ICF100196PR

ON ESTABLISHING FACTOR SIGNIFICANCE ON THE DELAMINATION FRACTURE TOUGHNESS OF A COMPOSITE LAMINATE

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ABSTRACT

Interlaminar fracture is a dominant mode of failure in advanced composite materials. The resistance to delamination is quantified using Critical Delamination Fracture Toughness, G_c. Here, a methodology is developed using Designed Experiments and Taguchi concepts to analyze the contribution of various factors such as delamination length, stacking sequence, etc. and their first order interactions to the pure mode I delamination fracture toughness of a composite laminate. All the factors are considered at two levels and a Fractional Factorial experiment is conducted. F tests help in culling the appropriate factors. Both balanced and symmetric specimens are used with a [0/90] ply setup. A study of the effects of main factors and means of the interactions was done to establish the combination of factors that would yield the highest fracture toughness within a given set of constraints of the factors. An idea of significant factors and contributions is then used to obtain a response surface equation that is used to predict the interlaminar fracture toughness.

KEYWORDS

Laminates, Fracture Toughness, Optimization, Taguchi, Design of Experiments, Response Surface

INTRODUCTION

Composite laminates have high stiffness to weight and strength to weight ratios compared to conventional materials. Hence, they are a favorite choice for many applications ranging from sports to aerospace. As such, it has become essential to test the reliability of laminated composites under different conditions, and manufacture them for high strength and fracture toughness. Delamination is one major mode of failure of composites and it may be assumed that a variety of factors affect the delamination fracture toughness of composite structures. Though many papers [1,2] have been published to optimize G_c based on one single factor, such as stacking angle using genetic algorithms, not one discusses the effect of the interactions of various factors, neither presents a technique to optimize G_c based on all factors.

In general, a delamination can occur in three modes, namely opening (mode I), shearing (mode II) and out-of-plane (mode III). The interlaminar fracture toughness is expressed in terms of strain energy release rate of mode I (G_I), mode II, and mode III (G_{III}). Of the three release rates, G_I is often the most critical, and hence a laminate fails by mode I delamination more often than the other two. Therefore, it becomes essential to optimize the mode I critical fracture toughness, G_{Ic}, for given set of constraints. This is achieved through use of statistical methods like Design of Experiments, Taguchi Arrays, and Response Surface Methodologies.

A variety of factors, all at two levels, will be assumed to be contributing to G_{Ic}. ASTM [3] standards for mode I interlaminar fracture toughness was followed. Since a factorial experiment would result in a huge number of runs, fractional factorial experimental concepts and Taguchi orthogonal arrays would be used in reducing the number of test runs. Highest order of interaction is used as the defining contrast. Two replications of the experiment will be conducted to gather more data and reduce error. Normality tests are performed on the data to assess the assumptions of normality. Yates [4] algorithm is then used to obtain the sum of squares. F tests on the sum of squares help in establishing the significant factors. Sum of squares will also be used to establish the percentage contribution of each factor and their first order interactions.

STATISTICAL METHODOLOGIES FOR OPTIMIZATION

The objective of the methodology developed here is to establish the factors that affect the delamination fracture toughness. In the simple beam theory analysis, the specimen can be assumed to consist of two identical cantilever beams with a built in end and split arm length equal to the length of the crack, that is the delamination length. According to beam theory, the energy release rate for a Double Cantilever Beam (DCB) test of a cross ply laminate is given by:

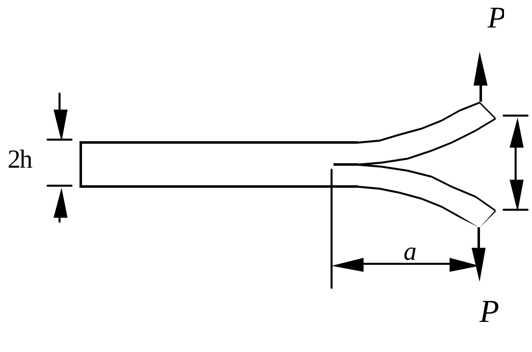

Figure 1: Double Cantilever Beam Analysis on Composite Laminate

$$G_I = \frac{12P^2}{b^2h^3E_{11}}\left[a^2 + \frac{2a}{l} + \frac{1}{l^2} + \frac{h^2E_{11}}{10G_{13}}\right] \text{ where } l = \frac{1}{h}\sqrt[4]{\frac{6E_{22}}{E_{11}}} \quad (1)$$

In the aforementioned mode I energy release rate, G_I, for quasi-static loading expression, the effect of shear deformation has been included.

Since composite laminates can be tailored to achieve required properties through different designs, one cannot take help of equation such as (1) to readily obtain the optimum combination of factors that can be used to tailor a high G_I composite laminate within the imposed constraints on all the factors.

To this extent, the following factors will be assumed to affect the fracture toughness: 1.Type of Loading, 2.Width of the specimen, 3.Length of the Specimen, 4.Thickness of the Specimen, 5.Lay up configuration, 6.Delamination length, 7.Material, 8.Ply thickness, 9.Thickness of Substrate, 10. Stacking Sequence.

Assuming each factor to be at two levels, one would need 2^{10} = 1024 experimental runs to study the effects of all the factors. Such huge number of runs is not practical and economical in laboratory conditions. Factors such as ply thickness, type of loading, material, thickness of substrate and lay up will be assumed to be constant while developing the model.

Model Equation

The model equation for the designed experiment would be assumed to be of the form,

$$Y_{ijklmn} = \text{Main Factors} + \text{Interactions} + \text{Error } (\ _{n}) \qquad (2)$$

where subscripts i, j, k, l, m and n denote the subscript of the five factors and error. The levels of each of the five factors are given in the table below. ASTM [3] standards for testing were used.

TABLE 1

VALUES OF THE FACTORS AT TWO LEVELS

Factor	Low Level -1	High Level 1
Width (A)	0.5 inches	0.8 inches
Length (B)	6.0 inches	8.0 inches
Thickness of specimen (C)	0.14 inches	0.18 inches
Delamination length (D)	2.4 inches	2.6 inches
Stacking Sequence (E)	Balanced	Symmetric

Using Taguchi's [5] linear graph for orthogonal array L_{16}, a 2^{5-1} fractional factorial experiment will be run with 16 runs. Two replications will be used to increase accuracy. The treatment condition for the 16 runs is tabulated below in Table 2.

TABLE 2

TREATMENT CONDITIONS FOR THE EXPERIMENT USING L_{16}

		Specimen Number															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Level of each factor	A	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
	B	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	1	1
	C	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	1	-1	1	1
	D	-1	1	-1	1	-1	1	-1	1	-1	-1	-1	1	-1	1	-1	1
	E	-1	1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1

Data to test the Methodology

Random data was generated using the data obtained in previous experiments to test the methodology. Two replications would be done to increase the accuracy.

TABLE 2

TEST DATA FOR THE METHODOLOGY

Specimen Number	Replication I G_{Ic} in N/m	Replication II G_{Ic} in N/m
1	304.44	303.16
2	296.87	299.75
3	306.88	308.73
4	316.82	312.70
5	301.01	300.94
6	311.81	310.74
7	299.88	298.98
8	309.82	312.29
9	309.56	307.45
10	301.54	303.42
11	307.63	306.29
12	299.56	300.42
13	312.87	311.67
14	272.31	276.45
15	300.84	298.90
16	278.24	272.23

Normality Tests

The Analysis of variance technique that is used assumes the data to be from random samples, equal variances and normality. Therefore,it is essential to check the normality of the data. A Normal Probability plot, Figure 2, will be used to test the data for any outliers. A linear normal probability plot indicates that data is normal. The figure below shows a normal probability plot of the random data in table 2.

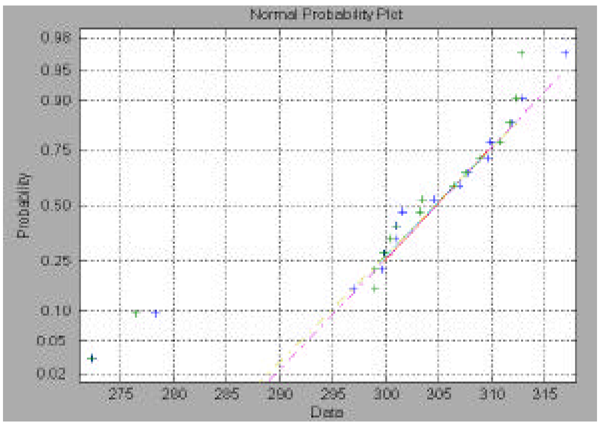

Figure 2: Normal probability of Test Data

Analysis of Variance

A two-way analysis of variance on the test data is shown in Table 3. The sum of squares of the treatment combinations were obtained using Yates [4] algorithm for fractional factorial experiments. Treatment combinations that has significantly smaller sum of squares compared to the other treatment combinations are pooled into the error sum of squares, thus increasing the degrees of freedom of error to approximately half of the total degrees of freedom. This increases the reliability of the results.

TABLE 3
ANALYSIS OF VARIANCE

Contrast	D.O.F	SS	MS	F	P
A	1	394.1028125	394.1028125	5.06	0.05104958
C	1	399.03125	399.03125	5.13	0.04977224
D	1	528.125	528.125	6.79	0.02846789
AD	1	1309.440313	1309.440313	16.84	0.00266212
CD	1	332.1753125	332.1753125	4.27	0.06876407
BE	1	535.4628125	535.4628125	6.88	0.02767558
Error	9	699.7838	77.75		

REPORT OF STATISTICAL ANALYSIS

It can be concluded based on the fictitious test data that factors A: Width, C: Thickness of Specimen, D: Delamination Length, and first order interactions AD: between width and delamination length, CD: between thickness of specimen and delamination length, and BE: between length and stacking sequence appear significantly contributing to the interlaminar fracture toughness. A study of two-way means on the interactions using reverse Yates [4] algorithm is done to further explore the interaction effects. The results of the two-way means are tabulated in Table 4 and 5.

TABLE 4
MEAN MODE I FRACTURE TOUGHNESS AT TWO LEVELS OF WIDTH, THICKNESS AND DELAMINATION LENGTHS.

Width	Delamination Length	
	Low	High
Low	302.43 N/m	307.72 N/m
High	308.83 N/m	287.91 N/m
Thickness of specimen	Low	High
Low	306.25 N/m	304.57 N/m
High	305.63 N/m	291.06 N/m

TABLE 5
MEAN MODE I FRACTURE TOUGHNESS AT TWO LEVELS OF LENGTH AND STACKING SEQUENCE

Length of specimen	Stacking Sequence	
	Balanced	Symmetric
Low	294.93 N/m	307.66 N/m
High	304.27 N/m	300.65 N/m

CONCLUSIONS

Based on the two-way means analysis on test data shown in tables 4 and 5, one can easily observe the combination of factors that can be used to tailor a composite laminate that would yield higher interlaminar fracture toughness G_{Ic} . Experiments can be performed to obtain real test data and the above methodology can be used to analyze the significant factors and interactions. A response surface equation (3) based on the significant factors and interactions can be obtained using techniques of Response Surface Methodologies. If the response equation has a discrete factor such as stacking sequence, two response equations can be derived for each level of the discrete factor as shown in equation (3). Here subscripts –1,1 indicate two levels of the discrete factor.

$$Y_{1,-1} = {}_{0} + {}_{1} A + {}_{2} C + {}_{12} A C + \qquad (3)$$

Equation (3) can also be used to predict the interlaminar fracture toughness G_{Ic} for a given set of values of the factors. Since each of the response equation consists only of continuous factors, optimization techniques based on gradient methods can be used to obtain the maximum fracture toughness within the constraints imposed. This reduces the amount of computational effort required tremendously, since discrete optimization techniques such as genetic algorithms need not be used.

ACKNOWLEDGMENTS

The authors would like to gratefully acknowledge the financial support of the National Science Foundation, Civil and Mechanical Systems Division, NSF Career Grant 99-82023. The authors are thankful to the grant monitor Dr. Jorn Larsen-Basse for his interest and encouragement.

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