ORAL/POST REFERENCE: ICF100759PR

RE-EXAMINATION OF OVER SPECIFIED CONDITION OF STRESS INTENSITY FACTOR FOR MULTI-MATERIAL WEDGES AND JUNCTIONS

Wei-Chung Wang[1] and Shin-Way Lin[2]

[1]Department of Power Mechanical Engineering, National Tsing Hua University, Hsinchu, Taiwan 30013, Republic of China

[2]Industrial Technology Research Institute, Materials Research Laboratories, Hsinchu, Taiwan 30040, Republic of China

ABSTRACT

Stress singularities often occur in wedges and junctions. Pageau and his colleagues [1] used the Airy's stress function to formulate the stress and displacement fields for the n-material wedges and junctions. In their analysis, it was found that "over specification" of two SIFs occurs while complex singularity appears. They proposed a standardized generalized stress intensity factor (GSIF) to characterize the stress field and avoid over specification. However, the proposed GSIF did not resolve over specification problem completely. Using complex variable technique, the over specified condition arisen from the traditional definition of complex SIF was disclosed in this paper.

A new definition of complex SIF was also introduced to additionally consider the conjugate part of the singularity and resolve the unconformity between the stress expansion and the traditional definition. By using the new definition, the mixed characteristics of SIFs for the multi-material wedges and junctions can be clarified while complex singularity can be degenerated into real singularity. Furthermore, the energy release rate for interfacial crack was re-analyzed to include the mixed mode effect. From the polar plots of $K_{\theta\theta}$ and $K_{r\theta}$, the modes corresponding to the singularities can be distinguished for the bi-material junction problens, in which Chen and Nisitani [2] solved. Due to geometrical symmetry, the singularities were decoupled for the tensile and shear modes. It was concluded that the patterns of the new defined SIFs can be used to characterize the stress fields and loading modes and be also applied to general multi-material structures without geometrical symmetry.

KEYWORDS
stress intensity factor, multi-material, wedge, junction

INTRODUCTION

Stress singularities are very commonly occurred in multi-material structures. Due to the infinite magnitude of stress near the tip of junction, the concept of stress intensity factor (SIF) based on the fracture mechanics is often used to characterize the singular stress fields. In general, three methods are mostly used to analyze the stress singularity. They are (i) Airy's stress function proposed by Williams [3]; (ii) Mellin transform used by Bogy [4] and Dunders [5] as well as applied by Hein and Erdogan [6] on the analysis for various wedge angles; (iii) Kolosov-Muskhelishvili [7] stress function proposed by Theocaris [8]. It is well known that stress oscillation, which is caused by complex stress singularities, arises in some multi-material structures and annoys researchers very much.

Only few reports have devoted to the expansion of stress field because of algebraic difficulties. There is no widely accepted SIF for the general stress singular field. Rice [9] discussed several possible definitions of SIF for interfacial cracks. He pointed out that tensile and shear effects near the crack tip are intrinsically inseparable into analogues of classical mode I and mode II conditions. This problem also exists in junction structures [8]. Yang and Munz [10] determined eigenfunctions for both real and complex singularities. Chen and Nisitani [2] showed that the mode I and mode II solutions are decoupled due to geometrical symmetry for real singularities in a particular case of bi-material junction. Pageau et al. [1] noticed that the definition of the generalized SIF (GSIF) defined by Chen and Nisitani [2] for bi-material junctions differs from that of Yang and Munz [10] for bi-material wedges. In the work of Pageau et al. [1], some traditional definitions were reviewed. They proposed a standardization scheme for the definition of SIF by the Airy's stress function approach of Williams [3]. Two real constants M_I and M_{II} were introduced in terms of GSIF. They found that only normal stress and shear stress can be respectively used to define mode I and mode II SIFs, i.e. K_I and K_{II}, because of the behavior of the eigenfunctions at θ=0, and θ is the polar angle. Herein, over specification of two SIFs corresponding to M_I and M_{II} was depicted. Although their scheme defines the GSIF to avoid over specification, whether the definition can be adopted on failure assessment due to the over specified condition is questionable and furthermore the definition depends on normal or shear stress conditionally. Specifically speaking, the GSIF proposed by Pageau et al. [1] cannot resolve the over specification completely.

In order to overcome the over specified condition and find a unified definition of GSIF, complex variable technique was applied to analyze the wedge and junction problems in this paper. A similar expression to that used by Pageau et al. [1] was derived. Obviously, the over specified condition is arisen from the traditional definition of the complex SIF. Hence, a new definition of SIF was proposed in this paper while conjugate singularity was taken account in the SIF expression. In the new definition, the mixed part of SIF was identified. The new definition of the SIF conforms to the stress field expansion; therefore, over specified condition no more occurs. The complex singularity can also be degenerated to the case of real singularity. Since the newly defined SIF overcomes the over specified condition and identifies the mixed part of modes, it can be properly used for failure assessments. On the other hand, the energy release rate for interfacial crack was re-analyzed to include the effect of mixed mode. It is found that the energy release rate can be composed by mode I, mode II and corresponding mixed parts.

ANALYSIS OF THE OVER SPECIFIED CONDITION

According to authors' previous work [11], stresses are expressed as

$$\begin{aligned}
&\sigma_{\theta\theta} + i\sigma_{r\theta} \\
&= (2\pi r)^{\eta}\{[\mathrm{Q}_1 E_{\theta\theta}(\theta) - \mathrm{Q}_2 F_{\theta\theta}(\theta)]\cos(\varepsilon \ln r) - [\mathrm{Q}_1 F_{\theta\theta}(\theta) + \mathrm{Q}_2 E_{\theta\theta}(\theta)]\sin(\varepsilon \ln r)\} \\
&\quad + i(2\pi r)^{\eta}\{[\mathrm{Q}_1 E_{r\theta}(\theta) + \mathrm{Q}_2 F_{r\theta}(\theta)]\cos(\varepsilon \ln r) + [\mathrm{Q}_1 F_{r\theta}(\theta) - \mathrm{Q}_2 E_{r\theta}(\theta)]\sin(\varepsilon \ln r)\}
\end{aligned} \tag{1}$$

where Q_1 and Q_2 are undetermined real constants, η is the real part of (λ-1), and ε is the so-called oscillation index and constant 2π is included in the expression. $E_{\theta\theta}$, $F_{\theta\theta}$, $E_{r\theta}$ and $F_{r\theta}$ are functions of θ. When ε becomes zero, the oscillation of trigonometric function disappears. It is seen that eqn (1), which is obtained through complex variables technique, is the same as that derived by Pageau et al. [1]. Based on this expression, the definition of SIF can be further discussed.

By following the definition of SIF for bi-material interfacial crack [12-14], the SIF is expressed as

$$\sigma_{\theta\theta} + i\sigma_{r\theta}\,|_{\theta=0} = (2\pi)^{-\frac{1}{2}} K r^{-\frac{1}{2}+i\varepsilon} = \frac{K r^{i\varepsilon}}{(2\pi r)^{1/2}} = \frac{K_I + iK_{II}}{(2\pi r)^{1/2}} r^{i\varepsilon} \tag{2}$$

The over-specified problem can also be easily disclosed by expanding eqn (2).

$$\begin{aligned} &\frac{K_I + iK_{II}}{(2\pi r)^{1/2}} r^{i\varepsilon} \\ &= \frac{1}{(2\pi r)^{1/2}} \{[K_I \cos(\varepsilon \ln r) - K_{II} \sin(\varepsilon \ln r)] \\ &\qquad + i[K_{II} \cos(\varepsilon \ln r) + K_I \sin(\varepsilon \ln r)]\} \end{aligned} \tag{3}$$

By comparing the expressions of K_I and K_{II} between eqns (1) and (3), it can be seen that the expressions of either K_I or K_{II} of the real part are different from those of the imaginary part. Therefore, if both the real and imaginary parts of eqns (3) are adopted to obtain the SIFs, they over specify the SIFs. In other words, K_I and K_{II} in eqns (3) represent SIFs of mode I and mode II, respectively, only when ε equals zero. They no longer mean pure mode I and mode II when ε is not zero, i.e., mode mixing occurs.

NEW DEFINITION OF STRESS INTENSITY FACTOR

As discussed in [11], the new SIFs are shown as follows:

$$\begin{aligned} &\sigma_{\theta\theta} + i\sigma_{r\theta} \\ &= (2\pi r)^{\eta} \{[K_{I\theta} \cos(\varepsilon \ln r) - K_{I\varepsilon\theta} \sin(\varepsilon \ln r)] \\ &\quad + i[K_{II\theta} \cos(\varepsilon \ln r) + K_{II\varepsilon\theta} \sin(\varepsilon \ln r)]\} \end{aligned} \tag{4}$$

where

$$\begin{aligned} K_{I\theta} &= Q_1 E_{\theta\theta}(\theta) - Q_2 F_{\theta\theta}(\theta) \\ K_{I\varepsilon\theta} &= Q_1 F_{\theta\theta}(\theta) + Q_2 E_{\theta\theta}(\theta) \\ K_{II\theta} &= Q_1 E_{r\theta}(\theta) + Q_2 F_{r\theta}(\theta) \\ K_{II\varepsilon\theta} &= Q_1 F_{r\theta}(\theta) - Q_2 E_{r\theta}(\theta) \end{aligned} \tag{5}$$

Note that the notation θ is additionally added into the subscript of SIFs and that $K_{I\theta}$, $K_{II\theta}$, $K_{I\varepsilon\theta}$ and $K_{II\varepsilon\theta}$ are functions of θ only. In the homogeneous case, both K_I and K_{II} are clearly defined along $\theta = 0^o$. In the general multi-material wedge and junction problems, however, it is not straightforward to define the SIFs. In other words, $\theta = 0^o$ can be chosen arbitrarily. Furthermore, the SIFs defined in eqns (4) and (5) reflect the mixing characteristics inherent in the multi-material system. Alternatively speaking, fracture modes become rather complicated everywhere a simple loading is applied on a multi-material system.

REPRESENTATIONS FOR THE SHEAR AND TENSILE MODES

Since the mixed parts of GSIF for multi-material system are clarified, the role of mixed characteristics is examined for the widely used traditional criteria. Stress oscillation always causes a difficult situation in the

definition of SIF. From the previous sections, the newly defined SIF was proposed to be compatible with stress field and well applied on the case of stress oscillation.
The singular stress comes from the exponent η . In mixed state of stress, oscillation index ε annoys the engineers very much. It seems not easy to find a strength index not including the characteristic length r. Oscillation index ε causes variation of stresses with varying frequencies. As r is near zero, the spatial frequency becomes higher. While the spatial frequency is high and "wave length" is smaller than material microstructure characteristic length, the amplitude of the "wave" is a proper expression for the SIF. Eqn (4) can therefore be rewritten as

$$\begin{aligned}\sigma_{\theta\theta} &= (2\pi r)^{\eta}\{K_{\theta\theta}\cos(\varepsilon\ln r+\phi_{\theta\theta})\}\\ \sigma_{r\theta} &= (2\pi r)^{\eta}\{K_{r\theta}\cos(\varepsilon\ln r+\phi_{r\theta})\}\end{aligned} \tag{6}$$

The factor $K_{\theta\theta}$ and $K_{r\theta}$ is a function of θ and is defined as

$$\begin{aligned}K_{\theta\theta} &= \sqrt{K_{I\theta}{}^{2}+K_{I\varepsilon\theta}{}^{2}}\\ K_{r\theta} &= \sqrt{K_{II\theta}{}^{2}+K_{II\varepsilon\theta}{}^{2}}\end{aligned} \tag{7}$$

$K_{\theta\theta}$ becomes $K_{I\theta}$ and $K_{r\theta}$ becomes $K_{II\theta}$ when real singularity is used.

On the other hand, energy release rate is often used to measure the fracture resistance of materials. Considering the case of a bi-material interfacial crack, energy release rate should be changed to include the conjugate part of the singularity. The traction, $\sigma_{yy}+i\sigma_{yx}$, and the displacement jumps, $\delta_y+i\delta_x$, across the crack face are expressed as follows:

$$\sigma_{yy}+i\sigma_{yx}\big|_{\theta=0}=\frac{K_1 r^{i\varepsilon}}{\sqrt{2\pi r}}+\frac{K_2 r^{-i\varepsilon}}{\sqrt{2\pi r}} \tag{8a}$$

$$\delta_y+i\delta_x=\frac{1}{(1+2i\varepsilon)\cosh(\pi\varepsilon)}\frac{4K_1 r^{i\varepsilon}}{E^*}\sqrt{\frac{2r}{\pi}}+\frac{1}{(1-2i\varepsilon)\cosh(\pi\varepsilon)}\frac{4K_2 r^{-i\varepsilon}}{E^*}\sqrt{\frac{2r}{\pi}} \tag{8b}$$

where $\frac{2}{E^*}=\frac{1}{E_1'}+\frac{1}{E_2'}$ and $E'=E$ for plane stress, $E'=E/(1-\nu^2)$ for plane strain. E_1 as E_2 are Young's moduli of materials 1 and 2, respectively.

The second terms of eqn (8) are the conjugated parts of the first terms, which are the traditional expressions [9]. Then the energy release rate G is derived by following Hutchinson's derivation [15]:

$$G=\frac{|K_1|^2}{\cosh^2(\pi\varepsilon)E^*}+\frac{|K_2|^2}{\cosh^2(\pi\varepsilon)E^*}=\frac{K_I{}^2+K_{II}{}^2+K_{I\varepsilon}{}^2+K_{II\varepsilon}{}^2}{2\cosh^2(\pi\varepsilon)E^*} \tag{10}$$

It is seen that the mixed parts appear in the energy release rate clearly. In a homogeneous crack, both K_I and $K_{I\varepsilon}$ contribute to mode I.

NUMERICAL DEMONSTRATION

Consider a model of bi-material junction. There exist two singularities, 0.623716 and 0.755233. By arbitrarily applying loads, $K_{\theta\theta}$ and $K_{r\theta}$ can be obtained for both singularities. Fig. 2 is the SIF distribution for λ=0.623716. It is found that $K_{\theta\theta}$ is not zero along line of symmetry, but $K_{r\theta}$ is. Obviously, this singularity 0.623716 corresponds to tensile mode. On the other hand, the SIF distribution for λ=0.755233 is shown in Fig. 3. $K_{\theta\theta}$ is zero along line of symmetry, but $K_{r\theta}$ is not. This singularity 0.755233 corresponds to tensile mode. Hence, it is possible to distinguish modes by SIF distribution rather than mathematical formulation as done in [2]. Similar cases are also found in multi-material structures.

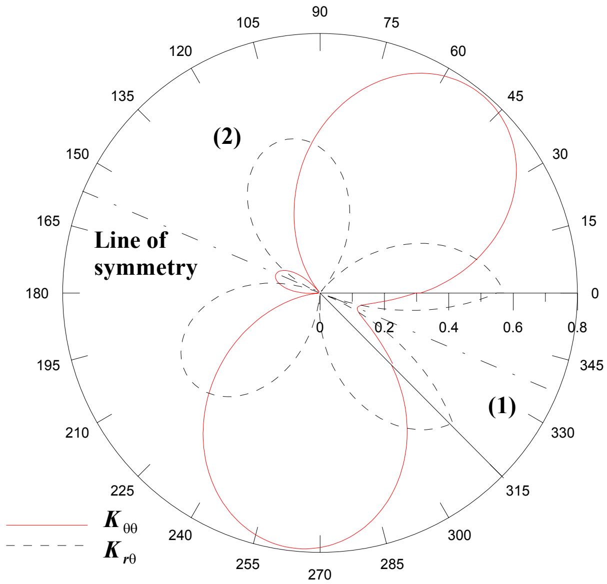

Figure 1. Distributions of $K_{\theta\theta}$ and $K_{r\theta}$ for an open wedge and λ=0.623716

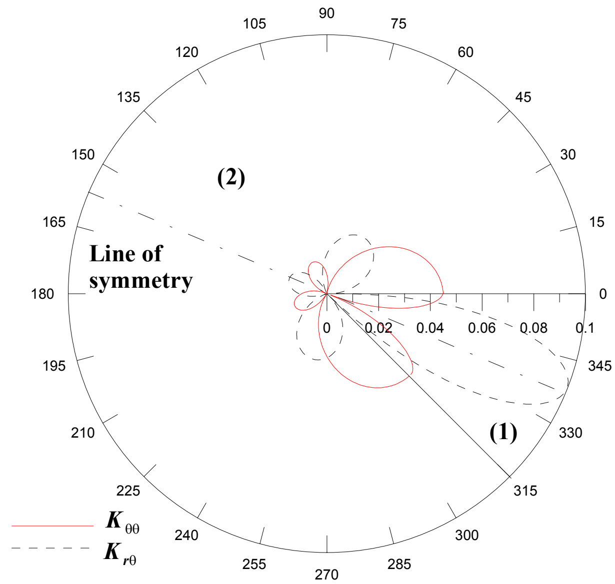

Figure 2. Distributions of $K_{\theta\theta}$ and $K_{r\theta}$ for an open wedge and λ=0.755233

DISCUSSIONS AND CONCLUSIONS

In this paper, a new definition of SIF was proposed by employing the complex variable method. The traditional expression of complex SIF is not complete due to the lack of inclusion of additional conjugate term and is over specified as pointed out in the paper by Pageau et al. [1]. An additional conjugate term was added and a new form of stress expression was derived in this paper. In the new form, the mixed parts of SIF were clarified. Consequently, the new form can be applied well on both real and complex singularities. Typically, the traditional energy release rate G was re-examined for the interface crack. And the mixing part $K_{I\varepsilon}$ and $K_{II\varepsilon}$ appears in the expression of G.

Here, the angular distrbution of $K_{\theta\theta}$ and $K_{r\theta}$ can be used to represent the loading mode. They are actually two different aspects for modes. It is doubtful if $K_{\theta\theta}$ and $K_{r\theta}$ are tensors as the stresses $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$. They are indeed not tensors, since they are defined asymtotically near the singular point, that is r=0, but not at it. That is, it characterizes the behavior near the singular point.

It was shown that the loading modes could be examined from the patterns of the angular distribution of $K_{\theta\theta}$ and $K_{r\theta}$ rather than from the geometry. The patterns of $K_{\theta\theta}$ and $K_{r\theta}$ angular distribution are characteristic of stress fields for different geometries. The maximum $K_{\theta\theta}$ and $K_{r\theta}$ can be determined from the patterns. Therefore, the pattern of the $K_{\theta\theta}$ distribution represents the characteristics of the loading modes more properly. It is useless to distingush the tensile and shear modes when the reference coordinates cannot be set definitely.

Considering the spatial frequency of the stress oscillation, the new factors $K_{\theta\theta}$ and $K_{r\theta}$, which are functions of θ, are defined. And $K_{\theta\theta}$ and $K_{r\theta}$ can be degenerated into $K_{I\theta}$ and $K_{II\theta}$ when the singularity is real. The mode mixing can also be found clearly from the $K_{\theta\theta}$ and $K_{r\theta}$ angular distribution. Since the proper form of stress expression was obtained, the stress field can be derived and SIF can be extracted. It should be then noted that the new form is more appropriately suitable for multi-material junction and wedge problems in which reference direction is not defined clearly.

ACKNOWLEDGEMENTS

This research was supported in part by the National Science Council of the Republic of China (Grant no.: NSC88-2212-E007-007)

REFERENCES

1. Pageau S. S., Gadi, K. S., Biggers, S. B. Jr. and Joseph, P. F., 1996, “Standardized Complex and Logarithmic Eigensolutions for N-material Wedges and Junctions,“ International Journal of Fracture, Vol.77, pp.51-76.
2. Chen, D. and Nisitani, H., 1993, “Singular Stress Field Near the Corner of Jointed Dissimilar Materials,” ASME Journal of Applied Mechanics, Vol. 60, pp.607-613.
3. Williams M. L., 1952, “Stress Singularities Resulting from Various Boundary Conditions in Angular Corners of Plates in Extension,” ASME Journal of Applied Mechanics vol.19 (1952) 526-528.
4. Bogy D. B., 1968, “Edge-bonded Dissimilar Orthogonal Elastic Wedge under Normal and Shear Loading,” ASME Journal of Applied Mechanics, Vol.35, pp.460-466.
5. Dunders, J., 1969, ‘discussion’, ASME Journal of Applied Mechanics, Vol.36, pp. 650-652.
6. Hein, V. L. and Erdogan, F., 1971, “Stress Singularities in a Two-material Wedge,” International Journal of Fracture Mechanics, Vol.7, pp. 317-330.
7. Muskhelishvili, N. I., 1953, “Some Basic Problems of Mathematical Theory of Elasticity,” P. Noordhoff and Company, New York.
8. Theocaris, P. S., 1974, ”The Order of Singularity at Multi-wedge Corner of a Composite Plate,” International Journal of Engineering Science, Vol.12, pp.107-120.
9. Rice, J. R., 1988,”Elastic Fracture Mechanics Concepts for Interfacial Cracks”, Journal of Applied Mechanics, Vol. 55, pp. 98-103.
10. Yang, Y. Y. and Munz, D., 1995, ”The Stress Distribution in Dissimilar Materials Joint for Complex Singular Eigenvalues under Thermal Loading,” Journal of Thermal Stresses, Vol.18, pp.407-419.
11. Lin, S. W. and Wang, W. C., 1999,” New Concept from the Standardization of Complex Eigensolutions for Wedges and Junctions,” Advances in Electronic Packaging, EEP-Vol. 26-2, pp.2053-2059.
12. England, A. H., 1965, “A Crack between Dissimilar Media,” ASME Journal of Applied Mechanics, vol. 32, pp. 400-402.
13. Erdogan, F., 1965, ”Stress Distribution in Bonded Dissimilar Materials with Cracks,” ASME Journal of Applied Mechanics, Vol. 32, pp. 403-410.
14. Rice, J. R., and Sih, G. C., 1965, “Plane Problems of Cracks in Dissimilar Media”, ASME Journal of Applied Mechanics, Vol. 32, pp. 418-423.
15. Hutchinson, J. W., Mear, M. E., Rice, J. R., 1987,”Crack Paralleling an Interface Between Dissimilar Materials”, ASME Journal of Applied Mechanics, Vol. 54, pp. 828-832.
16. Lin, S. W. and Wang, W. C., 1999, “General Transfer Method for the Multi-material Junction and Wedge Problems,” Computational Mechanics, Vol. 23, pp. 502-509.