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REVISITED ENERGY CRITERIA FOR INTERFACE CRACK DEFLECTION

D. Leguillon[1], C. Lacroix[1] and E. Martin[2]

[1] Laboratoire de Modélisation en Mécanique, CNRS - Université P. et M. Curie, 8 rue du Capitaine Scott, 75015 PARIS - France.
[2] Laboratoire de Génie Mécanique, IUT A, Université de Bordeaux 1, 33405 TALENCE Cedex – France.

ABSTRACT

Crack deflections by interfaces increase the apparent toughness of composites. Predictions are based on the comparison between the energy released during the crack growth along the interface or within the inclusions. We propose a review of our recent works in the topic. In a first step, using matched asymptotics and singularity theory, we exhibit a slightly modified form of the He and Hutchinson criterion. Next, an analysis of the residual thermal stresses leads to conclude that they have a little influence on the deflection criterion (but of course not on the further growth of the extensions). In a second step, we establish a revisited criterion, which deals with the excess of energy produced in some cases during the crack advance. Finally another mechanism is investigated: the interface is assumed to fail prematurely leaving initially an unbroken ligament between the primary crack tip and the debonded interface. Deflection results of the linking of the primary and the interface cracks.

KEYWORDS

Composites, interfaces, crack deflection, residual stresses.

INTRODUCTION

Fibers or other inclusions are inserted in brittle materials to promote toughening processes [2]. Cracks growing within the matrix are expected to kink along the interfaces and either to blunt the primary crack tip or to develop dissipative processes by friction. In ceramic matrix materials it is an essential mechanism which limits matrix cracks path and delays the final ruin of the structure. An efficient criterion able to predict such crack deflection is of course essential to tailor these composites. An approach, proposed by He and Hutchinson [3] (HH), is based on the analysis of energy release rates at the tip of virtual crack extensions either deflecting along the interface or penetrating into the fiber.

In a first step, using matched asymptotics and singularity theory, we propose a slightly modified form of the HH criterion involving the amount of energy that the primary crack requires to extend [11]. It emphasizes on the awkward role of the arbitrary extension lengths introduced in both models.

Next, an analysis of the residual thermal stresses leads to slightly different conclusions than that of He et al. [4]. At the leading order, it is drawn that they have few influence on the deflection criterion (but of course not on the possible further growth of the extensions) [10].

Prior to these extensions, the primary crack is assumed to impinge on the interface. In some cases this is questionable because of the non-classical character of the singularity, it makes the primary crack growth easier and easier as it approaches the interface. Thus, in a second step, we propose a revisited criterion that deals with the excess of energy (with respect to the Griffith criterion) produced during the crack advance. Surprisingly, arbitrary extension lengths definitions are no longer necessary; it is obviously a noticeable improvement of the HH criterion although its formulation remains very simple [8].
Finally another mechanism, first suggested by Cook and Gordon [1], is investigated: the interface is assumed to fail prematurely leaving initially an unbroken ligament between the primary crack tip and the debonded interface, as observed by Lee et al. [6]. Deflection results finally of the linking of the primary and the interface crack. Necessary conditions to such a mechanism are derived. In particular, in case of a stiff matrix, interface debonding ahead of the primary crack is shown to be almost inhibited [9].

MATCHED ASYMPTOTICS AND THE GRIFFITH CRITERION

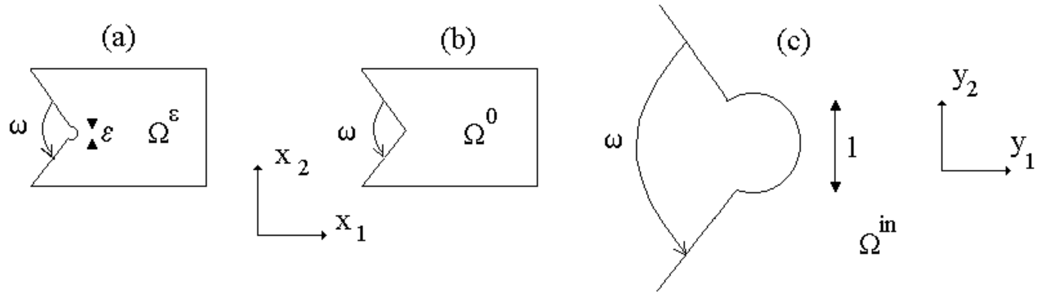

Figure 1: The perturbed (a), unperturbed (b) and stretched domains (c).

Matched asymptotics procedure is briefly recalled here. We consider a domain Ω^{ε} in which a corner is slightly perturbed by a flaw (a short crack, a small void,) with dimensionless diameter ε (figure 1(a)). The solution $\vec{U}(\varepsilon,x_1,x_2)$ to a plane elasticity problem in this domain can be expressed as

$$\vec{U}(\varepsilon,x_1,x_2)=\vec{U}(0,x_1,x_2)+\text{ smaller terms,} \tag{1}$$

where $\vec{U}(0,x_1,x_2)$ is solution to a similar problem but in the unperturbed domain Ω^0 (figure 1(b)). This term is singular in the corner and expands as

$$\vec{U}(0,x_1,x_2)=kr^{\lambda}\vec{u}(\varphi)+\vec{U}^R(x_1,x_2)\,. \tag{2}$$

r and φ are the polar co-ordinates; $0<\lambda<1$ is the singularity exponent, it is solution to an eigenvalue problem and $\vec{u}(\varphi)$ is the associated eigenvector, k is the generalized intensity factor. The remaining part $\vec{U}^R(x_1,x_2)$ is a smooth complement. For simplicity, we do not take into account multiple, complex or defective eigenvalues. Of course, the far field in eqn. (2), is valid out of a vicinity of the perturbation. To have information on the near field, one has to stretch the initial domain by $1/\varepsilon$ and then consider the limit domain Ω^{in} as $\varepsilon\to 0$ (figure 1(c)). The solution now writes

$$\vec{U}(\varepsilon,x_1,x_2)=\vec{U}(\varepsilon,\varepsilon y_1,\varepsilon y_2)=k\varepsilon^{\lambda}[\rho^{\lambda}\vec{u}(\varphi)+\vec{V}(y_1,y_2)]+\text{ smaller terms,} \tag{3}$$

where $y_i=x_i/\varepsilon$ and $\rho=r/\varepsilon$. The particular form of the first term of the expansion is due to the matching conditions; it behaves at infinity like the first term of eqn. (2) near 0. There is an intermediate area in which both outer (eqn. (1)) and inner (eqn. (3)) expansions hold.

The Betti's theorem allows expressing the change in potential energy between an initial (unperturbed) state and a next (perturbed) one

$$\delta W_p = \frac{1}{2}\int_\Gamma [\sigma(\vec{U}(\varepsilon,x_1,x_2))\vec{n}\vec{U}(0,x_1,x_2) - \sigma(\vec{U}(0,x_1,x_2))\vec{n}\vec{U}(\varepsilon,x_1,x_2)]dx , \quad (4)$$

where generically $\sigma(\vec{U})$ denotes the stress field associated to $\vec{U}$. Γ is any contour surrounding the corner and $\vec{n}$ its normal pointing toward the corner, it can be taken either in Ω^0 or Ω^{in} . Replacing the above expansions in eqn. (4) leads to the following relation

$$\delta W_p = k^2 K \varepsilon^{2\lambda} + \ldots , \quad \text{with} \quad K = \frac{1}{2}\int_\Gamma [\sigma(\vec{V}(y_1,y_2))\vec{n}\rho^\lambda \vec{u}(\varphi) - \sigma(\rho^\lambda \vec{u}(\varphi))\vec{n}\vec{V}(y_1,y_2)]dy . \quad (5)$$

Clearly, K depends on the shape of the perturbation but not on its actual size. Moreover it is independent of the applied loads, which appear in eqn. (3) only through the multiplicative coefficient k .
This procedure will be used in the following with a new short crack or a short crack increment as a perturbation. From eqn. (5), the incremental expression of the Griffith criterion takes the Irwin-like form

$$\delta W_p = k^2 K \varepsilon^{2\lambda} \geq G_c \varepsilon \;\Rightarrow\; G = k^2 K \varepsilon^{2\lambda - 1} \geq G_c . \quad (6)$$

CRACK DEFLECTION AT AN INTERFACE – THE HE AND HUTCHINSON CRITERION

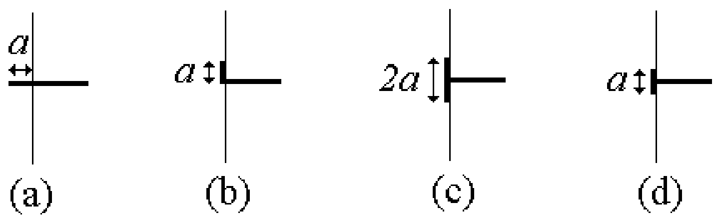

Figure 2: Penetration (a), single deflection (b), double deflection (c and d) of a crack at an interface.

A crack lies in material 1 and impinges on the interface with another component denoted material 2 and the problem is to determine the crack path. As a consequence of eqn. (6) the difficulty is the following, if material 2 is stiffer than material 1 then $\lambda > 1/2$ and $G = 0$, and in the opposite situation $\lambda < 1/2$ and $G \to \infty$. The usual differential form of the Griffith criterion is inappropriate. To avoid this obstacle, He and Hutchinson [3], using integral equations, compare the energy release rates G_p and G_d (the differential form) respectively at the tip of a penetrated (figure 2(a)) and a deflected (figures 2(b), 2(c)) crack at a short distance a of the impinging point. From their analysis, deflection is promoted if

$$\frac{G_d}{G_p} \geq \frac{G_{ic}}{G_{2c}} , \quad (7)$$

where the right hand side is the ratio of the interface and material 2 toughness G_{ic} and G_{2c} . The left hand side arises to be independent of the applied loads and of the increment length a .
Similarly G_p and G_d (the incremental form) can be computed from eqn. (6). They correspond now to the energy released during the crack growth and the equivalent to eqn. (7) leads to [11] (LS)

$$\frac{K_d}{K_p} \geq \frac{G_{ic}}{G_{2c}} , \quad (8)$$

where K_p and K_d are defined by eqn. (5) and correspond respectively to the penetrated and deflected geometries. Although they look similar, the two criteria are different. HH assume the penetration and deflection increment lengths and study the local fields at the tip of the new extensions. It is thus consistent to carry out the analysis at a same distance a of the primary crack tip (figures 2(a), 2(b), 2(c)). On the contrary, in the present approach, the question is to determine the energy balance that allows creation of crack extensions. In this context, it is consistent to examine equal crack extensions. It makes an important difference in case of symmetrical double deflection along the interface. In the HH case the total interface debonding length is $2a$ (figure 2(c)) whereas it must be a in present one (figure 2(d)). A comparison shows a good agreement between HH and LS criteria provided extension lengths are consistent. Nevertheless, there is no reason to take equal increments in both directions. But, otherwise the two criteria (eqns. (7) and (8)) become dependent on the ratio of the increment lengths that is unknown, making them questionable. It is discussed in a next paper by He et al. [4].

THE ROLE OF RESIDUAL THERMAL STRESSES

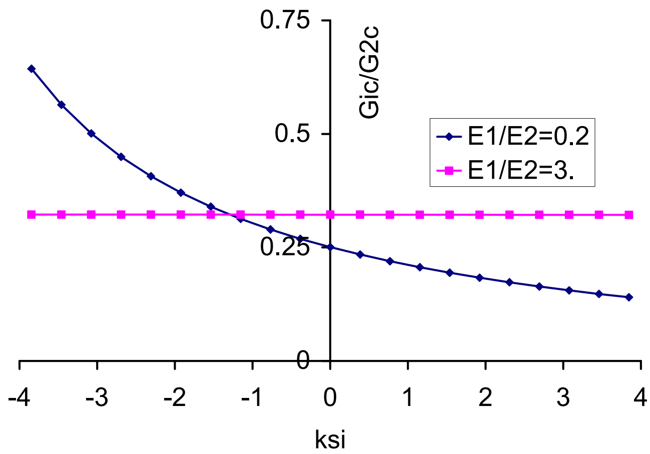

Figure 3: The influence of residual thermal stresses on the deflection/penetration criterion. The vertical axis corresponds to the absence of residual stresses.

In addition to the above discussion, He et al. [4] analyze the role of residual thermal stresses resulting of a cooling process. The problem can also be examined through the matched asymptotics procedure. As a first consequence, the generalized intensity factor in eqn. (2) splits in two parts

$$k = k_m + k_\theta \,, \tag{9}$$

which are respectively the contributions of the mechanical k_m and thermal k_θ loadings. Then the main role of residual thermal stresses is to modify the intensity factors, whereas the criterion (eqn. (8)) remains unchanged at the leading order from mechanical to thermal and to combined mechanical and thermal loadings. Residual stresses influence the load level at which the mechanism starts but not the mechanism itself. If secondary effects are accounted for, there is an additional term in the criterion [10] deriving from the generalization of the non-singular "T-stress"

$$\frac{K_d + H_d \, T/k \, a^{1-\lambda}}{K_p + H_p \, T/k \, a^{1-\lambda}} \geq \frac{G_{ic}}{G_{2c}} \,, \tag{10}$$

where T is a non-singular traction depending on the applied mechanical and thermal loadings and where H_p and H_d are coefficients similar to K_p and K_d. This criterion involves explicitly the increment length a. Moreover, it is non-local since it contains k and T which depend on the applied loads, on the processing temperature and on the geometry of the whole structure. T itself splits in two parts

$$T = T_m + T_\theta \,, \tag{11}$$

The mechanical contribution T_m is omitted in the He et al. [6] analysis. Figure 3 shows the trend of residual stress effects for two elastic contrasts between the materials. The dimensionless parameter ξ is proportional to the thermal expansion coefficients mismatch, $\xi = c(\alpha_1 - \alpha_2)$. When fiber is stiffer than the matrix, deflection is promoted if $\xi < 0$ (i.e. if $\alpha_2 > \alpha_1$). On the contrary, if $\alpha_2 < \alpha_1$ the deflection trend is lowered by the residual stresses. Moreover, the influence is almost negligible in the opposite situation of a stiff matrix and a soft fiber. It must be pointed out that these conclusions concern the prediction of the crack branching, i.e. the very beginning of the process, they do not inform on the possible further growth of the extensions.

THE PARTICULAR CASE $\lambda < 1/2$

In the above sections, it has been assumed that prior to any penetration or deflection, the crack impinges on the interface. This is questionable, especially when $\lambda < 1/2$, i.e. when material 1 is stiffer than material 2 as it is observed in reinforced ceramic matrix materials. Fibers are softer than the matrix and inserted only to promote toughening processes. Indeed, the energy release rate increases to infinity as the matrix crack approaches the interface (the Griffith criterion is more and more violated) and then decreases after penetration or deflection and finally drops below the critical toughness G_{2c} or G_{ic} (at a distance a_p or a_d known from eqn. (6)). Thus, the above assumption is not realistic. The kinetic energy δW_{kp} or δW_{kd} produced by the fracture process before the energy release rate drops below the critical toughness can be estimated in the two cases [6]

$$\delta W_{kp} = \delta W_{k1} + G_{2c} a_p \frac{1-2\lambda}{2\lambda} \quad \text{or} \quad \delta W_{kd} = \delta W_{k1} + G_{2c} a_d \frac{1-2\lambda}{2\lambda} \,, \tag{13}$$

where δW_{k1} is the kinetic energy produced before the crack reaches the interface. Thus we assume that deflection is promoted if $\delta W_{kd} \geq \delta W_{kp}$ which corresponds to a principle of maximum decrease in total energy as suggested by Lawn [5]. It leads to a slightly different form of eqn. (8)

$$\left(\frac{K_d}{K_p}\right)^{1/2\lambda} \geq \frac{G_{ic}}{G_{2c}} \,, \tag{14}$$

Which does not require the introduction of any arbitrary increment length. It is obviously a noticeable improvement of the HH criterion although the formulation remains very simple.

INTERFACE DEBONDING AHEAD OF THE PRIMARY CRACK

The premature failure of the interface can be invoked as another mechanism for crack deflection by an interface. It has been suggested by Cook and Gordon [1] and observed in many other situations [6]. The matched asymptotics procedure is particularly suited to study this situation.

The interface debonds on a length a while the primary crack tip is at a distance b of the interface (figure 4). The ligament is considered as the perturbation and is associated to the small parameter $\varepsilon = b/L \ll 1$ of the expansions (L is any characteristic length of the structure). The coefficient K in (5) depends on the local geometry, then if we set $\mu = a/b$, two coefficients $K(0)$ and $K(\mu)$ arise respectively from figure 4(a) and 4(b) and the change in potential energy between the two states (prior and after debonding) writes

$$\delta W_p = k^2 (K(\mu) - K(0)) \varepsilon^{2\lambda} + \ldots \tag{15}$$

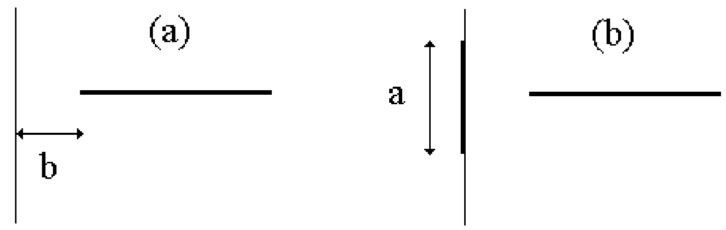

Figure 4: Interface debonding ahead of the primary crack.

The competition between the debonding and the primary crack growth within material 1 (without debonding) can be analyzed. It involves of course the material 1 toughness G_{1c}. It is shown in [9] that early interface debonding is promoted if

$$g(\mu) = \frac{K(0) - K(\mu)}{2\lambda K(0)\mu} \geq \frac{G_{ic}}{G_{1c}}. \tag{16}$$

$g(\mu)$ is plotted in figure 5 for different contrasts between the components. When there is no contrast (figure 5(b)) or when material 1 is stiffer than material 2 (figure 5(c)), $g(\mu)$ has a maximum $g_{\max}$ such that if $G_{ic}/G_{1c} > g_{\max}$, then the interface cannot debond prematurely. For high contrasts $E_1 >> E_2$, this maximum is very low and the early debonding process is almost inhibited except for very weak interfaces. On the contrary, when material 2 is stiffer than material 1 (figure 5(a)), the knowledge of the ratio G_{ic}/G_{1c} provides a lower bound $\mu_{\min}$ for the interface debonding length. However, the early failure cannot be predicted, eqn. (16) is a necessary condition but is not sufficient. A much thorough investigation shows that, in this latter case, it is necessary to invoke in addition a stress criterion to have a complete prediction of the mechanism [7]. The stress criterion gives an upper bound for the admissible debonding lengths and it must be checked that it is consistent with the lower bound $\mu_{\min}$.

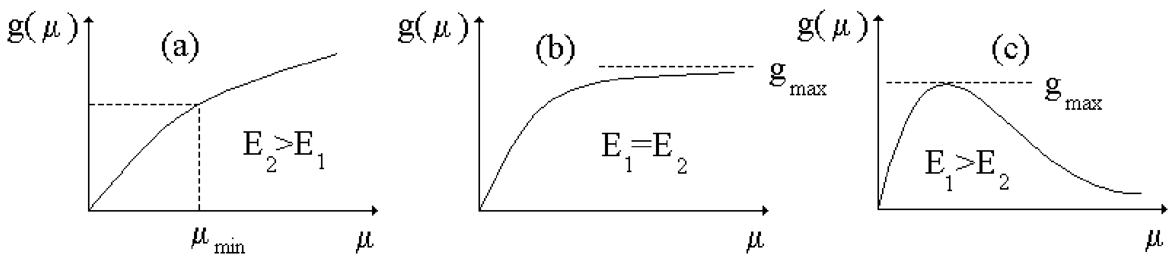

Figure 5: The function $g(\mu)$ for different contrasts between the components.

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