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RESONANCE AND CRACK PROPAGATION IN PRESTRESSED ORTHOTROPIC MATERIALS CONTAINING AN INCLINED CRACK

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ABSTRACT

In a previous paper [6] we have established for a prestressed orthotropic material containing a crack the direction of crack propagation and the critical incremental stresses, using Sih's generalised fracture criterion and considering all classical modes. Also, we concluded that in the case of prestressed isotropic materials acted by symmetrically distributed constant normal stresses the classical hypothesis used in Griffith-Irwin theory is justified according Sih's new fracture criterion. For all three-fracture modes we observed that Griffith-Irwin theory and Sih's new fracture criterion lead to the nearly same values of the critical incremental stresses producing crack propagation. In this paper we apply Sih's generalised fracture criterion to determine the critical incremental stresses producing crack propagation as well as the direction of the crack propagation in a prestressed orthotropic elastic material containing an inclined crack. Also, using numerical analysis we study the resonance phenomenon for two prestressed composite materials.

KEYWORDS

Inclined crack, prestressed composite materials, Sih's fracture criterion, resonance.

1. INTRODUCTION

We consider a prestressed material containing an inclined crack of a length $2a$ situated in x_1x_3 plane. We supposed that the material is unbounded and the crack faces are acted by constant normal incremental stresses p. The initial applied stress σ_0 is in direction of the crack.

Our first aim is to determine the elastic state produced in the body using Guz's representation theorem.

Our second aim is to determine the critical values of the incremental stresses and the direction of crack propagation. To do this, we use Sih's generalised fracture criterion for an orthotropic and for transversally isotropic materials assuming that the body in not initially deformed. We assume that the stress free reference configuration of material is locally stable and initial deformation is infinitesimal.

In the last part we verify that the critical value of the initial applied stress σ_0 for which the phenomenon of resonance can appear σ_0^c obtained by Guz [1] and Soós [2] is also available in the case of right crack parallel to the initial applied stresses.

2.GUZ'S REPRESENTATION THEOREM FOR INCREMENTAL FIELDS

We consider a prestressed, orthotropic, linear elastic material. We take as co-ordinate planes the symmetric planes of the material. We assume that the material is unbounded and contains a right crack of length *2a*, situated in a plane making with x_2x_3 an angle β. We supposed that the material is prestressed by an initial applied stress σ_0 acting in the direction of x_1 - axis see Figure 1. We assume that the magnitude of the initial applied stress σ_0 is sufficiently small and it produces only infinitesimal initial deformations. We suppose that the upper and lower faces o the crack are acted by distributed incremental stress having a constant value $p>0$, see Figure 1. Finally, we consider that the initial deformed equilibrium configuration of the body is locally stable.

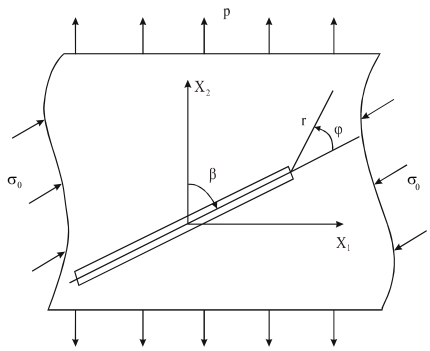

Figure 1: - Inclined crack in a prestressed orthtropic material

As it was shown by Guz [1,3,4] and independently by Soós [2] the incremental elastic state of the body can be expressed by two analytical complex potentials $\Psi_j(z_j)$ defined in two complex planes z_j, $j=1,2$. We denote by u_1,u_2 and respectively by θ_{11}, θ_{12}, θ_{21} and θ_{22} the involved components of incremental displacements, respectively incremental stresses.

According Guz's representation formulae we have:

$$u_1 = 2\operatorname{Re}\{b_1\Phi_1(z_1)+b_2\Phi_2(z_2)\} \quad ; \quad u_2 = 2\operatorname{Re}\{c_1\Phi_1(z_1)+c_2\Phi_2(z_2)\} \tag{2.1}$$

$$\theta_{11} = 2\operatorname{Re}\{a_1\mu_1^2\Psi_1(z_1)+a_2\mu_2^2\Psi_2(z_2)\} \quad ; \quad \theta_{12} = -2\operatorname{Re}\{\mu_1\Psi_1(z_1)+\mu_2\Psi_2(z_2)\} \tag{2.2}$$

$$\theta_{21} = -2\operatorname{Re}\{a_1\mu_1\Psi_1(z_1)+a_2\mu_2\Psi_2(z_2)\} \quad ; \quad \theta_{22} = 2\operatorname{Re}\{\Psi_1(z_1)+\Psi_2(z_2)\} \tag{2.3}$$

In this relations

$$\Psi_j(z_j) = \Phi_j'(z_j) = \frac{d\Phi_j(z_j)}{dz_j}, j=1,2 \tag{2.4}$$

and

$$z_j = x_1 + \mu_j x_2,\ j = 1,2. \tag{2.5}$$

The parameters a_j, b_j, c_j, $j=1,2$ have following expressions

$$a_j = (\varpi_{2112}\varpi_{1122}\mu_j^2 - \varpi_{1111}\varpi_{1212})/(B_j\mu_j^2);\ b_j = -(\varpi_{1122}+\varpi_{1212})/B_j;\ c_j = (\varpi_{2112}\mu_j^2+\varpi_{1111})/(B_j\mu_j) \tag{2.6}$$

$$B_j = \varpi_{2222}\varpi_{2112}\mu_j^2 + \varpi_{1111}\varpi_{2222} - \varpi_{1122}(\varpi_{1122}+\varpi_{1212}). \tag{2.7}$$

The instantaneous elasticities ϖ_{klmn}, k,l,m,n=1,2 can be expressed through the elastic coefficients C_{11}, C_{12}, C_{22} and C_{66} of the material and through the initial applied stress σ_0 by the following relations.

$$\varpi_{1111} = C_{11}+\sigma_0\ \varpi_{1212} = C_{66}\ ;\ \varpi_{2222} = C_{22}\ \varpi_{1221} = C_{66}+\sigma_0\ ;\ \varpi_{1122} = C_{12}\ \varpi_{2112} = C_{66}. \tag{2.8}$$

In their turn the elastic coefficients can be expressed using the engineering constants of the material and we have:

$$C_{11} = \frac{1-\nu_{23}\nu_{32}}{E_2E_3H} \qquad C_{22} = \frac{1-\nu_{13}\nu_{31}}{E_1E_2H} \qquad C_{12} = \frac{\nu_{12}+\nu_{32}\nu_{13}}{E_1E_2H} \qquad C_{66} = G_{12} \tag{2.9}$$

with

$$H = (1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{31}\nu_{13}-\nu_{21}\nu_{32}\nu_{13}-\nu_{12}\nu_{23}\nu_{31})/(E_1E_2E_3) \tag{2.10}$$

In this relations E_1, E_2, E_3 are Young's moduli in the corresponding symmetry directions of the material ν_{12},..., ν_{32} are Poisson's ratios and G_{12}, G_{23}, G_{31} are the shear moduli in the corresponding symmetry planes. We recall that by σ_0 we have designed the initial applied stress acting in x_1 direction. The parameter μ_j can be obtained determining the roots $\nu_j, j=1,2$ of algebraic equation :

$$\nu^2 + 2A\nu + B = 0 \tag{2.11}$$

with

$$A = [\varpi_{1111}\varpi_{2222} + \varpi_{1221}\varpi_{2112} - (\varpi_{1122} + \varpi_{1212})^2]/(\varpi_{2222}\varpi_{2112}) \quad B = (\varpi_{1111}\varpi_{1221})/(\varpi_{2112}\varpi_{2222}). \tag{2.12}$$

As was shown by Guz [1,3,4] the above equation (2.11) can not have real roots if the initial deformed equilibrium configuration of the body is locally stable. The complex parameters μ_j can be calculated through the roots $\nu_j, j=1,2$ using equations (see Guz [1, 3, 4] and Soós [2])

$$\mu_1 = \sqrt{\nu_1}, \mu_2 = -\sqrt{\nu_2} \text{ if } \operatorname{Im}\nu_j \neq 0, j = 1,2 \tag{2.13}$$

and

$$\mu_1 = \sqrt{\nu_1}, \mu_2 = -\sqrt{\nu_2} \text{ if } \operatorname{Im}\nu_j = 0 \text{ and } \operatorname{Re}\nu_j < 0, j = 1,2. \tag{2.14}$$

It can be shown that the parameters μ_j, $j=1,2$ satisfy the relations

$$\operatorname{Im}(\mu_1\mu_2) = 0 \quad \text{and} \quad \operatorname{Re}(\mu_1 + \mu_2) = 0. \tag{2.15}$$

In what follows we assume that parameters μ_1 and μ_2 are not equal i.e.

$$\mu_1 \neq \mu_2. \tag{2.16}$$

The above condition is satisfied for orthotropic materials as well as for prestressed isotropic materials.

The expressions of the complex potentials $\Psi_j(z_j)$, $j=1,2$ corresponding to our incremental boundary value problem were determined by Guz [1,3,4] and later independently by Soós [2] and have the following equations

$$\Psi_j(z_j) = \frac{(-1)^{j-1}}{\chi_j(\varphi)} \cdot \frac{a_j\mu_j K_I + K_{II}}{2\Delta\sqrt{2\pi r}}, j = 1,2 \tag{2.17}$$

where

$$\Delta = a_2\mu_2 - a_1\mu_1 \tag{2.18}$$

$$\chi_j(\varphi) = (\cos\varphi + \mu_j \sin\varphi)^{1/2}, j = 1,2 \tag{2.19}$$

and

$$K_I = p\sin^2\beta\sqrt{\pi a}, \; K_{II} = p\sin\beta\cos\beta\sqrt{\pi a} \tag{2.20}$$

are the stress intensity factors corresponding to the first respectively second mode of fracture for an applied incremental stress $p>0$.

3. SIH'S ENERGETICAL CRITERION IN A MIXED FRACTURE MODE

We denote by W *the incremental strain energy density* i.e.

$$W = \frac{1}{2}\theta_{kl}u_{l,k}, \quad k,l = 1,2 \tag{3.1}$$

where

$$u_{l,k} = \frac{\partial u_l}{\partial x_k}$$

The expressions (2.17) of the complex potentials $\Psi_j(z_j), j = 1,2$ show that near a crack tip the incremental strain energy density W has a singular part as well a regular part. We design by r and φ the radial distance from considered crack tip and the angle between radial direction and the line ahead the crack, see Figure 1.

Using the expressions (2.17) of complex potentials and Guz's formulae (2.1) – (2.3) after long but elementary calculus we obtain that near, the considered crack tip the strain energy density have the following structure

$$W(r,\varphi) = \frac{S(\varphi)}{r} + a\ regular\ part \tag{3.2}$$

Here $S(\varphi)$ is *Sih's incremental strain energy density factor* and is given by the following equation

$$S(\varphi) = \frac{(a_1\mu_1K_1 + K_{11})(a_2\mu_2K_1 + K_{11})}{4\pi} s_m(\varphi) \tag{3.3}$$

where the function $s_m(\varphi)$ depends on the elastic constants of the material and on the initial applied stress having the following expression :

$$s_m(\varphi) = \mathrm{Re}\left[\frac{a_1a_2\mu_1\mu_2}{\Delta}\left(\frac{\mu_1}{\chi_1(\varphi)} - \frac{\mu_2}{\chi_2(\varphi)}\right)\right]\mathrm{Re}\left[\frac{1}{\Delta}\left(\frac{a_2\mu_2b_1}{\chi_1(\varphi)} - \frac{a_1\mu_1b_2}{\chi_2(\varphi)}\right)\right] + \ldots.. \tag{3.4}$$

In above equation

$$\chi_j(\varphi) = \sqrt{\cos\varphi + \mu_j\sin\varphi},\ j = 1,2. \tag{3.5}$$

We extend the validity of Sih's fracture criterion, (see [5]) for orthotropic or isotropic prestressed elastic materials assuming that:

H1: Crack propagation will start in a radial direction φ_c along with the incremental strain energy density factor $S(\varphi)$ is a minimum, *i.e.*

$$\frac{dS}{d\varphi}(\varphi_c) = 0, \qquad \frac{d^2S}{d\varphi^2}(\varphi_c) > 0. \tag{3.6}$$

H2: The critical intensity

$$S_c = S_{\min} = S(\varphi_c) \tag{3.7}$$

governs the onset of the crack propagation and it represents a material constant independent of the crack geometry, loading and initial applied stress. The assumed Sih's type fracture criterion is based on local density of the incremental strain energy near the crack tip and requires no apriori assumptions concerning the direction in which energy is released by separating crack surfaces. Using the equations (2.19), (3.3) and (3.7) we obtain the incremental stress p_c for which crack propagation starts at critical direction φ_c

$$ap_c^2 = 4S_c / s_m(\varphi_c). \tag{3.8}$$

In above relation S_c is Sih's new material parameter, which takes the place of Griffith's specific surface energy γ in the new theory of brittle fracture. Once S_c is known, the relation (3.8) can be used to get p_c.

4.CRACK PROPAGATION FOR UNPRESTRESSED ORTHOTROPIC MATERIALS

In this section we suppose that our material is not initially deformed *i.e.*:

$$\sigma_0 = 0. \tag{4.1}$$

Long, but elementary calculus shows that in this case we have

$$a_1 = a_2 = 1. \tag{4.2}$$

Hence, according to equation (2.18) we get

$$\Delta = \mu_2 - \mu_1. \tag{4.3}$$

Obviously, the relation (3.3) giving Sih's strain energy factor $S(\varphi)$ rest valuable, but now the function $s_m(\varphi)$ will have a simplified form. In the following we present the results of our numerical analysis concerning the possible values of critical angle φ_c versus angle β for two anisotropyc materials. For simplicity we have assumed transversally isotropic materials, $x_2\ x_3$ being isotropy plane. In this case we have

$$E_2 = E_3, \nu_{12} = \nu_{13}, \nu_{23} = \nu_{32}, G_{12} = G_{31}, \nu_{21} = \nu_{31} \tag{4.4}$$

and

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}, \quad G_{23} = \frac{E_2}{2(1+\nu_{23})}. \tag{4.5}$$

We consider two transversally isotropic materials characterised by the following values

(i) $E_2 = 5.56GPa\,, \nu_{23} = 0.2\,, G_{23} = 2.3GPa\;\; E_1 = 75GPa\,, \nu_{12} = 0.22\,, G_{12} = 2GB$ (4.6)

and

(ii) $E_2 = 10.3GPa\,, \nu_{23} = 0.28\,, G_{23} = 4.02GPa\,, E_1 = 181GPa\,, \nu_{12} = 0.46\,,\;\; G_{12} = 2GPa$ (4.7)

The set (i) corresponds to a fibber reinforced aramid / epoxy composite and the set (ii) corresponds to a fibber reinforced graphite / epoxy composite. The dependence of the crack propagation angle φ_c versus angle β for the

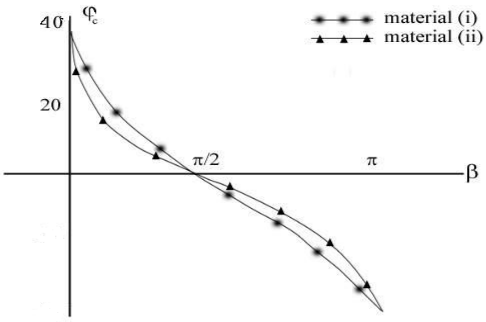

Figure 2 - Angle φ_c versus angle β

two materials (i) and (ii) is presented see in Figure 2. We observe that critical angle φ_c decrease when crack's angle β increases. Also, a remarkable result is obtained in the case when, $\beta = \dfrac{\pi}{2}$ which corresponds to the first classical fracture mode. In this case we obtain a well-known result that the crack will propagate along its line (see [6]).

5.RESONANCE PHENOMENON FOR AN INCLINED CRACK

As Guz [3] shows the phenomenon of internal stability or resonance has a well-defined physical meaning and can be elucidated taking into account the fact that all materials have an internal structure. The phenomenon concerns the loss of stability of the structure and depends on the geometrical and mechanical characteristics of the body as a whole . A rigorous study of this phenomenon has to take into account explicitly the parameters describing the internal structure of the body. Takings into account that such phenomenon can take place arise the following question: May exist a critical value σ_0^c of the initial applied stress σ_0 such that when σ_0 starting from zero converges to σ_0^c, the incremental stress p_c converges to zero?. For a fibber reinforced composite the answer to this question was given by Guz and it is positive (see [1], Chap. 2). How for a fibber reinforced composite material Young's modulus E_1 is greater than E_2 and than shear modulus G_{12}, Guz was able to show that the critical value σ_0^c is given by the following relation:

$$\sigma_0^c \approx -G_{12}\left\{1 - \frac{G_{12}^2}{E_1 E_2}\left(1-\nu_{13}\nu_{31}\right)\left(1-\nu_{23}\nu_{31}\right)\right\} < 0. \qquad (5.1)$$

Since $E_2 << E_1$ and $G_{12} << E_1$ the critical compression stress σ_0^c produces only infinitesimal strains in the prestressed material. In what follows we verify in the case β is constant, for our reinforced materials (*i*) and (*ii*) characterised by the values given by eqs (4.6) respectively (4.7) that phenomenon of resonance can appear when σ_0^c converges to $\sigma_{0(i)}^c$ for material (*i*) and to $\sigma_{0(ii)}^c$ for material (*ii*). Here we denoted by $\sigma_{0(i)}^c$ and respectively $\sigma_{0(ii)}^c$ the critical value for materials (*i*) and respectively (*ii*) from expression (5.1) and we obtained the following values :

$$\sigma^c_{0(i)} = -1.9805 GPa \quad \text{and} \quad \sigma^c_{0(ii)} = -1.9957 GPa \,. \tag{5.2}$$

Using equation (3.8) we obtain for the critical incremental stress following expression:

$$p_c = 2\sqrt{\frac{S_c}{a s_m(\varphi_c)}} \,. \tag{5.3}$$

Table 1 presents the dependence of the incremental stress φ_c and of critical fracture angle φ_c for materials (*i*) and (*ii*). How Sih's parameter S_c and the fracture's length are constant we shall study the variation of the new stress function $\pi_c = \pi_c(\sigma_0)$ given by :

$$\pi_c = \frac{p_c}{2}\sqrt{\frac{a}{S_c}} \,. \tag{5.4}$$

TABLE 1

THE VALUES OF π_c AND φ_c VERSUS σ_0 FOR MATERIALS (*i*) AND (*ii*)

Material	σ_0	-1.99	-1.98	-1.96	-1.94	-1.92	-1.90	-1.75	-1.50	-1.25	-1.00	-0.50	-0.25	-0.10	0
(*i*)	π_c (GPa)		0.03	0.22	0.28	0.34	0.38	0.56	0.72	0.82	0.91	1.03	1.09	1.12	1.14
	φ_c (°)		44.4	43.8	43.6	43.5	43.4	42.5	41.8	41.4	41.1	40.9	40.5	40.4	40.3
(*ii*)	π_c (GPa)	0.05	0.26	0.33	0.39	0.43	0.47	0.62	0.77	0.87	0.94	1.05	1.10	1.14	1.15
	φ_c (°)	44.5	44.3	44.1	43.9	43.8	43.7	43.0	42.4	42.1	41.8	41.2	41.1	41.0	40.9

6. FINAL REMARKS

From our numerical analysis we conclude that:

- critical fracture angle φ_c decreases when crack's angle β increases;
- when crack's angle β is equal with $\pi/2$ case corresponding to the first classical fracture mode we obtain a well-known result, that $\varphi_c = 0$, *i.e.* the crack propagates along its line;
- when σ_0 decreases to the critical value σ_0^c also the incremental stress p_c decrease. When σ_0 converges to σ_0^c we observe that p_c converges to zero, *i.e* the crack will start to propagate even that p_c is approximately zero. In this case resonance phenomenon occurs.
- critical fracture angle φ_c decreases when incremental stress σ_0 decreases. In the case $\beta = \pi/2$, which corresponds to the first classical fracture mode, for $\sigma_0 = 0$ we obtain that critical fracture angle φ_c is in a vicinity of 40°, (see [6]).

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