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QUASI-MICROMECHANICAL MODELING OF EVOLUTIONARY DAMAGE IN QUASI-BRITTLE SOLIDS

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ABSTRACT

Presented in this paper is a quasi-micromechanical model for simulating the constitutive response of microcrack-weakened materials subjected to complex loading. A novel effective medium scheme is first suggested to calculate the microcrack interaction effects on the effective elastic moduli in a convenient manner. The microcracking damage is characterized in terms of the orientation domain of microcrack growth (DMG) as well as a scalar microcrack density parameter. The DMG describes the complex damage and its evolution associated with microcrack growth, while the scalar microcrack density factor yields an easy estimation of the effects of microcrack interaction. Considering various micromechanisms of microcracking damage, the overall effective constitutive relation in different deformation stages including linear elasticity, pre-peak nonlinear hardening, stress drop and strain softening are expressed in a unified form.

KEY WORDS

Constitutive relation, quasi-micromechanical model, damage evolution, brittle material, microcracking, microcrack interaction

INTRODUCTION

In spite of significant development of damage mechanics during the past decades, some fundamental and important issues in this field have not yet been resolved. The description of evolutionary damage, the calculation of microcrack interaction, and the formulation of the effective constitutive relation of microcracked materials, among others, are still widely argued in the literature [1]. To establish a relatively comprehensive and applicable constitutive relation model, a promising approach is to combine the methods and strategies of both phenomenological and micromechanical damage mechanics. To this end, two alternatives seem worth considering. The first is to construct a model within the framework of continuum damage mechanics, while the definition of damage variable, the formulation of evolution law as well as the determination of key parameters are specified on the basis of micromechanical analysis. Such a damage model may be referred to as a micromechanics-based phenomenological model, or a quasi-phenomenological model [2]. The second is to build the basic framework of the model directly from micromechanical analysis, while some concepts and skills of continuum damage mechanics are incorporated to lead to a simple but more exact model with solid physical background. We refer to such a model as quasi-micromechanical. An attempt is made here to establish a quasi-micromechanical damage theory for calculating the overall constitutive relation of brittle materials with interacting and evolutionary microcracks.

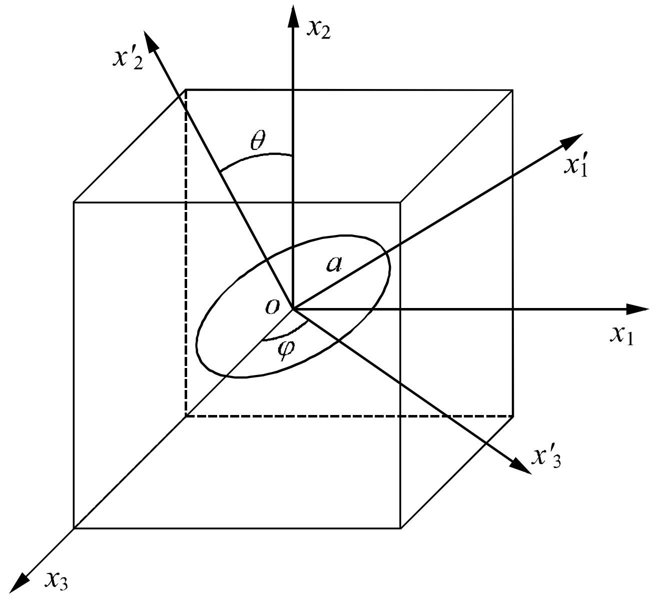

Figure 1: Global and local coordinate systems

ANALYSIS OF DAMAGE EVOLUTION

Domain of microcrack growth

Consider a quasi-brittle material weakened by randomly distributed microcracks of penny shape. Assume that the number density of microcracks is n_c, and that in the initial state all microcracks have the same statistically-averaged radius a_0. Choose a representative volume element (RVE), whose boundary is subjected to tractions in equilibrium with a uniform overall stress $\boldsymbol{\sigma}$. First, consider a single microcrack, say the α-th, in it. Refer to a global Cartesian coordinate system $(o - x_1x_2x_3)$ and a local Cartesian coordinate system $(o - x'_1x'_2x'_3)$, as shown in Fig. 1, in which the x'_2-axis is parallel to the normal **n** of the microcrack, and the x'_3-axis is coplanar with x_1 and x_3. Then the orientation of the microcrack is expressible in terms of the angles $(\theta,\ \varphi)$.

Adopt the mixed-mode fracture criterion for a penny-shaped microcrack in an isotropic medium as

$$\left(\frac{K'_{\mathrm{I}}}{K_{\mathrm{IC}}}\right)^2 + \left(\frac{K'_{\mathrm{II}}}{K_{\mathrm{IIC}}}\right)^2 = 1, \tag{1}$$

where K'_{I} and K'_{II} represent the mode-I and II stress intensity factors (SIFs), K_{IC} and K_{IIC} their intrinsic critical values, respectively. K'_{I} and K'_{II} are defined by

$$K'_{\mathrm{I}} = 2\sigma'_{22}\sqrt{\frac{a}{\pi}}, \qquad K'_{\mathrm{II}} = \frac{4}{2-\pi}\sqrt{\frac{a}{\pi}\left[(\sigma'_{21})^2 + (\sigma'_{23})^2\right]}\ , \tag{2}$$

where $\sigma'_{ij} = g'_{ik}g'_{jl}\sigma_{kl}$ is the stress tensor in the local coordinate system, g'_{ij} is the transformation matrix between the two systems [3].

Once a microcrack satisfies the criterion (1), it will propagate in a stable fashion, increasing the radius from the initial value a_0 to a characteristic value a_u and being arrested by energy barriers (such as grain boundaries of different directions) with higher strength. The same assumption of high energy barriers that serve as a crack trapping mechanism was adopted by Krajcinovic [1], Ju and Lee [4] and some others.

Under loading, more and more microcracks may meet the criterion in (1) and then propagate. Thus, the microcracking damage state can be characterized in terms of the *orientation domain of microcrack growth* (DMG) [3], which is defined as the possible orientation scope in the orientation space (θ, φ) of all microcracks that have propagated in the aforementioned fashion. In other words, all microcracks whose orientations are within the orientation scope of DMG must have propagated and have the radius a_{u}. The concept of DMG is defined on the basis of solid physical consideration and has clear geometrical meaning in the orientation space. A DMG can also be considered as a set of all microcracks that have propagated.

The description of damage evolution in a brittle material under complex loading is generally a hard task for micromechanical damage models. Employing the concept of DMG, however, we can analyze easily the damage evolution using the calculation rules in set theory. As the applied stresses vary with time *t*, the evolution equation of DMG, $\Omega(t)$, is expressed by the summation of sets as [3]

$$\Omega(t+\Delta t) = \Omega(t) \bigcup \Omega(\sigma_{ij}(t+\Delta t)), \tag{3}$$

where $\Omega(\sigma_{ij})$ denotes the DMG corresponding to the stress tensor σ_{ij} under the condition of monotonically proportional loading. The detailed formulas for calculating $\Omega(\sigma_{ij})$ were given in [3].

Secondary growth of microcracks

With further increase in applied stresses, some microcracks normal or nearly normal to the maximum principal tensile stress may pass through the high-energy barriers and experience secondary growth. Similar to (1), the criterion of secondary growth of a circular microcrack may take the following form [5]

$$\left(\frac{K'_{\mathrm{I}}}{K_{\mathrm{ICC}}}\right)^2 + \left(\frac{K'_{\mathrm{II}}}{K_{\mathrm{IICC}}}\right)^2 = 1, \tag{4}$$

where K_{ICC} and K_{IICC} are respectively the critical values of mode I and II SIFs of energy barriers, often taken as the values of fracture toughness of the pristine matrix. When some microcracks have experienced secondary growth, the damage and deformation will be localized in the material causing the rapid stress drop and strain softening phenomena in the stress-strain curve [5].

AN ESTIMATION METHOD FOR EFFECTIVE MODULI

For an RVE as shown in Fig. 2(a), the overall average strain $\bar{\boldsymbol{\varepsilon}}$ can be decomposed as

$$\bar{\boldsymbol{\varepsilon}} = \bar{\boldsymbol{\varepsilon}}^{\mathrm{m}} + \bar{\boldsymbol{\varepsilon}}^{\mathrm{c}}, \tag{5}$$

where $\bar{\boldsymbol{\varepsilon}}^{\mathrm{m}} = \mathbf{S}^{\mathrm{m}} : \boldsymbol{\sigma}$ denotes the matrix strain tensor averaged over the RVE, $\bar{\boldsymbol{\varepsilon}}^{\mathrm{c}}$ the microcrack-induced variation in the overall average strain, and $\mathbf{S}^{\mathrm{m}}$ the compliance of the matrix.

Assuming that all microcracks are planar, the variation of the volume-averaged strain can be calculated by

$$\bar{\boldsymbol{\varepsilon}}^{\mathrm{c}} = \frac{1}{2V} \sum_{\alpha=1}^{N} S^{(\alpha)} (\bar{\mathbf{b}}\mathbf{n} + \mathbf{n}\bar{\mathbf{b}})^{(\alpha)}, \tag{6}$$

where $N = n_{\mathrm{c}} V$ is the total number of microcracks in the RVE, the superscript (α) stands for a quantity of the α-th microcrack, $S^{(\alpha)}$, $\bar{\mathbf{b}}^{(\alpha)}$ and $\mathbf{n}^{(\alpha)}$ denote the surface area, the average opening displacement discontinuity vector and the unit vector normal to the crack faces, respectively.

Thus, the key problem becomes how to calculate the opening displacement of a microcrack embedded in a solid containing many disordered microcracks. For such a problem, some simplifications or approximations are necessary [6]. On one hand, the medium surrounding a microcrack is weakened by the numerous microcracks, and then has a stiffness lower than the pristine matrix. On the other hand, the stress field to which the microcrack is exposed is perturbed due to the existence of other microcracks. As a straightforward approximate model, the microcrack is assumed to be surrounded by an effective medium, referred to also as the comparison or reference matrix, with compliance $\mathbf{S}^0$ and subjected to an effective stress $\boldsymbol{\sigma}^0$ in the far field, as shown in Fig. 2(b). This approximation is common to almost all the effective medium methods and the effective field methods, e.g. the DCM, SCM, DM and GSCM, although the definitions of $\mathbf{S}^0$ and $\boldsymbol{\sigma}^0$ in them are different [6].

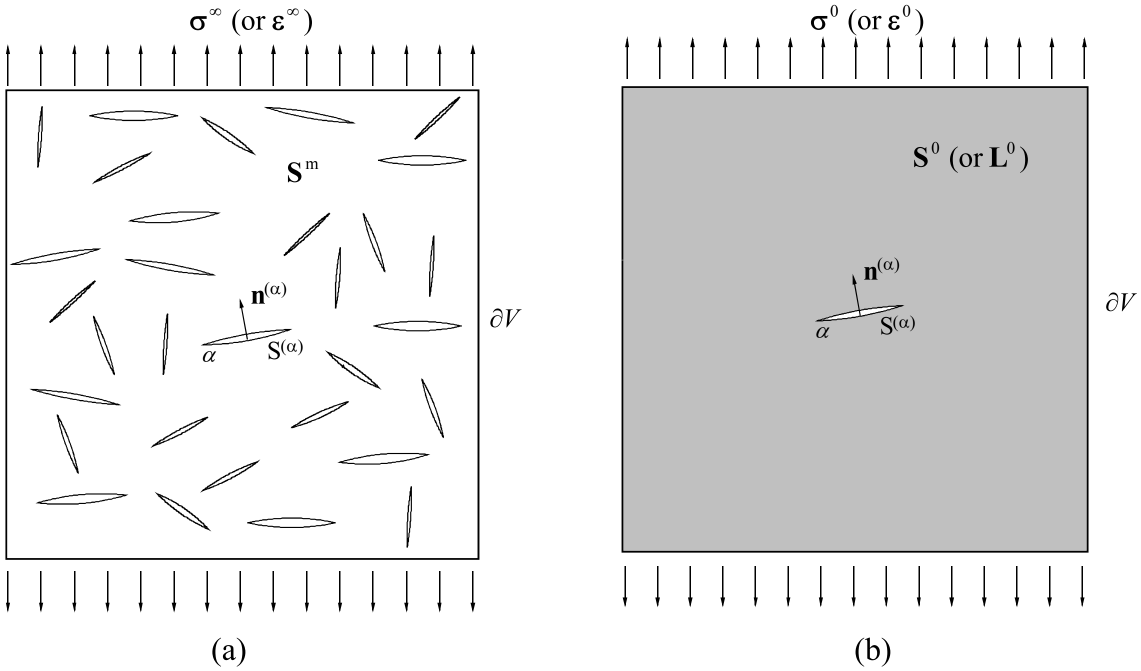

Figure 2: (a) RVE, and (b) approximate model for calculating microcrack opening displacements

In the approximate model in Fig. 2(b), the average opening displacement vector of a microcrack can be expressed as

$$\overline{\mathbf{b}} = \mathbf{B}(\mathbf{S}^0, G, \boldsymbol{\sigma}^0) \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{n}\,, \tag{7}$$

where the second-rank symmetric tensor $\mathbf{B}$ is called the crack opening displacement tensor, and G signifies the geometry of the microcrack. Define a fourth-order tensor $\mathbf{H}$ by

$$\boldsymbol{\sigma}^0 = \mathbf{H} : \boldsymbol{\sigma}\,, \tag{8}$$

which relates the effective stress $\boldsymbol{\sigma}^0$ to $\boldsymbol{\sigma}$. Assume that the tensor $\mathbf{H}$ is same for all microcracks, as is consistent with the inherent assumptions of the effective medium methods.

Thus, the effective compliance increment due to the α-th single microcrack is obtained from (6)–(8) as

$$\overline{S}_{ijkl}^{\mathrm{c}(\alpha)} = \frac{S^{(\alpha)}}{4V} H_{stkl} \left(n_i B_{js} n_t + n_j B_{is} n_t + n_i B_{jt} n_s + n_j B_{it} n_s \right)^{(\alpha)}. \tag{9}$$

Then, the overall compliance tensor $\mathbf{S}$, defined by $\overline{\boldsymbol{\varepsilon}} = \mathbf{S} : \boldsymbol{\sigma}$, is arrived at

$$S_{ijkl} = S_{ijkl}^{\mathrm{m}} + \frac{1}{4V} H_{stkl} \sum_{\alpha=1}^{N} S^{(\alpha)} \left(n_i B_{js} n_t + n_j B_{is} n_t + n_i B_{jt} n_s + n_j B_{it} n_s \right)^{(\alpha)}. \tag{10}$$

Almost all the estimation techniques available in the literature, based on an effective medium or effective field, can be formulated in the form of (10), and, in other words, have a complete formal similarity. Apart from their heuristic foundations, the conventional methods for estimating the effective moduli are each developed from several possible choices of $\mathbf{S}^0$ and $\boldsymbol{\sigma}^0$, which are by no means better than others.

To define a simple and applicable estimation scheme, we specify here

$$\boldsymbol{\sigma}^0 = \boldsymbol{\sigma}, \; \mathbf{S}^0 = (1-\xi f^{\eta})^{-1}\mathbf{S}^{\mathrm{m}}, \tag{11}$$

where ξ and η are two adjustable parameters, which can be determined by fitting experimental results or other theoretical results of good accuracy. By comparing the present method with the GSCM for the two extreme cases of isotropy and complete anisotropy, we suggest that ξ=4/9 and η=1.0.

Provided that the pristine matrix is isotropic, the suggested method calculates the opening displacement of a microcrack by assuming that it is embedded in an isotropic, infinite effective medium, analogously to Taylor's model. Their difference is that the effective medium in the presented method is, instead of the pristine matrix, an approximate reference medium with degraded effective elastic moduli depending upon the actual damage. The suggested scheme combines the advantages of both the DCM and first-order effective medium methods.

QUASI-MICROMECHANICAL DAMAGE MODEL

Description of damage

The problem of evolutionary damage in a brittle material subjected to complex loading is too complicated to be solved with such methods as SCM and DM. For this reason, little work has been done in the filed of micromechanics to consider the overall constitutive relation of brittle materials under complex loading. A promising approach to achieve this aim is to combine phenomenological and micromechanical damage mechanics. Such an attempt is made here to present a quasi-micromechanical damage model. Beside the concept of DMG, the scalar microcrack density parameter f [7] is adopted to describe the damage in a microcracked solid. The DMG describes exactly the anisotropic microcracking damage state, while f, which defines merely the magnitude of the isotropic part of the damage, is introduced to render the calculation of effective moduli much easier. In this way, the new estimation scheme proposed above can be implemented into the micromechanics-based DMG damage model. The parameter $f(t)$ is related to the DMG $\Omega(t)$ by

$$f(t) = n_{\mathrm{c}}\left[\bar{a}(t)\right]^3 = n_{\mathrm{c}}a_0^3 + n_{\mathrm{c}}\iint_{\Omega(t)} p(a,\theta,\varphi)\left(a_{\mathrm{u}}^3 - a_0^3\right)\sin\theta \,\mathrm{d}\theta \,\mathrm{d}\varphi \;, \tag{12}$$

where $p(a,\theta,\varphi)$ denotes the probability density function describing the distribution of the orientations and sizes of microcracks in the material.

Then, the effective compliance tensor of the damaged solid can be estimated by embedding each microcrack into an effective medium with compliance $\mathbf{S}^0$ defined in (11) and subjected to the far-field stress $\boldsymbol{\sigma}$. In this case, the nonzero components of the crack opening displacement discontinuity vector $\mathbf{B}$ are

$$B'_{11} = B'_{33} = \frac{16(1-\nu^2)}{\pi E(2-\nu)(1-4f/9)}, \qquad B'_{22} = \frac{8(1-\nu^2)}{\pi E(1-4f/9)}. \tag{13}$$

Thus, the compliance tensor in (9) induced by the elastic deformation of a single microcrack is rewritten as

$$\bar{S}_{ijkl}^{\mathrm{c}(\alpha)}(a) = \frac{\pi a^3}{6} B'_{mn}(g'_{2i}g'_{mj} + g'_{2j}g'_{mi})(g'_{2k}g'_{nl} + g'_{2l}g'_{nk}) \,. \tag{14}$$

Constitutive relation

In this paper, only open microcracks are considered. The overall effective constitutive relations for all the four stages including linear elasticity, pre-peak nonlinear hardening, stress drop and strain softening [5,8] can be expressed in the following unified form:

$$\varepsilon_{ij} = \left[S^{0}_{ijkl} + S^{\mathrm{c1}}_{ijkl} + S^{\mathrm{c2}}_{ijkl} + S^{\mathrm{c3}}_{ijkl}\right]\sigma_{kl} + \varepsilon^{\mathrm{R}}_{ij} \tag{15}$$

where S^{c1}_{ijkl}, S^{c2}_{ijkl}, S^{c3}_{ijkl} denote the increment of the compliance tensor due to microcracks that have not propagated ($a = a_0$), that have experienced the first growth ($a = a_\mathrm{u}$) and that have undergone the secondary growth ($a > a_\mathrm{u}$), respectively, $\varepsilon^{\mathrm{R}}_{ij}$ denotes the strains due to the residual deformation of microcracks,

$$S^{\mathrm{c1}}_{ijkl} = \int_0^{2\pi}\int_0^{\pi/2} n_\mathrm{c} p(a,\theta,\varphi)\bar{S}^{\mathrm{c}}_{ijkl}(a_0)\sin\theta \mathrm{d}\theta \mathrm{d}\varphi - \iint_{\Omega} n_\mathrm{c} p(a,\theta,\varphi)\bar{S}^{\mathrm{c}}_{ijkl}(a_0)\sin\theta \mathrm{d}\theta \mathrm{d}\varphi \tag{16}$$

$$S^{\mathrm{c2}}_{ijkl} = \iint_{\Omega} n_\mathrm{c} p(a,\theta,\varphi)\bar{S}^{\mathrm{c}}_{ijkl}(a_\mathrm{u})\sin\theta \mathrm{d}\theta \mathrm{d}\varphi - \int_0^{2\pi}\int_0^{\theta_{\mathrm{cc}}} n_\mathrm{c} p(a,\theta,\varphi)\bar{S}^{\mathrm{c}}_{ijkl}(a_\mathrm{u})\sin\theta \mathrm{d}\theta \mathrm{d}\varphi \tag{17}$$

$$S^{\mathrm{c3}}_{ijkl} = \int_0^{2\pi}\int_0^{\theta_{\mathrm{cc}}} n_\mathrm{c} p(a,\theta,\varphi)\bar{S}^{\mathrm{c}}_{ijkl}(a_\mathrm{s})\sin\theta \mathrm{d}\theta \mathrm{d}\varphi \tag{18}$$

$$\varepsilon^{\mathrm{R}}_{ij} = \iint_{\Omega} n_\mathrm{c} p(a,\theta,\varphi)\bar{\varepsilon}^{\mathrm{R}}_{ij}(a_\mathrm{u})\sin\theta \mathrm{d}\theta \mathrm{d}\varphi + \int_0^{2\pi}\int_0^{\theta_{\mathrm{cc}}} n_\mathrm{c} p(a,\theta,\varphi)\left[\bar{\varepsilon}^{\mathrm{R}}_{ij}(a_\mathrm{s}) - \bar{\varepsilon}^{\mathrm{R}}_{ij}(a_\mathrm{u})\right]\sin\theta \mathrm{d}\theta \mathrm{d}\varphi \tag{19}$$

where θ_{cc} is an angle parameter related to the number of microcracks that have experienced secondary growth [5], $\bar{\varepsilon}^{\mathrm{R}}_{ij}(a)$ the residual strains induced by a microcrack of radius a [8], and a_s the radius of a microcrack during the secondary growth.

CONCLUSIONS

The quasi-micromechanical damage model developed here has the following main features. First, the microcracking damage is characterized in terms of both the DMG and the scalar microcrack density, which function as an exact representation of anisotropic damage and a key parameter in the proposed scheme for calculating the impacts of microcrack interaction, respectively. Second, the damage evolution under complex loading can easily be analyzed with the aid of set theory. Third, the constitutive relation formulated can be applied to the whole deformation process of quasi-brittle materials, including the stages of linear elasticity, nonlinear damage hardening, post-peak stress drop and strain softening. Fourth, the effects of microcrack interaction on effective moduli can be calculated as easily as the DCM. The present attention is focused on material behavior under tension, though the main idea can be extended to the case of compression.

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