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ON THE DEPENDENCE OF THE WEIBULL EXPONENT ON GEOMETRY AND LOADING CONDITIONS AND ITS IMPLICATIONS ON THE FRACTURE TOUGHNESS PROBABILITY CURVE USING A LOCAL APPROACH CRITERION

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ABSTRACT

The present study addresses the issue of the dependence of the Weibull exponent *m* on geometry and loading conditions. It is shown that the amplitude and shape of the notch tip stress field and, in particular, the triaxiality characterising the stress state determines the value of the exponent *m*. Tests performed on RNB(T) specimens of carbon steel 22NiMoCr37, type A 508 Cl 3, at temperatures ranging from -18 ^{o}C to -196 ^{o}C actually indicate that *m* varies from ~ 6 to 40, depending on the notch depth and root radius while for specimens carrying sharp cracks its value drops down to ~ 4. This last result seems to be consistent with the Wallin hypothesis of a theoretical value equal to four, for fracture mechanics specimens with high constraint, such as C(T) or SE(B), with positive values of the Q-stress or T-stress and a triaxiality factor, TF, approaching 2.5. Temperature, in as long as it does not modify the stress state from plane strain to plane stress and the TF, has no effect on the value of *m* which is independent of the material as well.

Key words: *K* field, *HRR* field, *Q*-stress, triaxiality factor, process zone, fracture probability distribution.

INTRODUCTION

Local approach to fracture has been widely applied by many authors. Fontaine *et al.* [1] used local approach to predict brittle fracture in welded joints. Amar and Pineau [2] proposed a modified definition of the probability of failure to asses the behaviour of a structural steel in the brittle-ductile transition region. Saillard *et al*. [3] used this model to predict brittle fracture in a thick pressurised shell under thermal shock. In all these applications, the characteristic material parameters, *i.e.* the *m* exponent, the Weibull stress σ_w and the element volume V_o , have been inferred without a great accuracy. As to the Weibull exponent *m*, for instance, it was assumed equal to 22, as suggested by Mudry [4]. This value was also confirmed in a study by the authors [5] as part of an international round robin on local approach promoted by the European Structural Integrity Society (ESIS). Recently, the conclusion of the second round robin on local approach [6] has pointed out the difficulties in the determination of the material parameters and the large differences in their estimation among the laboratories involved. In particular, the m exponent was found to range from 10 to 50 with a mean value of 16. This latter value is presently considered to be the most suitable for the steel considered, a low alloy carbon steel 22NiMoCr37, type A 508 Cl 3. The fact that needs to be pointed out is that in all the analyses the value of the Weibull exponent m was derived through a numerical-experimental procedure based on the use of round notch bars in traction, RNB(T), carrying notches of different root radii, generally from 0.2 to 20 mm. The question arises as to whether or not the value of the Weibull exponent assessed on RNB(T) specimens can be used in an analysis referring to a structure carrying a real crack rather than a notch. The difference between a sharp or natural crack and a notch is in the shape and amplitude of the stress field set up ahead of the tip: the blunter the notch the smother the stress field which, for larger root radii, can even lose the characteristic stress singularity $1/\sqrt{r}$ typical of the crack. We know that these differences in the stress field amplitude and profile have an impact on the attitude of the material towards brittle fracture, which is revealed by the well known fact that a sharp crack is much more dangerous than a blunted one or a notch and that a circular hole is used as a crack arrester. Then, the previous question becomes whether the stress field has an effect on the exponent m. If it does, a value of m derived from a particular specimen carrying a notch of given tip radius ρ cannot be used to asses the probability of failure of another specimen or geometry with a different ρ. This would actually imply that the Weibull exponent depends on the crack geometry. The present study is devoted to answering this question. At variance with the present assumption, it appears that, indeed, the value of the m exponent depends on the type of stress field set up ahead of the crack or notch tip and is not a characteristic of the material.

THE PRESENT STUDY

Wallin (7, 8) argued that in a specimen carrying a sharp crack, where the stress field is represented by K_I, the extension x_{eff} of the effective process zone ahead of the crack tip, where the existence of a brittle phase can trigger the fracture, must necessarily be proportional to $K_I^{\ 2}$. Its volume ~ $(x_{eff})^2 \cdot t$, then, shall be proportional to the forth power of K_I. If we assume that the failure probability distribution P_f depends on that volume size and we write:

$$P_f \quad = \quad 1 - exp\left[-\left(\frac{K_I}{K_o}\right)^m\right] \tag{1}$$

then the m exponent in eq. (1) has to be equal to 4. If we move from a real sharp crack, to a blunted one, we can use the same arguments to derive a possible value of the m exponent. We can conveniently represent the stress field σ_{ij} ahead of a blunted crack using the HHR field with the introduction of a Q-stress:

$$\sigma_{ij} \quad = \quad \sigma_{ys}\left(\frac{EJ}{\alpha\sigma_{ys}^2 I_n x}\right)^{\frac{1}{n+1}} \quad + \quad Q\sigma_{ys}\delta_{ij} \tag{2}$$

where E is the Young's modulus, J the J-integral that determines the amplitude of the crack tip singularity, σ_{ys} the yield strength, x the distance from the tip, n the strain hardening coefficient, α and I_n are material constants. The Q stress in eq. (2) quantifies the constraint exerted over the crack tip plastic zone and is a measure of the degree of triaxiality existing ahead of a blunted crack. For a sharp crack under small scale yielding (SSY) conditions, Q is equal to zero. Therefore the extension of the process zone x_{eff} is proportional to J, as eq. (2) indicates, and its volume to J^2. Under these circumstances it is:

$$J \quad = \quad \left(1 - \nu^2\right)\frac{K_I^2}{E} \tag{3}$$

and, again, we find that the exponent m in eq. (1) is equal to 4. As the crack blunts, however, the system starts to lose constraint and triaxiality, resulting in growing negative Q values. A point is reached when SSY conditions are lost and the equivalence between J and K_I starts to vanish. This happens when the plastic enclave ahead of the blunted crack swells so much that K_I loses its physical meaning, since it is a linear elastic fracture mechanics parameter based on the assumption of a negligible plastic zone, typical of SSY. Now, even though the volume of the process zone still remains proportional to J^2, we cannot use eq. (1) any longer. In these conditions we should replace in eq. (1) K_I with J and the exponent 4 with 2. However, also a formulation of the probability of failure P_f in terms of J has a limit for at least two reasons. First, the J approach, even that of eq. (2) with the introduction of the *Q-stress*, is valid as long as the crack is stationary or undergoes a small stable growth. This may not be the case for a blunted crack when it experiences large stable growth before failure occurs. Secondly, as the blunting increases and the entire ligament becomes plastic, the J may no longer describe the stress field, even in the event of no-stable growth. Now, new tools are needed, such as the slip lines solution, except that the slip lines theory applies to rigid-plastic materials under plane strain conditions. In particular, round bars with notches having a tip radius greater than one millimeter, such as those used in the ESIS round robin, fail when the entire section is plastic. Even at −190 °C the specimens used in that program failed when the entire cross section was plastic. This is why a more general formulation of the probability of failure is required, using the true rupture stress σ_r rather than K or J as shown by eq. (4):

$$P_f = 1 - exp\left[-\left(\frac{\sigma_r}{\sigma_o}\right)^m\right] \tag{4}$$

In this case, however, the m exponent will no longer be necessarily equal to 4, as for the sharp crack in SSY, and this deviation can be seen already for a J controlled stress field when the Q stress assumes high negative values. This can be understood by observing that J is composed of a linear elastic component and a plastic one:

$$J = J_{el} + J_{pl} \tag{5}$$

The linear component J_{el} is precisely the energy release rate **G** and is proportional to the square of the applied σ, and therefore to K_I^2 as per eq. (3). The nonlinear one J_{pl} is not. J_{pl} is related to the applied stress σ through a power that depends on the compliance of the system, i.e., on the plastic load-deformation response, P-δ_{pl}, of the system containing the crack. This is because the area A under the P-δ_{pl} curve is proportional to J_{pl} and that area is also proportional to σ^{n+1} where n is also the exponent of the power low $\delta_{pl} = C \cdot P^n$ representing the P-δ_{pl} curve. Using the Rice definition of J [9], in fact:

$$\begin{aligned} J_{pl} &= \eta \cdot \frac{A}{t(w-a)}; \\ A &= \int_0^{\delta_{pl}} P d\delta \\ &= C \int P^n dP \;\rightarrow\; C\,\frac{n}{(n+1)} P^{n+1} \end{aligned} \tag{6}$$

where w is the specimen width and C its 'plastic' compliance. The plastic component J_{pl} will overwhelm J_{el} more and more as Q assumes larger and larger negative values and the SSY conditions are lost, i.e., as the blunting increases. Eventually, when the stress field will no longer be described by J, as in the case of a RNB(T), the value of m will depend on the compliance of the system, i.e., on the notch blunting. The fact that increasing negative values of Q can affect the value of the m exponent is actually addressing the issue of triaxiality. The confinement of the plastic zone set-up at the notch tip and, therefore, of the process zone that determines the value of the Weibull exponent *m*, is, in fact, provided by the triaxial stress state existing ahead of the tip. The higher the triaxiality the smaller the plastic zone size. Let's consider the triaxiality factor, *TF*:

$$TF = \frac{\sqrt{2}}{3} \cdot \frac{\sigma_1 + \sigma_2 + \sigma_3}{\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}} \tag{7}$$

to asses the degree of triaxiality existing at the notch tip. The stresses σ_1, σ_2 and σ_3 in eq. (7) are the principal stresses on the notch plane. The maximum theoretical value of *TF* is infinity for $\sigma_1=\sigma_2=\sigma_3$, but in practice a full plane strain condition with $\sigma_1=\sigma_2$ and $\sigma_3=2\nu\sigma_1$, where ν is the Poisson's coefficient, results in a *TF* equal to about 2.8, that will be considered as the limiting value. FE calculations made by the authors, indicate that the *TF* for a CT specimen is equal to ~2.3 at the plastic-elastic border, with 2.5 at the crack tip, in the plastic zone. FE calculations, made by Milella, Bonora and Gentile [10] yielded a TF = 0.5 for ρ = 20 mm and 1.7 for ρ = 1.2 mm. Similar calculations presented in the paper by Milella e Bonora [11] indicate that for geometries like those used in Round Robin 1, the *TF* is 1.5 for a root radius ρ=2 mm, 1.2 for ρ=4 mm, 1 for ρ=6 mm and 0.9 for ρ=10 mm. This latter value is approaching the lowest one: TF = 0.33 pertaining to a smooth round bar in traction, RB(T), where $\sigma_2=\sigma_3=0$. In all cases the *TF* was computed at the center of the specimen section where triaxiality reaches its maximum. Finally, for a RNB(T) with a sharp notch of 0.2 mm, the TF is equal to 2.18 for an ideally elastic-plastic material. This is a rather high value very close to the one obtained for the CT specimen carrying a sharp crack. It must be pointed out that the round bar with the root radius equal to 0.2 mm was, in fact, the only one to fracture in a brittle fashion. If the same notch were present in a flat thin plate under uniaxial traction, where triaxility is much lower than in the RNB(T), the corresponding TF would drop sharply making the system behave in a completely different way to which it is associated a higher value of the *m* exponent. The same notch in a Charpy-VN specimen would yield a different TF. Since triaxiality introduces, as we have shown, a more general way to define a complex stress state, the original question of whether or not the root radius has an effect on the Weibull exponent *m*, now becomes whether the TF has an influence on *m*. Theoretical arguments presented so far have been checked against experimental results obtained on CT specimens, Charpy-VN, RNB(T) of different root radii and on smooth traction specimens, as well. As to the CT and in general to any specimen carrying a sharp or real crack, indeed there is not much to add to the results obtained by Wallin [7, 8] which clearly indicate that the Weibull exponent *m* is equal to 4. An experimental assessment of the *m* exponent for a notch having a tip radius ρ equal to 0.2 mm can be done using both Charpy-VN (ρ = 0.25 mm and *a* = 2 mm) or RNB(T) specimens. The results obtained by Milella on Charpy-VN specimens of A 508 Cl3 steel at −30 °C lead to a Weibull exponent *m* = 7.65. As to RNB(T) of root radius equal to 0.2 mm, the results obtained by the authors reanalyzing the tests performed by Beremin [12] on 22NiMoCr37 steel at −196 °C. The bar diameter was 14 mm and 7.5 mm in the reduced section. In their analysis of the experimental data, the authors have used eq. (4) in the evaluation of the failure probability distribution where the stress σ_r, as already explained, is the true failure stress on the reduced section of the specimen. The value of the exponent *m*, therefore, is inferred from the slope of the experimental data in a plot $ln\{ln[1/(1-P_f)]\}$,where P_f is frequency of failure, versus $ln\sigma_r$. A detailed description of the methodology can be found in ref. [10]. Using the same procedure, the authors have reanalyzed the experimental data available in the open literature relative to RNB(T) having notches of root radius equal to 1.2, 2, 2.4, 4, 6, 10, 20 mm and ∞, i.e. RB(T), [12, 13, 14]. The results are presented in table 1. Let's now plot *m* as versus the corresponding triaxiality factor TF. This is shown in figure 1 that clearly indicates the existence of a linear relationship between *m* and TF that can be expressed by the equation:

$$m = 55.4 - 22.4 \cdot TF \tag{9}$$

TABLE 1

m	ρ (mm)	Specimen	m	ρ (mm)	Specimen
4.69	crack	CT	44.66	2	RNB(T)
5.6	crack	CT	24.26	4	RNB(T)
5.44	0.2	RNB(T)	31.26	4	RNB(T)
7	0.2	RNB(T)	56.03	4	RNB(T)
7.6	0.25	CVN	30.22	6	RNB(T)
22	1.2	RNB(T)	38.37	10	RNB(T)
18.6	2	RNB(T)	40.99	10	RNB(T)
20.75	2	RNB(T)	44.73	20	RNB(T)
26.28	2	RNB(T)	46.29	∞	RB(T)

We have shown that the *m* exponent depends on the notch root radius ρ that determines the amplitude and shape of the stress fields generated at the notch tip and, more precisely, on the triaxiality existing in the cross section of the specimen. Therefore, we shall say that as long as temperature does not affect the stress field and triaxiality, it will not have any impact on the value of the Weibull exponent *m*. A detailed discussion of this conclusion can be found in reference [11]. Indeed, the analysis made by the authors on Charpy-VN specimens of A 533 B steel tested at temperatures equal to −75, −25, 0, 150 and 300 °C has shown that the *m* exponent was equal to 5.2 at −75 °C, 6 at −25 °C, 8 at 0 °C, jumping to 18 at 150 °C and even 23 at 300 °C, as expected. In general, we can say that, as long as temperature does not affect the triaxiality factor of a particular geometry, the Weibull exponent *m* will not be affected by temperature and this statement can be transferred to materials: as long as the change of material does not affect TF, *m* will be independent of the material used. The general statement therefore remains: equal *m* values will correspond to equal *TF*.

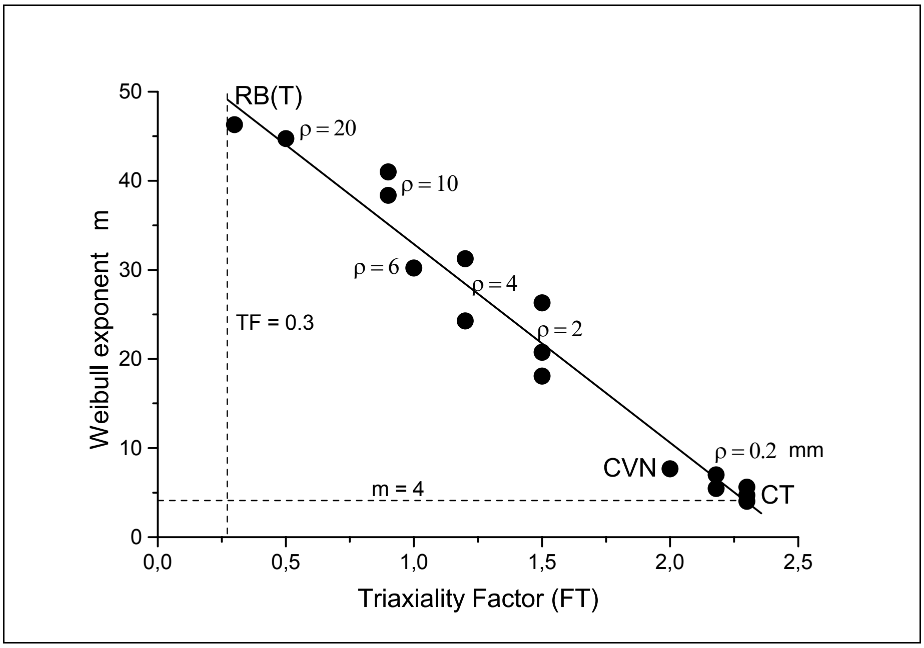

Figure 1 - Trend of the Weibull exponent *m* versus the triaxiality factor TF for the geometries considered. The vertical dashed line indicates the TF pertaining to a smooth round bar in traction RB(T). The horizontal one marks the value of *m* found by Wallin for specimens containing a sharp crack.

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