is implemented by means of scaling analyses of the crack front roughness, of the stress distribution character and of the free-energy release. It is shown that spatial scaling parameters can be linked to the type of material's randomness (thus being material parameters), while temporal scaling parameters are independent of it and characterise the fracture process.

KEYWORDS

Crack propagation, brittle fracture, randomness, numerical simulations, scaling

INTRODUCTION

Real brittle materials usually exhibit – to a different extent – a spatial randomness in their properties. This can result from processes of genesis for natural materials (rocks) or of manufacturing for artificial ones (ceramics, composites, etc.). The most common reason for such randomness is non-uniformity in distributions either of phases/components or of defects. A macroscopic manifestation of this randomness is the vivid difference in a crack trajectory and/or in a fragmentation type (number and shape of fragments) of identical specimens of the same material under identical loading conditions. Another sign of it is a well-known scatter in mechanical parameters at the moment of fracture observed for twin specimens of brittle materials. Studies of their microstructure usually show considerable variations in distributions of phases or defects, which result in different scenarios of generation and evolution of failure. Utilisation of micro- or nanoindentation also traditionally demonstrates a high sensitivity to the indenter's position; indentation measurements performed in neighbouring microscopic regions belonging to a weak matrix and to a rigid inclusion/precipitate will show considerably different results.

There is a distinct difference in types of mechanical analysis of brittle random materials. Their deformational properties can be obtained with the use of various homogenisation schemes provided the specimen size is considerably larger than the characteristic dimension of microstructure elements. These methods are mostly effective for dilute concentration of such (non-interacting) elements but modifications of these methods can also deal with systems with high degrees of randomness or even with spatially changing properties (functionally graded materials). The fracture analysis of these materials, in contrast, cannot be reduced to a study of their effective properties, since the critical and post-critical behaviour is unique for each specimen of brittle materials because of its microstructure. So, one possibility is a statistical (Weibull-like) analysis of large sets, or, a direct introduction of randomness into simulations. This paper deals with the latter approach.

MODEL OF STOCHASTIC BRITTLE SOLID WITH MODE-I CRACK

Modelling Stochastic Materials

An introduction of spatial randomness of material properties into the model can be implemented by various methods but they all exploit any type of the distribution of material's properties over different parts of the analysed area. These parts should normally fulfil requirements for a representative volume element (unit cell). It means that, on the one hand, they should be sufficiently large to contain a large number of microscopic elements and, on the other hand, their dimensions should be sufficiently small compared to dimensions of the whole specimen/structure. The character of spatial distributions should reflect the scatter of respective properties, experimentally obtained either on twin specimens or from different parts of the same specimen. Obviously, different materials have different reasons for spatial randomness, which should be respectively accounted for in a model.

In this paper a spatial non-uniformity in the stiffness level is considered to be the main stochastic property of the modelled brittle material. This can be true for rocks [1, 2] and composites [3]; another approach is used, for instance, in modelling of sintered ceramics where stochasticity in a distribution of pores can be treated as a main source of randomness [4, 5]. The studied area of such material is divided into M elements with varying stiffness, the level of which for the jth element is determined by the following relation:

$$G_j = G_r\left(1+\left(\sin\frac{\pi k}{M}\right)^r\right)K_{\text{ren}}, \quad \left(j,k=\overline{1,M}\right). \tag{1}$$

Here, G_r is the minimal local stiffness, k is a random number. Parameter r determines the material's extent of randomness: its higher magnitude corresponds to higher non-uniformity. This parameter is linked with the effective coefficient of the distribution's half-width $K_{0.5}$:

$$K_{0.5}(r) = \frac{1}{\pi}\operatorname{arcsec}\sqrt[r]{2}\,. \tag{2}$$

It is obvious that the ratio of maximal stiffness to minimal one is equal 2 for any r. The last multiplier in Eqn. 1 is introduced in order to exclude the influence of the total stiffness of the area $G^* = \sum_{j=1}^{M} G_j$ on results of simulations and it is determined from the requirement $G^* = \underset{r}{const}$. Three various distributions of stiffness for different levels of $K_{0.5}$ are shown in Figure 1.

Mode-I Crack and Local Stress-Concentration Factors

A specimen with a V-shaped notch (with its edge EF and its symmetry plane xOy containing this edge laying in the middle cross-section ABCD of the specimen) is shown in Figure 2. It can be used for the mode-I fracture analysis. Let us note that a standard analysis within the framework of fracture mechanics is usually carried out for another plane, namely, a plane orthogonal to the edge EF of the notch (*yOz*). This reduction of the crack front to its trace – a single point in *yOz* – cannot be applied here since material properties change along the front.

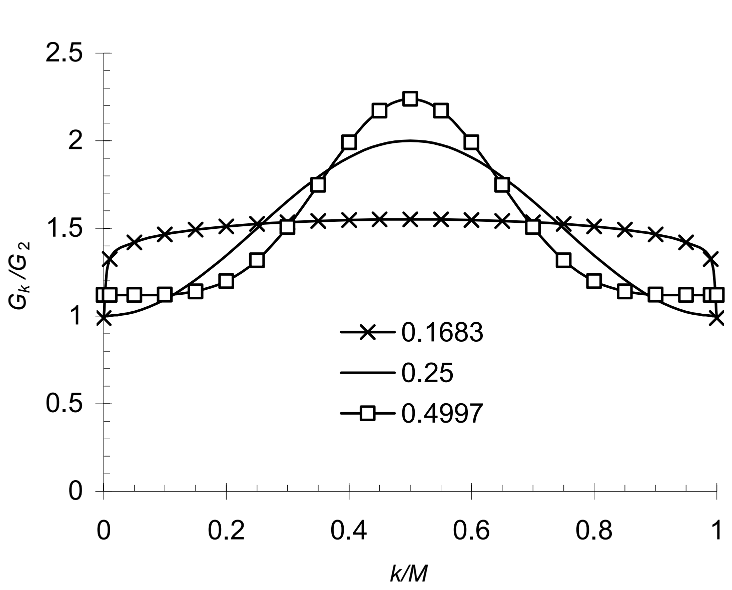

Figure 1: Stiffness distributions for various levels of $K_{0.5}$

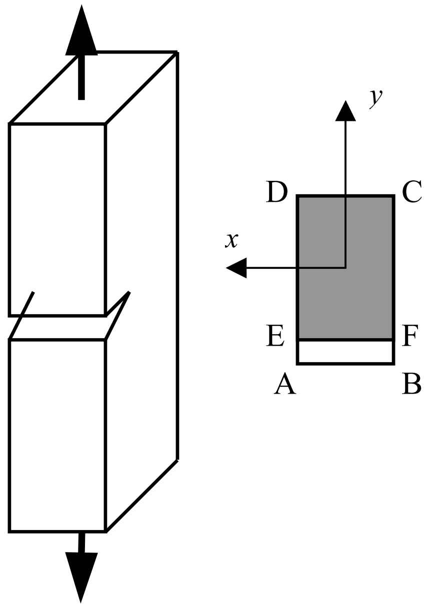

Figure 2: Mode-I crack and specimen's middle cross section

Cross-section ABCD is discretised into an orthogonal network of *M* rectangular elements with a pair of indices *p* and *q* denoting the element belonging to the *p*th row and *q*th column of elements (rows being parallel to EF and columns orthogonal to it). Then the level of stresses in elements can be calculated by integrating the well-known relation for a mode-I crack [6, 7] $\sigma_{zz} = \sqrt{2\pi y}$ over elements in rows and accounting for their different stiffness in the form

$$\sigma^{pq} = \frac{MG^{pq}}{G^*}\frac{\overline{G}^q}{\widetilde{G}^q} K_{\mathrm{I}}^q \sqrt{\pi\, l_y}\left(\sqrt{p+1-n^q} - \sqrt{p-n^q}\right)\sigma_{zz}^{\infty}, \quad p \ge n^q . \tag{3}$$

Here $\overline{G}^q$ and $\widetilde{G}^q$ are initial and current levels of stiffness of the *q*th column; the local stress-intensity factor K_{I}^q can be approximated for a given geometry/loading by known relations from handbooks on stress-intensity factors (e.g., [8]); l_y is an elements' dimension along the *y* axis; n^q is a number of elements occupied by crack in the *q*th column; σ_{zz}^{∞} is the uniformly-distributed stress acting far from cross section ABCD.

Damage Accumulation and Crack-Damage Interaction

A deterioration of material under loading is described in terms of continuum damage mechanics by means of introduction of additional variables into the model. They describe macroscopic effects of microscopic evolution in an ensemble of defects. The choice of these additional macroscopic parameters should be related to respective mechanisms responsible for failure evolution. Some damage parameters for different deformation/fracture modes with respective damage accumulation laws are discussed in [9, 10]. Another approach utilizing a different damage parameter was used in [4, 5]. The damage level is also treated as a field function distributed over the network of elements described before.

The presence of crack naturally influences evolution of failure processes at lower length scales and should be reflected in respective model schemes. It is implemented in terms of stress concentration in the vicinity of the crack front: higher stress levels correspond to higher local damage accumulation rates [9].

An additional element in modelling crack-damage interaction is a local failure criterion: an element is considered to fail (with its stiffness diminishing from the initial level to the near-zero residual one) if its damage level exceeds a threshold value. If a locally failed element adjoins the crack front then the crack is considered to increase its length in this column of elements by l_y. The local crack growth in the qth column causes stress redistribution linked with two factors – a local stiffness decrease under the unchanged level of the external load and an increasing level of K_I^q. Both factors are accounted by Eqn. 3.

Spatial non-uniformity in material properties obviously results in different response of various regions to external loading. Both initial non-uniformity in internal stresses even under uniform external loading and difference in distribution of microscopic defects result in spatially varying rates of failure process development. Thus, crack propagation and stress redistribution lead to the stress growth in elements, accelerating, in its turn, the damage accumulation process and the consequent failure of these elements in front of the crack. In the case of a totally isotropic material this trend would result in a shift of crack front as a straight line parallel to its initial position (EP)– the standard case for fracture mechanics. But material's spatial non-uniformity and random processes of stress redistributions and elements' failures cause the difference in local levels of the crack propagation rate, which results in a complicated morphology of crack fronts (Figure 3).

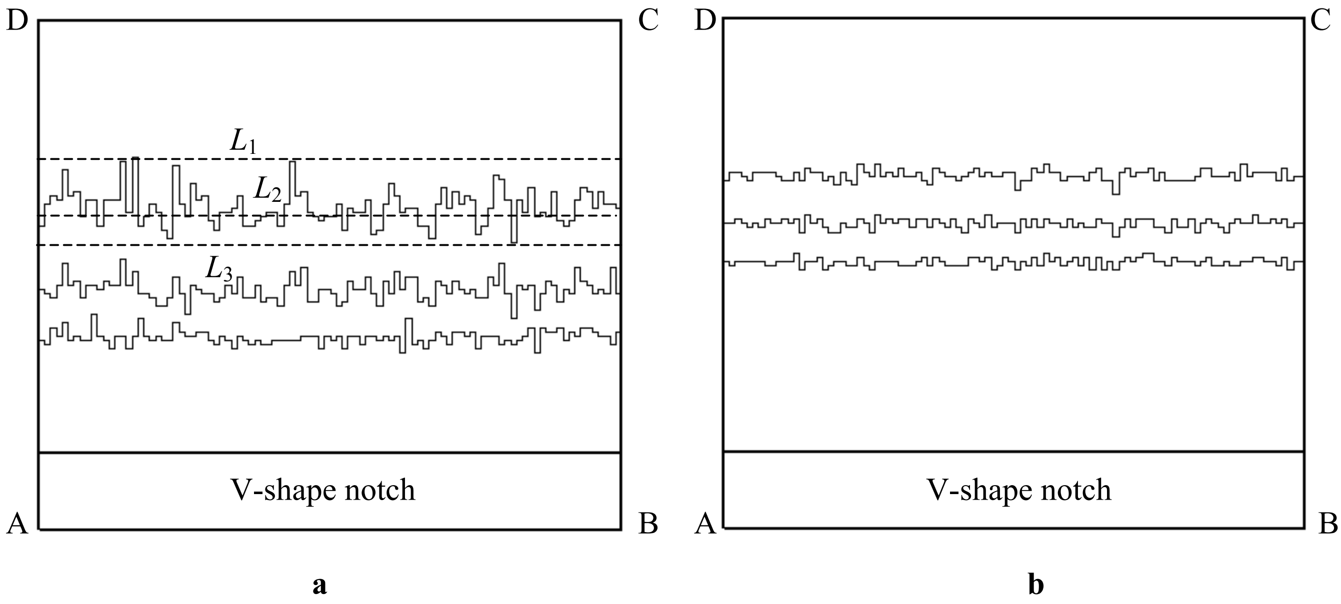

Figure 3: Position of crack fronts at different stages of crack propagation in specimens with different extents of material's randomness: (a) $K_{0.5} = 0.1683$; (b) $K_{0.5} = 0.4997$

SCALING CHARACTERISTICS OF CRACK PROPAGATION

Each statistical realisation of the spatial distribution of material's properties should result in the unique scenario of crack propagation and the unique crack-front morphology. This presupposes both a modification of traditional characteristics of fracture and introduction of new ones. To characterise a position of the crack front three different parameters are introduced instead of the traditional crack length. They characterise positions of the farthest – with respect to the initial notch – rows containing (a) at least one element belonging to the crack (shown as L_1 in Figure 3a), (b) half elements occupied by the crack (L_2) and (c) only failed elements (L_3). So, different moments of fracture (linked with respective time-to-fracture parameters) of a specimen can be considered: between the first exit of the crack front on its opposite – to the V-shape notch – edge (the crack "touches" it) and the final rupture of the specimen when all the elements of the cross-section fail.
Two types of scaling are used to describe the crack's tortuousity and randomness in crack propagation

dynamics: spatial parameters and temporal ones.

Spatial scaling parameters

The theory of fractals [11] can be effectively used in scaling analysis. The *fractal dimension* of cracks calculated by the box-counting method shows that the fractal dimension D increases with the growth of material's homogeneity (growing $K_{0.5}$) with a natural limit $D \to 2$ for the uniform stiffness distribution and crack propagation as a straight line parallel to its initial position.

Another scaling parameter characterising tortuousity of crack fronts can be obtained by means of the in-plane roughness analysis [12, 13], based on the general scheme for characterization of self-affine profiles [11]. The *roughness index* (also known as the *Hurst exponent*) ζ is determined from the scaling relation $y_{\max} \propto r^{\zeta}$, where

$$y_{\max}(r) = l_y \left\langle \max n^q(r')\Big|_{x<r'<x+r} - \min n^q(r')\Big|_{x<r'<x+r} \right\rangle\Big|_x . \quad (4)$$

Here $y_{\max}(r)$ is a distance along the y-axis between the farthest – with respect to the initial V-notch – locally failed element, belonging to a propagating crack, and the nearest one within the given window of size r along the crack front. The level of ζ obtained from microscopic observations was found to be equal 0.60±0.04 for 8090 Al-Li alloy and 0.54±0.03 for a Superα_2 Ti_3Al-based alloy on the length scale from 1 µm to 1.5 mm [19]; the detailed analysis of universality of ζ is given in [13]. The roughness of fracture surfaces (*out-of-plane roughness*) measured, for instance, by mechanical profilometry is a material-sensitive parameter: for sandstone the roughness exponent ζ was found to be close to 0.5 (0.47±0.03) and for basalt it was higher (0.80±0.04) for scale length from 25 µm up to several cm [14]. Results of numerical simulations based on the introduced description of spatially random brittle media show the effect of the extent of non-uniformity in material properties on the in-plane roughness: ζ = 0.56±0.03 for $K_{0.5} = 0.1683$ and ζ = 0.62±0.01 for $K_{0.5} = 0.4997$.

The multifractal analysis [15] can be used to quantify the load distribution character in the vicinity of tortuous crack fronts. It was shown [16] that the material's stochasticity type influences the *multifractal properties* of the load distribution: an increase in spatial uniformity causes the decrease of the width of multifractal spectra. The same formalism was applied to multiple matrix cracking of fibre-reinforced laminate composites under tensile fatigue [17-19]. It was shown that spatial distributions of both matrix cracks and of their length possess multifractal properties. Both the loading conditions (level of matrix stresses and number of cycles) and the structure of laminates (their stacking order) affect scaling characteristics of these distributions [17].

Temporal scaling parameters

The most important temporal features of the fracture process is time to fracture t_{fr} and its change with loading conditions (stress level σ and initial length of the V-shape notch). Detailed numerical modelling shows that *time-to-fracture scaling*, determined by the relation $t_{fr} \propto \sigma^{-n}$ is independent of material's spatial randomness. All types of non-uniformity in material properties (characterised by $K_{0.5}$) have the same level of the temporal scaling exponent n for all introduced parameters of the crack front position (see above) [20]. The same is true for the time-to-fracture dependence on the initial crack length.

Free-energy release is one of the standard parameters of fracture mechanics [6, 7]. The proposed model allows an estimation of energy release linked with local failures of elements at crack propagation. This process is characterised by rather non-trivial dynamics [21]. Multifractal formalism discussed in the previous subsection can also be applied to the study of this dynamics. It is shown that multifractal spectra for energy release at crack propagation in brittle materials are considerably more sensitive to the type of material randomness than respective scaling characteristics used in analysis of random events.

CONCLUSIONS

Crack propagation in brittle heterogeneous materials is a considerably more complicated case of fracture development than that of isotropic materials treated within the framework of standard fracture mechanics. Non-uniformity both of the spatial distribution of mechanical properties and of the evolution of ensembles of defects can result in multiple cracking, tortuousity of crack fronts and roughness of fracture surface. All these features are well known from fractography and should be adequately reflected in modelling schemes. One of the variants of such a scheme unifying ideas of fracture mechanics and continuous damage mechanics and utilising the spatial discretisation of the analysed region into elements with different properties is suggested. The fracture development obtained in numerical simulations within the framework of this model resembles real fracture processes. Complicated morphology of these processes needs an introduction of additional parameters. Various scaling parameters are shown to be useful in quantification of both spatial and temporal features of crack propagation. They also allow an effective comparison between experimental data obtained by precise measurements of crack roughness and results of numerical simulations.

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