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OPTIMIZATION OF A DYNAMIC FRACTURE PROCESS WITH A 1D MODEL

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ABSTRACT

This study presents a simplified model of a dynamic fracturation process. It is based on a 1-Dimensional mechanical model of a cracked structure under dynamic loading. Fracture parameters are the crack velocity and the energy release rate as in general but also the cohesion force which is the equivalent of the dynamic stress intensity factor in 2 dimensions. With these parameters we define two different criteria of propagation, the critical energy release rate and the critical cohesion force. Loading is generated by a striker of variable length, mechanical properties and initial velocity. Then optimization consists in finding the striker giving the maximum crack length for a given kinetic energy. The main results are that the striker should be of same mechanical impedance of the impacted bar and the optimal strike velocity depends on the criterion but should be high compared to the minimum required to initiate the propagation.

KEYWORDS

Crack Propagation, Dynamic Fracture

INTRODUCTION

Cracks in structures are always sources of failure and the aim of mechanics of fracture is to understand and generally prevent this kind of failure. In some particular case the objective is to make cracking easier. This will be the case in fracturing processes used in ballistic penetration, high speed machining or crushing of rocks. We can see that dynamic loading is preferred in these processes. In our present study we will try to find out the best loading condition to perform this fracturing. As dynamic fracture and dynamic crack propagation are very complicated subjects [1, 2] it is very difficult to perform a precise analysis of a dynamic fracturing process and it is even more difficult to perform its optimization. So we have decided to develop a 1-Dimensional model of dynamic fracture [3, 4] for which we can do exact optimization. With this model the relation between the applied loading and the mechanical state at the crack-tip is obtained analytically and the crack-tip location is a solution of a scalar differential equation depending only on the dynamic criteria of propagation. The optimization has been done using two kind of criteria.

1-DIMENSIONAL MODEL

Modelisation

We consider an elastic bar stuck on a rigid substratum with an initial debonding (the crack) of length a. The dynamic loading is generated by the impact of a second bar onto the first one as shown on Figure 1.

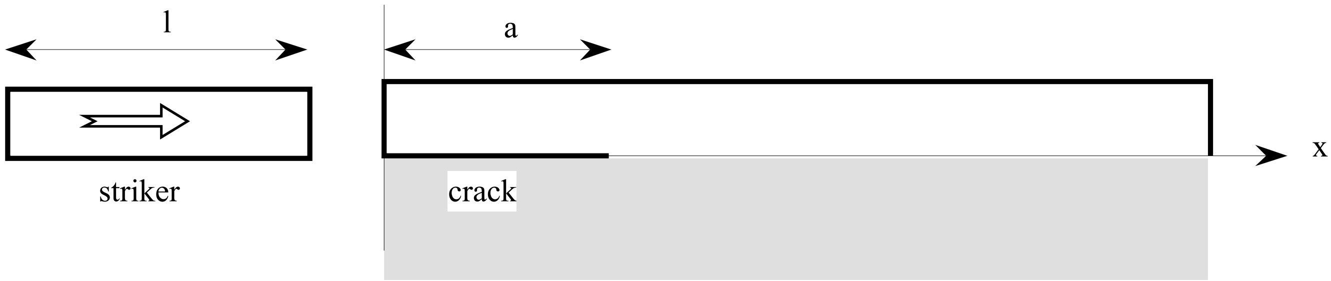

Figure 1: Dynamic fracture process model

We assume that the mechanical state is restricted to the longitudinal displacement u(x, t) and the normal force N(x, t) which are depending only on the abscissa x and time t. In this case the equations governing the problem are the elastic behaviour Eqn. 1. and the equation of motion Eqn. 2 :

$$N(x, t) = ES\, u_{,x}(x, t) \quad (1)$$
$$N_{,x}(x, t) = \rho S\, u_{,tt}(x, t) \quad (2)$$

where E, ρ, S are respectively the Young modulus, the density and the section of each bar.

Initial and boundary conditions

Before solving the problem we have to fix initial and boundary conditions. Initially we suppose the impacted bar at rest and time t = 0 the onset of the impact. So we have:

$$N(x, t) = 0 \quad ; \quad u(x, t) = 0 \quad \forall x > 0, \forall t < 0 \quad (3)$$

For the striker we assume a flying at uniform and constant velocity V :

$$N(x, t) = 0 \quad ; \quad v(x, t) = V \quad \forall x < 0, \forall t < 0 \quad (4)$$

where v(x, t) denotes the velocity field $u_{,t}(x, t)$.

For the boundary conditions, we prescribe a free boundary on the left side of the striker and a perfect unilateral contact between the striker and the bar:

$$N(-l, t) = 0 \quad ; \quad \begin{cases} N(0^-, t) = N(0^+, t) < 0 & ; \quad u_{,t}(0^-, t) = u_{,t}(0^+, t) \\ N(0^-, t) = N(0^+, t) = 0 & ; \quad u_{,t}(0^-, t) < u_{,t}(0^+, t) \end{cases} \quad (5)$$

Normally we should prescribe a boundary condition at the right end of the bar . In our case the impacted bar is free along the crack and totally fixed after the crack-tip:

$$u(x, t) = 0 \quad \forall x > a(t) , \forall t \quad (6)$$

This last boundary condition is then written like this:

$$u(a(t), t) = 0 \quad \forall t \tag{7}$$

A special attention have to be paid on this condition because this a moving condition with the crack propagation depending on the crack length which is also an unknown variable of the problem.

Criteria of propagation

The evolution of the crack length depends on the criterion of propagation chosen at the crack-tip. With this 1-D model there is no stress singularity and therefore no stress intensity factors to quantify the loading of the crack. To define this loading we calculate the energy release rate G(t) which is defined whatever the mechanical model of fracture is. In our case we finally obtained the following expression:

$$G(t) = \frac{1}{2ES} N^{-}(t)^2 \left(1 - \frac{\dot{a}^2}{c^2}\right) \tag{8}$$

Where $N^{-}(t) = N(a^{-}(t),\ t))$ is the normal force just before the crack-tip and $c = \sqrt{E/\rho}$ the celerity of the longitudinal elastic waves in the bar.

The normal force is not continuous at the moving crack-tip and the jump is obtained using the shock wave relation which leads to the definition of the normal force in front of the crack-tip:

$$N^{+}(t) = N(a^{+}(t), t) = N^{-}(t)\left(1 - \frac{\dot{a}^2}{c^2}\right) \tag{9}$$

Putting this result in the definition of G we obtain :

$$G(t) = \frac{1}{2ES} N^{-}(t)\, N^{+}(t) \tag{10}$$

This expression is very similar to the expression given in the 2-dimensional model with two stress intensity factors, the dynamic stress intensity factor and the kinematic intensity factor [5].

So in our model a criterion of propagation will appear as a function of these four quantities (G, N^{-}, N^{+}, $\dot{a}$) in which only two of them are actually independent.

OPTIMIZATION PROCEDURE

Equation of propagation

The general solution of Eqn. 1. and Eqn. 2. is a combination of two unknown scalar functions f and g for the striker and the impacted bar:

$$u(x, t) = f(t - x/c) + g(t + x/c) \tag{11}$$

Putting this expression in the initial and the boundary conditions we find a explicit relation between the loading and the criterion of propagation. This relation gives a non linear differential equation of the first order on a(t) [6]. To solve this equation we have to define the criterion of propagation. Generally one chooses the critical energy release rate criterion G_C:

$$\dot{a} \geq 0 \;;\; G = G_C \quad \text{or} \quad \dot{a} = 0 \;;\; G < G_C \tag{12}$$

Another choice is the critical normal force criterion N_C:

$$\dot{a} \geq 0 \; ; \; |N^+| = N_C \quad \text{or} \quad \dot{a} = 0 \; ; \; |N^+| < G_C \tag{13}$$

With this criterion we assume that the failure occurs when the cohesive force N^+ acting at the right side of the crack-tip reach the limit N_C. For identical toughness in quasi-static, fast crack propagation needs higher level of loading compared with the energy release rate criterion.
In the case of our impact and for both of the previous criteria the coefficients of the differential equation are stepwise functions of time and the crack propagates by step at constant velocity.

Case of striker and impacted bar made of identical material
We present here the results for a striker and a bar of same section S and made of the same material. To simplify we take E, S and ρ equal to unity.
Whatever the length of the striker the evolutions of the system are as follow:

1 - A compressive wave propagates backward in the striker and forward in the bar.
2 - The last wave reflects at the crack-tip making the crack propagates.
3 - The reflection of the first wave at the free end of the striker wave arrives to the crack-tip and stops the propagation.
4 - The striker goes back at uniform velocity and looses contact with the bar in which remain elastic waves with no more propagation.

The main results are reported in Table 1.

TABLE 1
RESULTS OF FRACTURATION BY IMPACT

Criterion	Energy release rate	Cohesion force
Loading parameter	$\beta = \frac{\sqrt{2\, G_C}}{V}$	$\gamma = \frac{N_C}{V}$
Condition for propagation	$\beta < 1$	$\gamma < 1$
Crack velocity $\dot{a}$	$\frac{1-\beta^2}{1+\beta^2}$	$1-\gamma$
Total length of propagation Δa	$\frac{1-\beta^2}{\beta^2}\, l$	$2\,\frac{1-\gamma}{\gamma}\, l$
Dissipated energy	$l\, V^2 \frac{1-\beta^2}{2}$	$l\, V^2 \frac{1-\gamma}{2-\gamma}$

In the optimization procedure we fix the initial kinetic energy W of the striker and we calculate the maximum length of fracture by changing the length and the velocity of the striker. The results are reported in Table 2.

As we can see the optimal solution depends strongly on the criterion. With the energy release rate criterion we can transform all the kinetic energy in fracture if the striker is short and thrown at a velocity must higher than the minimum required to initiate fracture. With the critical cohesion force criterion the impact velocity should be the double of the minimum required and at the most the half of the kinetic energy is used for fracturation.

TABLE 2

OPTIMAL FRACTURATION

Criterion	Energy release rate	Cohesion force
Initial impact energy W	$\frac{1}{2}V^2 = \frac{1}{2}\frac{G_C}{\beta^2}$	$\frac{1}{2}V^2 = \frac{1}{2}\frac{N_C^2}{\gamma^2}$
Total length of propagation Δa	$(1-\beta^2)\frac{W}{G_C}$	$4\gamma(1-\gamma)\frac{W}{N_C^2}$
Optimal loading parameter	$\beta = 0$	$\gamma = 1/2$
Maximum length of propagation Δa	$\frac{W}{G_C}$	$\frac{W}{N_C^2}$
Maximum dissipated energy	W	W/2

Case of striker and impacted bar made of different material

Here the properties of the impacted bar are kept equal to unity and those of the striker will be defined by the two following parameters, the mechanical impedance z and the elastic celerity of wave c:

$$ES = z\,c \quad \text{and} \quad \rho S = z/c \tag{14}$$

If the striker is lighter than the bar ($z < 1$) there will be just one impact. At the opposite for heavier striker ($z > 1$) there be several impacts and several fracturation stages. Main results are shown in Table 3.

TABLE 3

OPTIMAL FRACTURATION ACCORDING TO IMPENDANCE OF THE STRIKER

Criterion	Energy release rate	Cohesion force
Impedance parameter	$k = \frac{z-1}{z+1}$	$k = \frac{z-1}{z+1}$
Loading parameter	$\beta = \frac{\sqrt{2\,G_C}}{V\,(1+k)}$	$\gamma = \frac{N_C}{V\,(1+k)}$
Total length of propagation Δa if one step advance	$(1-\beta^2)(1-k^2)\frac{W}{G_C}$	$4\gamma(1-\gamma)(1-k^2)\frac{W}{N_C^2}$
Total length of propagation Δa if two steps advance	$(1+k^2-2\beta^2)(1-k^2)\frac{W}{G_C}$	$4\gamma(1+k-2\gamma)(1-k^2)\frac{W}{N_C^2}$
Optimal parameters	$\beta = 0$; $k = 0$	$\gamma = 1/2$; $k = 0$

Changing the constitutive material of the striker is not efficient to increase the fracturation and the best way to proceed is to take a striker of same impedance of the bar whatever the criterion.

CONCLUSIONS

We have presented a simplified model of dynamic fracturation. This 1-Dimensionnal model is rich enough to reproduce dynamic phenomena coupled with fracture mechanics but still simple to perform all calculation analytically. It has been used to find the optimal conditions of fracturation that is the maximum crack propagation for a fixed loading energy. In our study the loading was the kinetics energy of a striker and the optimization has been done for two criteria of propagation, the energy release rate criterion and the cohesion force criterion.

This model can not treat precisely real problems because it does not include three dimensional effects like stress singularities at the crack-tip. Moreover we have also simplified loading condition but the general conclusions about the nature of the striker and the influence of the criterion should apply to real problems. We can also do the optimization with more realistic criteria taking into account for example the thermo-mechanical coupling [7, 8], different crack criteria for initiation, propagation and arrest, ...

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