ORAL/POSTER REFERENCE: 0846

SELF-CONSISTENT CONSTITUTIVE MODELING OF PARTICLE DISPERSED COMPOSITE MATERIALS WITH MICRO-CRACK TYPE DAMAGES

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ABSTRACT

The treatment of perfectly bonded interface between the particles and matrix of composite is sometimes considered to be inappropriate in describing the physical nature. The effect of imperfect interface on the overall behavior of particle-reinforced composites is studied. In order to investigate the detrimental effects of the slightly weakened interface on the overall material properties, a rigorous constitutive model was schemed, which uses a self-consistency scheme based on the Eshebly's equivalent inclusion method, and is capable to reflect the meso-local damage effects even in the range where the volume fraction of particles is high. Both the tangential and normal discontinuities at the interface are independently modeled, and these relative displacements are directly proportional to the corresponding components of tractions at the interface. The numerical results are also shown. It is found that the imperfect interface conditions of debonding and/or sliding give detrimental effects on the overall properties of composites. Thus, the establishment of the most appropriate model describing properly the meso-local phenomena.

KEYWORDS

Composites, Imperfect Interface, Sliding and Debonding, Constitutive Model

INTRODUCTION

The inhomogeneity problem has received considerable attention since Eshelby[1] published his well-known paper on the treatment of ellipsoidal inclusion. Based on the Eshelby's equivalent inclusion model, quite a few works have been performed, which assume perfect bonding at the interface between the inhomogeneity and the matrix. However, the solutions of perfect bonding condition have not been considered to be sufficient in describing the mechanical properties of the meso-local inhomogeneity problem of situations involving debonding and sliding. It is obvious that the interface conditions dramatically affect the mechanical behavior of the composites. Therefore, there have been considerable interests in imperfect interface problems as may be appropriate in the case of either pre-existing defects or interface damage due to, for example, the cyclic loading. The most popular model for quantifying this imperfection is the linear spring-layer model, in which a relationship at the interface between the traction vector and the displacement jump is assumed. This model of the linear interface has been employed by Hashin[2,3], Qu[4,5], Zhong[6,7], and Gao[8] among others in the development of the relevant problems. One of the interesting works in this area is by Gao[8], who has modeled the circular inclusion in the matrix imperfect interface under a uniform tension to find the equivalent eigen strain values. In meso-mechanics of solids, elastic solutions to the inhomogeneity problem are often used to relate the overall deformation and the corresponding stress field in the composites. In this paper, an Eshelby tensor, is derived accounting for the mechanics of the imperfect interface. The Eshelby tensor is averaged over the entire area of an inclusion to obtain the average effect that is concerned for evaluating the overall properties of composites. On the local study inside and around an inclusion, a constitutive law that accounts for the meso-local imperfection, based on a self-consistent scheme is developed, which applies the Eshebly's equivalent inclusion method to the particle-matrix domain, thus an averaged compliance was determined rigorously. The present constitutive modeling was carefully devised to be capable of reflecting the effect of slightly-weakened interface on the overall properties of the composites, even in the range where the volume fraction of particles is high.

MODELING IMPERFECT INTERFACE

As pointed out by some researchers, the conditions of initially perfect bonding at the interface between the inhomogeneities and the matrix sometimes may deteriorate. Boundary-sliding as well as debonding are normal phenomena seen after a period of service for some composites. Especially under cyclic loadings, defect and damage may occur on the particle-matrix interface. They would incur the imperfections of the interface. Consider a circular inclusion Ω embedded in an infinitely extended elastic domain (matrix) D-Ω. $\partial\Omega$ represents the imperfect surface area, as depicted in Fig. 1. The defected interface is modeled by a spring layer with its vanishing thickness. It is assumed that the tractions on the interface remain continuous (may not be equal to that of the perfect bonding case), but the displacements are not. Furthermore, the normal and tangential displacement-discontinuities on the interface are assumed directly proportional to the corresponding traction components. Then, the interfacial conditions can be expressed as

$$\left(\sigma_{ij}\big|_{\partial\Omega^+} - \sigma_{ij}\big|_{\partial\Omega^-}\right)\boldsymbol{l}_j = 0 \tag{1}$$

$$\boldsymbol{u}_i\big|_{\partial\Omega^+} - \boldsymbol{u}_i\big|_{\partial\Omega^-} = \eta_{ij}\sigma_{jk}\boldsymbol{l}_k \tag{2}$$

where, l_i denotes the unit outward normal vector of the interface $\partial\Omega$, the second order tensor η_{ij} indicates the compliance of the hybrid spring layer at the interface, which is usually expressed as

$$\eta_{ij} = \delta_{ij}/\boldsymbol{t} + (1/\boldsymbol{n} - 1/\boldsymbol{t})\boldsymbol{l}_i\boldsymbol{l}_j\,. \tag{3}$$

In the above, δ_{ij} is the Kronecker delta, the two scalars t and n represent the tangential and the normal stiffness of the spring, respectively.

Gao[8] derived a solution to this problem by assuming an Airy stress functions for both the matrix and the inclusion. After the stress field inside of the inclusion was found, the total strain in the inclusion were derived using Hooke's law. Then the Eshelby tensor is determined from the eigen strain so that the strain would be equivalently generated due to the corresponding part of the eigen strain. Since for evaluating the overall properties of the composite with the imperfect interface, only the average effect is concerned, thus the Eshelby tensor is defined such that

$$\bar{\boldsymbol{S}}_{ijkl} = \frac{1}{\Omega}\int_{\Omega}\boldsymbol{S}_{ijkl}(\boldsymbol{r},\theta)\boldsymbol{d}\Omega \tag{4}$$

The expressions of S^M for circular inclusions are given in Appendix in detail.
The effects of the imperfect interface on overall properties of composite have been taken into account by the use of the obtained average Eshelby tensor. Thus, the effective properties of the composite with imperfect interfaces can be obtained by further averaging the effect to multi-particle dispersed domain problem.

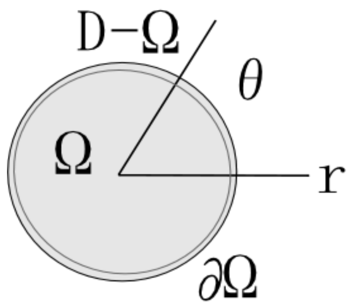

Fig.1 Schematic of particle with imperfect interface

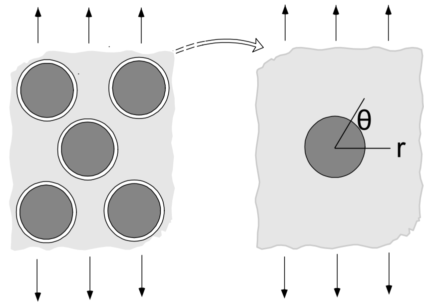

Fig.2 Composite with imperfect interface and equivalent domain

AVERAGING METHOD

The macro-local average properties of a composite with the imperfect interface can be obtained more reasonably by employing the self-consistent compliance (SCC) method. Based on Eshelby's equivalent inclusion theory, the stress and strain state of a virtual inclusion in the medium can be made equal to that of the real inclusion by imposing some proper amount of eigen strain to be solved. Mori and Tanaka[9] took the medium to be the matrix material, while later Mura[10] as well as Wu and Nakagaki[11] assumed it to be the averaged material of the composite. The latter is called the SCC model in the present report. It is assumed that a macro-local property of the composite with the inclusions with imperfect interface can also be treated by the same manner. Thus, the meso-local average stress and strain state as well as the properties of the macro-composite materials(Fig. 2). Following the approach used by Wu and Nakagaki (1999), and considering the present condition of the imperfect of interface, the equivalent relationship of stress can be expressed as in the following:

$$\mathbf{E}_2(\hat{\boldsymbol{\varepsilon}} + \boldsymbol{\varepsilon}^c) = \hat{\mathbf{E}}(\hat{\boldsymbol{\varepsilon}} + \boldsymbol{\varepsilon}^c - \boldsymbol{\varepsilon}^{*a}) \tag{5}$$

The **Bold** type characters stand for a second or fourth order tensor. The subscript 2 indicates the variable is of the inclusion, and the superscript (^) indicates that the quantity is of an averaged value. $\boldsymbol{E}_2$ represents the material tensor for the inclusion material, $\hat{\mathbf{E}}$ is the average material tensor for the composite material, $\hat{\boldsymbol{\varepsilon}}$ stands for the average strain of the composite, $\boldsymbol{\varepsilon}^c$ indicates the strain mismatch between the inclusion and the composite, and $\boldsymbol{\varepsilon}^{*a}$ is the total eigen strain that is the sum of the eigen strain($\boldsymbol{\varepsilon}^*$) for the material inhomogeneity and the eigen strain($\boldsymbol{\varepsilon}^{*i}$) due to the effect of the imperfection at the interface between particles and the matrix. According to the Eshelby theory and by considering the conditions of imperfect interface, we have,

$$\boldsymbol{\varepsilon}^c = \overline{\boldsymbol{S}}\ \boldsymbol{\varepsilon}^{*a}\ . \tag{6}$$

The role of the Eshelby tensor in imperfectly bonded inclusion problems now assumes the effects of the material inhomogeneity and the imperfection, and the treatment of the rest is similar to the case of the perfectly bonded-inclusion composite problems. Substituting Eq. (6) into Eq. (5), the following will be found,

$$\boldsymbol{\varepsilon}^* = \boldsymbol{A}_0\, \hat{\boldsymbol{E}}\, \hat{\boldsymbol{\varepsilon}} \tag{7}$$

where,

$$\mathbf{A}_0 = [\mathbf{E}_2(\hat{\mathbf{E}} - \mathbf{E}_2)^{-1}\hat{\mathbf{E}} - \hat{\mathbf{E}}(\overline{\mathbf{S}} - \mathbf{I})]^{-1} \tag{8}$$

Here, $\mathbf{I}$ stands for the identity tensor of the fourth order.
The stress $\boldsymbol{\sigma}_2$ in the inclusion can also be obtained as in the following,

$$\boldsymbol{\sigma}_2 = \mathbf{B}\ \boldsymbol{\sigma}_0 \tag{9}$$

$$\mathbf{B} = \mathbf{I} + \hat{\mathbf{E}}(\overline{\mathbf{S}} - \mathbf{I})\mathbf{A}_0 \tag{10}$$

where, $\boldsymbol{\sigma}_0$ is the average stress of the composite.
Once determining the stresses in the inclusion phase, it is easy to establish the constitutive law of the composite as follows.

$$\boldsymbol{\varepsilon}_0 = \hat{\mathbf{L}}\ \boldsymbol{\sigma}_0 \tag{11}$$

$$\hat{\mathbf{L}} = \mathbf{L}_1[\mathbf{I} - f\mathbf{B})] + f\mathbf{L}_2\,\mathbf{B} \tag{12}$$

where $\mathbf{L}_1$ and $\mathbf{L}_2$ are the elastic compliances for the matrix and the inclusions, respectively, $\hat{\mathbf{L}}$ is the global average compliance of the composite material, which is equal to the inverse of the global average property tensor $\hat{\mathbf{E}}$. Because the right hand of Eq. (12) contains the yet-unknown tensor $\mathbf{E}$, the present scheme is of the self-consistent compliance(SCC) method. To solve those averaged values, an iterative algorithm is undertaken.

NUMERICAL RESULTS AND DISCUSSIONS

In order to investigate the effect of imperfect interfaces on the overall properties of composites, and to verify the performances of the spring-layer model with the averaging theory, numerical analyses were conducted. The used values for the Young's modulus and the Poisson's ratio of the matrix material and the particles are such that:

Matrix $\boldsymbol{E}_1$=2.0Gpa, ν_1=0.35

Particles $\boldsymbol{E}_2$=40Gpa, ν_2=0.18

Both the matrix and particles are assumed to be isotropic. The particles are spherical in shape. Two typical cases of plane problems, i.e. the plane strain and the plane stress were studied.

Variation for volume fractions of particles

Fig. 3 shows the computed equivalent Young's modulus of the composite with the use of Eq. (12) under three conditions with imperfect interfaces: i.e. pure sliding, pure debonding, and combined sliding and debonding, under the plane strain condition. Normalized Young's modulus defined by the following is shown for the variation of the particle volume fraction.

$$E_{nd} = \frac{\hat{E} - E_1}{E_2 - E_1} \tag{13}$$

The considered interface conditions are regulated the tangential and the normal stiffness of the spring, *t* and *n*. In the present, either 0, 1, or ∞ is considered for these factors and denoted so hereafter, whereas the unit value of the sliding/debonding parameter is equal to the spring stiffness of the matrix material evaluated per unit area. The solid line indicates the case of the perfect interface. It is quite clear that all of the imperfect interface conditions give a detrimental effect to the composite stiffness, as expected. Furthermore, the perfect debonding(short-dashed line) is more detrimental than the sliding with no friction, where other parameters are unchanged. The totally debonding case, that is when both the tangential and the normal stiffness of these springs tend to zero, the rigidity of the composite will become lowest. But it should be noted that the stiffness of the totally debonnded case is still much higher than that of the void inclusion case. When the sliding and debonding parameters have certain finite values(e.g. *t*=*n*=1 in Fig. 3), the line will drop somewhere in between the two ultimate conditions of the interface. Another interesting result is that the detriment effect of the imperfect interface becomes greater as the volume fraction of particles gets larger. For instance, when $f = 0.3$, the equivalent Young's modulus of the composite with the imperfect interface(totally debonding) is 79.2% of that of the corresponding perfect interface. While, this value will be approximately 76.1% when $f = 0.7$. Fig. 4 shows the similar results of Young's modulus in case of the plane stress.

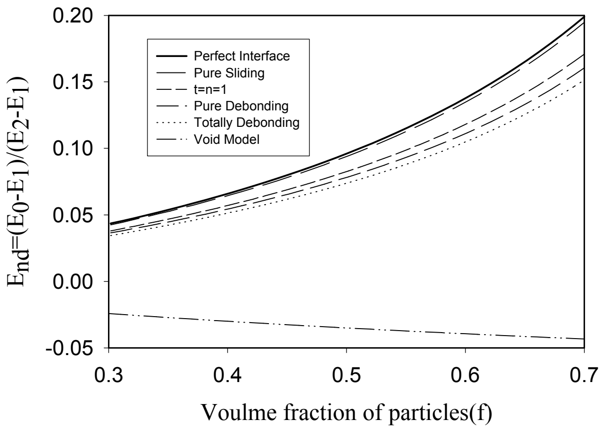

Fig.3 Young's modulus of composite with various interface conditions(Plain Strain)

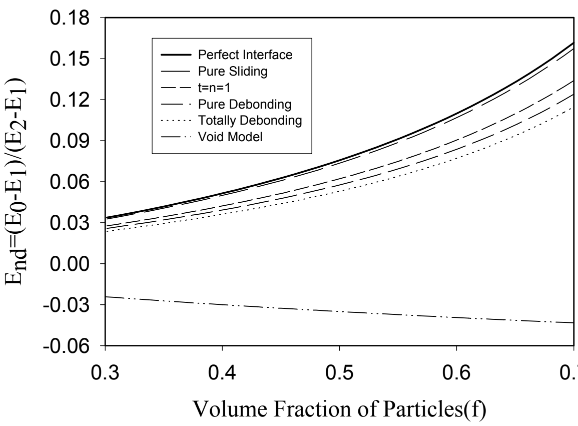

Fig.4 Young's modulus of composites with various interface conditions(Plain Stress)

Effect of sliding and debonding parameters

In order to investigate the effect of the two imperfect parameters in detail, i.e. sliding and debonding, on the overall properties of composite, two typical cases of only sliding and only debonding were studied. Fig. 5

and 6 show the results obtained for the plane strain case, with f =0.5. It is learned from these results that there is a range of transition in these imperfection springs for both the sliding and the debonding from the completely defective to the perfect interface. It is noteworthy that the range of the transition is spanned on approximately 0.1 to 10 for both the cases.

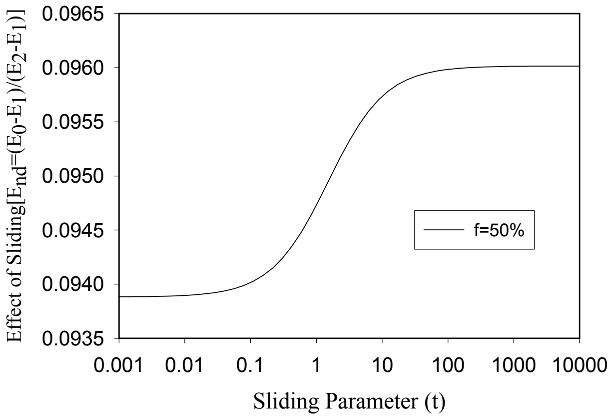

Fig.5 Effect of sliding parameter on overall properties

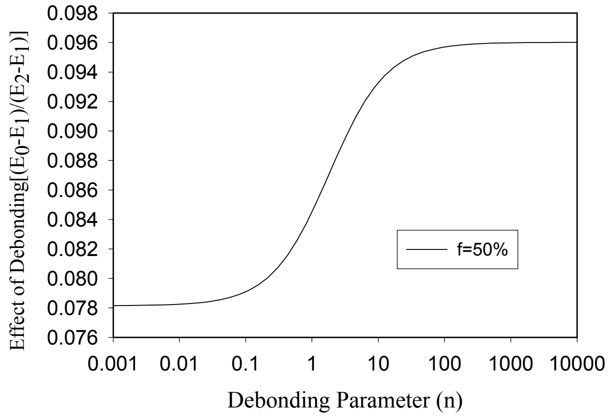

Fig.6 Effect of spring parameter on overall properties

Effect of sliding or debonding for particle volume fractions

For investingating the effect of the sliding and the debonding parameters on the overall Young's modulus with various particle volume fractions, the average Young's modulus of the composite is normalized by that with perfect interface of the same volume friction of particles that,

$$\boldsymbol{E}_{nor} = \frac{\boldsymbol{E}_{imperfect}}{\boldsymbol{E}_{perfect}} \tag{14}$$

Fig. 7 shows the results for the case of debonding for f = 0.3, 0.5, and 0.7. It is obvious that the relative detrimental-effect of debonding is more serious at higher volume fraction of particles. However, it is interesting to know that this is not the case for the sliding. Fig. 8 shows the results of sliding for the same selected particle volume fractions as in the previous problem. Unlike the debonding case, the sliding around the particle does not give marked effects to the macroscopic characteristics of the composite. The effect is small, but the sliding effect for the low volume fraction of the inclusion stands over that of the higher fractions.

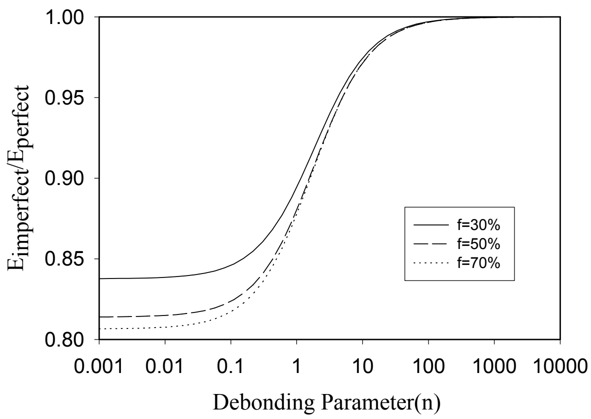

Fig.7 Comparison of effect of interface debonding for various volume fractions

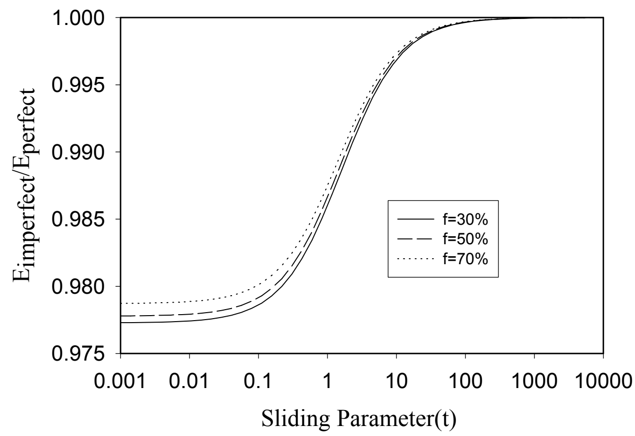

Fig.8 Comparison of effect of interface sliding for various fractions

CONCLUSIONS

The effect of imperfect interface on the overall behavior of particle-reinforced composites is studied. The interface is modeled as a spring layer with vanishing thickness. By assuming that, (a) the tractions on the interface remain continuous, but (b) the displacements are discontinuous, and (c) the normal and tangential displacement-discontinuities on the interface are directly proportional to the corresponding traction components, an averaged solution of Eshelby's S-tensor were obtained. This Eshelby tensor was applied to the carefully schemed Self-consistent Compliance model in order to develop the constitutive model for the composite incurring particle-matrix damages in the meso-mechanics level. The present model is used to investigate the effect of a slightly weakened interface. The following conclusions could be made by the numerical analysis. (1) The imperfect interface conditions give a detrimental effect on the overall properties of composites, where debonding is more detrimental than sliding. (2) The detrimental effect of the debonding interface is getting higher as the volume fraction of particles becomes larger. (3) Although the effect is small, the sliding effect in the composite is rather marked for the low volume fraction case of the inclusion than the higher fractions.

REFERENCES

J.D. Eshelby, The determination of the elastic field of an ellipsoidal inclusion and related problems, Proc. Roy. Soc., Vol. A241, pp.376-396.

Z. Hashin, Thermoelastic properties of particulate composites with imperfect interface, J. Mech. Phys. Solids, Vol. 39, No. 6, pp. 745-762, 1991.

Z. Hashin, The spherical inclusion with imperfect interface, Journal of Applied Mechanics, Transactions of ASME, Vol. 58, pp.444-449, 1991.

J. Qu, Eshelby tensor for an elastic inclusion with slightly weakened interface, Journal of Applied Mechanics, Transactions of ASME, Vol. 60, pp.1048-1050, 1993.

J. Qu, The effect of slightly weakened interface on the overall elastic properties of composite materials, Mechanics of Materials, Vol. 14, pp.269-281, 1993.

Z. Zhong, S. A. Meguid, On the imperfectly bonded spherical inclusion problem, Journal of Applied Mechanics, Transactions of ASME, Vol. 66, pp.839-846, 1999.

Z. Zhong, S. A. Meguid, On the eigenstrain problem of a spherical inclusion with an imperfectly bonded interface, Journal of Applied Mechanics, Transactions of ASME, Vol. 63, pp.877-883, 1996.

Z. Gao, A circular inclusion with imperfect interface: Eshelby's tensor and related problems, Journal of Applied Mechanics, Transactions of ASME, Vol. 62, pp.860-866, 1995.

T. Mori and K. Tanaka, Average stress in matrix and average energy of materials with misfitting inclusions, Acta, Metallurgica, Vol. 21, pp.571-574, 1973.

T. Mura, Micromechanics of Defects in Solids, Martinus Nijhoff, Boston, 1987.

Y. Wu, M. Nakagaki, Constitutive model for fiber dispersed composite, Transactions of the Japan Society of Mechanical Endineers, Vol. 65, No. 633, pp.1003-1009, 1999.

APPENDIX: THE MODIFIED ESHEBLY'S S-TENSOR

$S_{1111}=S_{2222}=Q_1-Q_2-Q_3+1$, $S_{1122}=S_{2211}=Q_1+Q_2+Q_3$, $S_{1212}=-Q_2-Q_3$

Other $S_{ijkl}=0$.

where $Q_1=\dfrac{nr(1-k_2)}{2(nrk_2-nr+4+2n)}$, $Q_2=\dfrac{rn(rtk_2+t+6)}{2P}$, $Q_3=\dfrac{6r(t-n)}{4P}$

and $P=3p_1p_2+ntp_2p_3+p_3p_1+12$, $p_1=n+t$, $p_2=r+k_1$, $p_3=rk_2+1$

$r=\mu_1/\mu_2$ k_i=3-4υ_i for plain strain, and k=(3-υ_i)/(1+υ_i) for plain stress, i=1,2.