REFERENCE: ICF1001003OR

Symposium on *Computation & Probabilistic Fracture Mechanics*
10th International Congress on Fracture, Honolulu, HI, Dec 2-7, 2001

PROBABILISTIC FAILURE ASSESSMENT OF A CARBON-CARBON SHELL FOR THE EARTH ENTRY VEHICLE CONCEPT

R. E. Kurth and F.W. Brust

Battelle Memorial Institute, 505 King Avenue, Columbus, OH 43201-2693

ABSTRACT

The goal of the first phase of this program is to demonstrate that it is feasible using currently accepted analyses procedures to estimate the probability of failure of the Earth Entry Vehicle (EEV) to be less than 1 in 1,000,000. Clearly, an experimental-only demonstration program is not feasible from either a cost or a time standpoint. Alternatively, an analyses-only effort is adequate only in the situation in which there is significant historical confidence in the methodologies. While titanium may have a sufficient history the Carbon-Carbon (C-C) composite material for the fore body and aft structure does not. Therefore, a two step process has been adopted. First, the combination of existing NASA Langley Research Center (NASA LaRC) mechanics analyses tools for composite structural analyses with probabilistic analyses tools is performed. This tool has been developed to demonstrate that a complete probability of failure calculation is feasible. Secondly, the probability of failure of the EEV C-C structure has been quantified with this tool. The important parameters for damage in the C-C aero-shell were determined and a statistical design for the development of a response surface fit to the finite element code was developed. Finite Element (FE) analyses of the EEV structure for the nominal entry condition with varying degrees of composite material damage were performed. These results were combined with the statistical design parameters to calculate a response surface fit to the FE code. The response surface was then analyzed using the fast probability integration method to obtain an estimate of the failure probability of the C-C.

KEYWORDS

Composite material, structural failure, risk assessment of structures

INTRODUCTION

The development of an estimate of the probability of failure of the C-C aero-shell proceeded in four steps. First, the existing data was reviewed. Second, a statistical design was developed for the fitting of a response surface* to the FE model. Next, the FE model was run as specified in the statistical design to obtain the response surface coefficients. These response surface coefficients

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* A response surface is a polynomial fit to a complex model.

were then input to the fast probability integrator to determine the failure probability for the C-C composite structure.

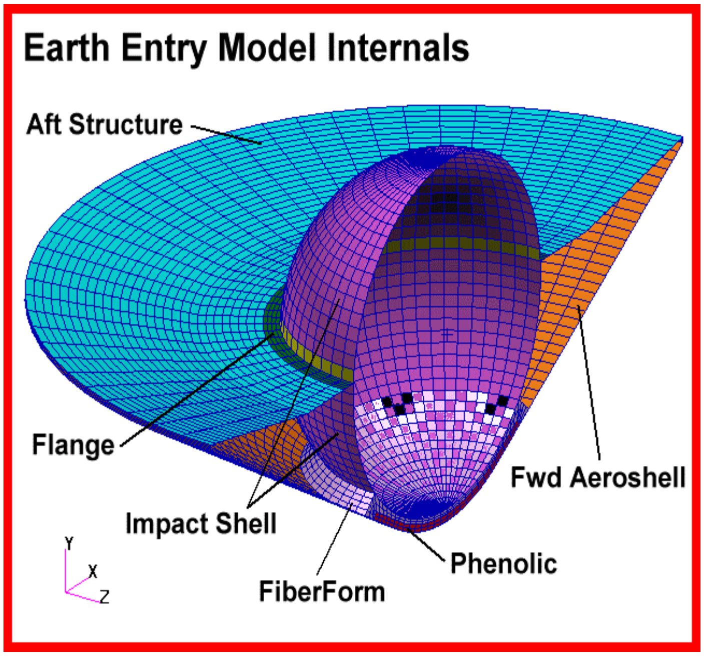

Figure 1. EEV Configuration Used In FE Analysis

FINITE ELEMENT (FE) MODELING

The finite element model was provided by NASA LaRC. The initial analyses were limited to the entry configuration for this phase. The reason is that the results of damage caused during the manufacturing process and the launch phase can be treated as inputs to the entry model. This will cut in half the number of FE analyses needed and allow the demonstration of the failure probability calculation. Provided that the damage that is assumed for the entry analysis is conservative (i.e. it bounds what the results of a launch analysis would provide) then the failure probability should only get lower as the more detailed launch analysis results are integrated into the final analyses.

RESPONSE SURFACE MODELING

The goal of the response surface modeling is to eliminate the need to perform FE analysis. By a response surface we mean a polynomial approximation to the FE code. Assuming that the maximum stress is the response of interest then:

$$\sigma_{\max} = a_0 + \sum_{i=1}^{N} a_i x_i + \sum_{i<j} a_{ij} x_i x_j + \sum_{i<j<k} a_{ijk} x_i x_j x_k + \ldots + a_{123\cdots N} x_1 x_2 x_3 \cdots x_n$$

For the random variables represented by x_i we can calculate a distribution of $\sigma_{\max}$. By comparing this distribution to the distribution of the ultimate strength we obtain the probability of C-C failure. How do we obtain this response surface? For the problem at hand we used a factorial experimental design.* The reason for this is two-fold. One this design will cover a large fraction of the input space. Secondly, it minimizes the number of runs needed for the response surface development.

FAILURE CRITERION DEVELOPMENT

The development of a failure criterion for the C-C composite does not actually involve a failure of the C-C material in the classical sense. Rather, the concern is that the strain in the C-C becomes high enough to cause the Thermal Protection System (TPS) material to crack the result of which is viewed as a failure of the EEV system. Because the failure data collected to date is in terms of stress we first need to establish a stress-strain relationship. This was done through classical statistical analysis, but is not reported in this paper. Rather, the results of these calculations are simply stated. For the C-C tensile strength data the mean is 21.18 ksi and the standard deviation is

* The "experiment" in this case is the FE analyses.

4.06 ksi. We obtain a value of β equal to 6.1 for the tensile strength data using a Weibull distribution to describe the strength.

We can now use these values for β and κ to calculate the probability that the load exceeds the strength. The algorithm for this calculation is taken from the TRACLIFE[1] program originally written for residual strength calculations in aircraft and first programmed by Wirsching and Wu.[2] On today's personal computers this algorithm gives almost instantaneous answers for very low probability events (on the order of 10^{-9}). Let us examine the current analyses.

In this analysis it is necessary to specify the *limit state equation*. This is simply a mathematical statement of the surface at which the system will fail. This is a very simple equation for this analysis since we are simply interested in the event that the strength, denoted S, is less than the stress, denoted L. Therefore, $S - L < 0$, is the limit state equation. For the shape parameter values, β, given such that the first two moments, i.e. the mean and variance, of the two-parameter Weibull distribution are matched to the "data" then the Rackwitz-Fiessler or Chen Lind algorithms can be used to calculate the probability that the limit state has been exceeded. The next section will produce preliminary P_F numbers. For now we move to the FE inputs needed to develop the response surface.

FINITE ELEMENT CODE ANALYSES FOR RESPONSE SURFACE DEVELOPMENT

The development of a response surface for the determination of the probability of failure for the C-C composite material is based on a fold-over factorial design. This design allows for future expansion of the response surface without losing the information contained in these original runs. A total of sixteen finite element analyses are performed to determine the peak strain values. Thus, five of these runs are used to determine the average value and the impact of the independent variables, also called "main effects" on the probability of failure, the remaining eleven runs are used to determine all of the interaction terms impacts on the failure probability. So what are the variables to be included in the analyses? Based on a review of nominal FE analyses the following four variables are recommended for inclusion:

Delamination of the C-C
Fiber cracking of the C-C
Matrix cracking of the C-C
Temperature

We recognize that there are other inputs to the analyses that can impact the failure probability but we believe they are of secondary importance to these variables. For example, there is a significant question about the entry angle because of the asymmetric loading that would be imposed. This is a variable that can be added at a later time by using the fold-over feature of the statistical design. It would require an additional 16 runs. All of the damage is recommended to be placed in the zone shown in Figure 1.

C-C FAILURE PROBABILITY CALCULATIONS

There were two types of failure probability assessments made during this study. First, a simple comparison of the predicted strains to the strength was studied. Secondly, the response surface analyses were performed.

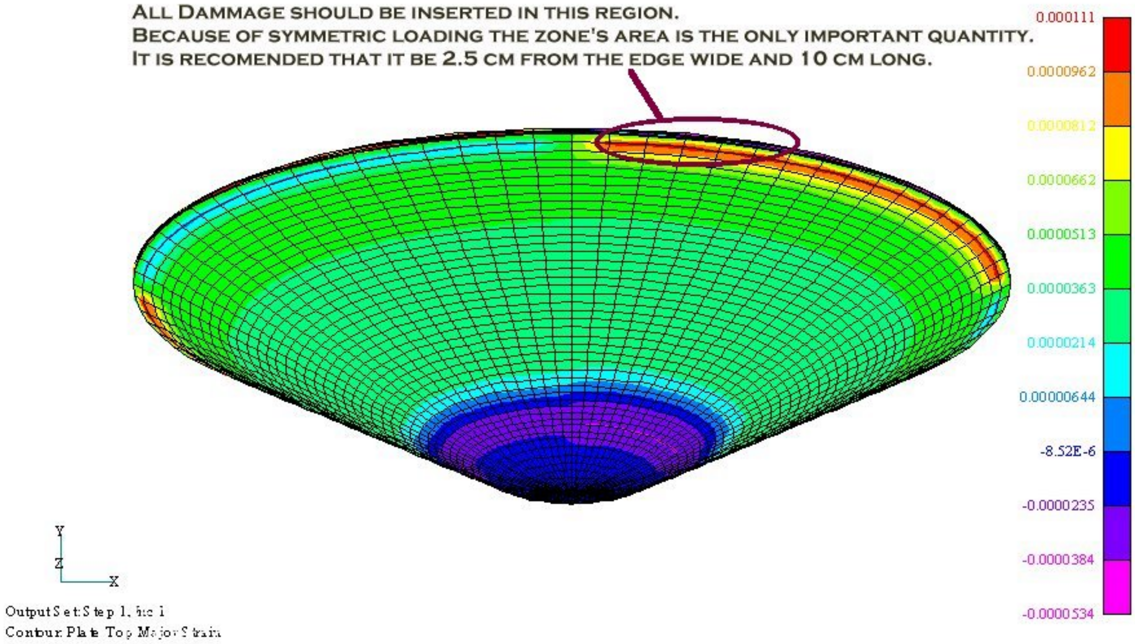

Figure 1. Recommended Damage Locations

The first step in the assessment of the structural strength was to examine the point estimate of the probability of failure. To do this we examine the calculated maximum principal strain calculated in the base case analysis and compare it to the (assumed) Weibull distribution for the strength. Thus, the maximum principal strain of 8.97 x 10^{-5} is compared to the TPS allowable strain distribution with a shape factor, β, equal to 2 and 4. When the value of 4 is used the maximum principal strain calculated by the FE code falls at a point in the strength distribution that states the strength would be less than this *point value* is equal to a probability of 4.5 x 10^{-7}. This is encouraging but we must recall that the Weibull distribution has very long.

We know look at the estimated failure probability for the C-C composite shell for all eight of the response surface calculations. These are shown in Table I. The estimated failure probabilities rang from 9 x 10^{-8} to 1.3 x 10^{-5}. However, we must remember that these probabilities are *conditional* probabilities, i.e. they are dependent on the inputs to the finite element model. Since these are *not* equally weighted we cannot simply take the arithmetic average of the numbers to determine the overall failure probability. Rather we must perform the FPI analysis of the fitted response surface to obtain our final result.

C-C RESPONSE SURFACE ANALYSES

The FE code analyses were performed using ABAQUS. The results of the FE analyses are reported as the maximum principal strain within the C-C shell structure. These results are given in Table I. Here we take these eight values and calculate the coefficients of the response surface equation.* A cursory examination of the values in Table I shows that the maximum principal strain changes by a factor of 2.2 from the highest to lowest value ratio. Table I indicates that the failure probability can

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* Recall that since the temperature is no longer a variable all columns in Table VI involving the factor 4 must be removed from the analysis.

change by a factor of 144 or 2 and ½ orders of magnitude! This is the impact of the long tails in the assumed Weibull distribution.

An examination of the response surface coefficients shows that matrix cracking is the driver for the maximum principal strain with fiber cracking a factor of 3 lower. The impact of delamination on the maximum principal strain is an order of magnitude less than fiber cracking.

EEV Conceptual Design Assessment Battelle - NASA LaRC Strain Failure Criterion C-C Composite					
Load Condition	NASA Value	NASA Safety Factor	β=4.00		Strain Failure Safety Factor
50 g entry	Maximum principal stress	9.7	7.25E-06	1 in 137,894	6.2
CASE 1	8.970E-05 mm-mm	20.9	4.53E-07	1 in 2,206,302	13.4
CASE 2	7.700E-05 mm-mm	24.4	8.95E-08	1 in 11,169,403	15.6
CASE 3	1.000E-04 mm-mm	18.8	4.53E-07	1 in 2,206,302	12.0
CASE 4	1.040E-04 mm-mm	18.1	4.53E-07	1 in 2,206,302	11.5
CASE 5	1.530E-04 mm-mm	12.3	3.50E-06	1 in 285,937	7.8
CASE 6	1.690E-04 mm-mm	11.1	3.50E-06	1 in 285,937	7.1
CASE 7	1.940E-04 mm-mm	9.7	7.25E-06	1 in 137,894	6.2
CASE 8	2.000E-04 mm-mm	9.4	1.34E-05	1 in 74,432	6.0

Table I. Response Surface Results Point Estimates for Failure Probability of the C-C Aero-Shell

We can also examine how the variability in the individual effects impacts the overall response variability. To do so we note that the variance of the linear combination of two variable is given as:

$$Var(Z) = a_1^2 Var(x_1) + a_2^2 Var(x_2)$$

Repeatedly applying this equation to the response surface equation allows the variance in the maximum principal strain to be partition among the main effects (delamination, fiber cracking, and matrix cracking) and all of their interactions. The result of this partitioning demonstrates that over 90% of the variance in the maximum principal stress is derived from the variability in the assumed matrix cracking damage.

FPI ANALYSIS OF FE CODE RESULTS

The response surface coefficients derived in the previous section were input to the FPI analysis procedure. For the base case analysis we see the estimated failure probability is 3.4 x 10^{-6}.

These results are obtained by assuming a COV for the damage areas of 12.5%. If we increase this COV we will increase the probability of failure. As an example, if we increase the COV from 12.5% to 18.75% the probability of failure increases from 3.4 x 10^{-6} to 6.5 x 10^{-6}. We believe that the base case analysis is the most representative but that opinion is based on expertise not on data.

While the failure probability of 3.4 x 10^{-6} is exceptionally low, it is still too high for the PRA. Because this was a scoping PRA there are several conservative assumptions built into the analyses. These assumptions were reviewed via an expert panel. The failure probability number was then revised, *based on expert opinion without further analyses*, to provide the input to the full PRA model.

CONCLUSIONS

When collecting all of these factors together it was the consensus that the C-C failure probability would be a factor of 4 to 6 lower in a best estimate analyses. Further, if it can be shown that the carbon phenolic material will be used for the TPS and that the bond line maintains integrity then there would be another factor of 4 to 10 reduction from the increased TPS allowable strain. However, without changes it was agreed that a factor of 5 reduction (the mid-range value) could be applied for the best estimate of the C-C failure probability or a value of 6.8×10^{-7}.

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[1] Kurth, R.E. and Bigelow, C.A., *Transport Risk Assessment Containing Widespread Fatigue Damage: TRACWFD Analysis of Longitudinal and Circumferential Splice Joints to Determine the Onset of Widespread Fatigue Damage and Its Probability of Occurrence*, Second Annual Joint DoD/FAA/NASA Conference on Aging Aircraft, August, 1998, Williamsburg, VA

[2] Wirsching, P. and Wu, J.