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NUMERICAL SIMULATIONS OF DYNAMIC INTERFACIAL FRACTURE PHENOMENA

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ABSTRACT

In this paper, first, to extract mixed-mode stress intensity factors for interfacial cracks subject to impact loading as well as for dynamically propagating interfacial cracks, the component separation method of the dynamic J integral is developed. This method is more advantageous than the M integral method often used for interfacial crack problems, since the present method requires no auxiliary solution field that is sometimes not possible to construct in the M integral method. Next, dynamic interfacial crack propagation is simulated by using a moving finite element method. The dynamic J integral, and the separated dynamic J integrals which have the physical significance of the energy flow rate from individual material side, and the stress intensity factors are evaluated.

KEYWORDS

Bimaterial, Interface Crack, Dynamic Crack Propagation, Dynamic J Integral, Separated Dynamic J Integrals, Moving Finite Element Method, Component Separation Method, Mixed-Mode Stress Intensity Factors

INTRODUCTION

In recent years, dynamic fracture mechanics for nonhomogeneous materials has been a focus of much attention because of the broad applications of composite materials and jointed materials to important structures. For dynamic fracture mechanics, Nishioka and Atluri [1] derived the path-independent dynamic J integral, which has the physical significance of energy release rate. Furthermore, for dynamic interfacial fracture mechanics, Nishioka and Yasin [2] have recently developed the separated dynamic J integrals, which are equivalent with the separated energy release rates from individual material sides.

Also, some fundamental features of dynamic interfacial cracks have been discussed by Yang et al. [3] and more recently by Shen and Nishioka [4]. These two aspects set up the important basis of current component separation method. In early works on extracting mixed-mode stress intensity factors for interfacial cracks, Yau and Wang's M integral method [5] is commonly used. However, it is sometimes difficult to set up the auxiliary solution field that is necessary in the application of their method. For some complicate conditions such as crack kinking and branching, it is hard to get the auxiliary solution. Recently, the component separation method [6] has been extended to static interfacial crack problems [7]. This method has great advantages over the M integral method, since no auxiliary solution field is needed.

In this paper, first, for dynamic interfacial fracture mechanics, the component separation method of the dynamic J integral is developed based on the theoretical studies of the interfacial crack tip field [3][4]. By choosing appropriate characteristic length and applying J-K relationship, explicit formulas for extracting the stress intensity factors from the dynamic J integral are derived. Next, numerical solutions of stress intensity factors are presented for dynamically propagating interfacial cracks. To cope with the propagating crack, a moving finite element method is used to comply with the dynamically moving property of crack tip.

DYNAMIC J INTEGRAL AND SEPARATED DYNAMIC J INTEGRALS

The well-known Eshelby-Rice static J integral has been widely used in static fracture mechanics. For dynamic fracture mechanics, Nishioka and Atluri [1] derived the path-independent dynamic J integral for a homogenous material as

$$J'_k = \lim_{\Gamma_\varepsilon \to 0} \int_{\Gamma_\varepsilon} [(W+K)n_k - t_i u_{i,k}] dS = \lim_{\Gamma_\varepsilon \to 0} \left\{ \int_{\Gamma+\Gamma_c} [(W+K)n_k - t_i u_{i,k}] dS + \int_{V_\Gamma - V_\varepsilon} [\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}] dV \right\}, \quad (1)$$

where W and K are the strain and kinetic energy densities. Γ_ε denotes the near field integral path, while Γ and Γ_c are the far field path and the crack face integral path, respectively (see Fig. 1(a)). The crack axis components of the dynamic J-integral J'^0_k can be obtained by the coordinate transformation: $J'^0_k = \alpha_{kl} J'_l$, where α_{kl} is the coordinate transformation tensor. The tangential component of the dynamic J integral J'^0_1 has the physical significance of the dynamic energy release rate G. Thus we have

$$J'^0_1 = J' = G. \quad (2)$$

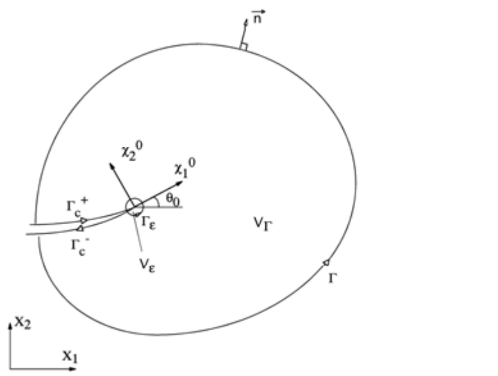
Considering a nonhomogeneous plate with a dynamically propagating interfacial crack as shown in Fig. 1(b), recently Nishioka and Yasin [2] have derived the separated dynamic J integrals as

$$J'^{(m)}_k = \lim_{\Gamma^{(m)}_\varepsilon \to 0} \int_{\Gamma_\varepsilon} [(W+K)n_k - t_i u_{i,k}] dS = \lim_{\Gamma^{(m)}_\varepsilon \to 0} \left\{ \int_{\Gamma^{(m)}+\Gamma^{(m)}_c+\Gamma^{(m)}_I} [(W+K)n_k - t_i u_{i,k}] dS + \int_{V^{(m)}_\Gamma - V^{(m)}_\varepsilon} [\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}] dV \right\} \quad (m=1,2), \quad (3)$$

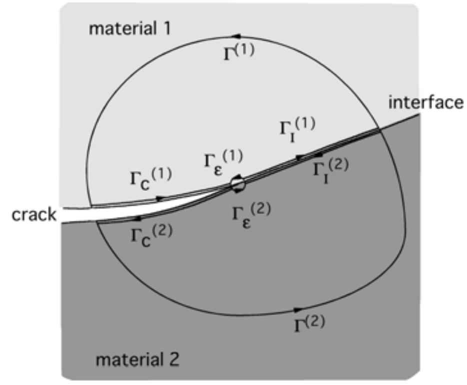
where $\Gamma_I^{(m)}$ (m=1,2) are the integral paths along the interface in the sides of the material 1 and 2. Also the crack axis components can be obtained by applying the coordinate transformation: $J'^{0(m)}_k = \alpha_{kl} J'^{(m)}_l$.

The separated dynamic J integrals also have the physical significance of the separated energy release rates $G^{(m)}$ (m=1,2) which are the energy flow rates from material m (m=1,2) into the propagating interfacial crack tip per unit crack extension. Thus, we have the following relations:

$$J'^{0(m)}_1 = G^{(m)} = J'^{(m)}_1 \cos\theta_0 + J'^{(m)}_2 \sin\theta_0, \qquad (m=1,2). \quad (4)$$



(a) Crack in homogeneous material



(b) Interface crack in nonhomogeneous material

Figure 1: Definition of integral paths

Furthermore, the dynamic J integral and the energy release rate can be obtained by the sum of the separated dynamic J integrals:

$$J'^{0}_{1} = J'^{0(1)}_{1} + J'^{0(2)}_{1} = G = G^{(1)} + G^{(2)}. \quad (5)$$

COMPONENT SEPARATION METHOD OF DYNAMIC J INTEGRAL

Recently the component separation method [6] was extended to static interfacial fracture mechanics [7]. Furthermore, in this paper, using the dynamic J integral and the separated dynamic J integrals, the component separation method is extended to evaluate dynamic stress intensity factors.

The dynamic J integral and the dynamic energy release rate can be related to the stress intensity factors [3][4] as follows:

$$J'^{0}_{1} = J'^{0(1)}_{1} + J'^{0(2)}_{1} = G = \Lambda\,(K_1^2+K_2^2) = \Lambda(1+\alpha^2)K_1^2, \quad (6)$$

and

$$\Lambda = \overline{\mathbf{w}}^T(\mathbf{H}+\overline{\mathbf{H}})\,\mathbf{w}\,(K_1^2+K_2^2)/(4\cosh^2\pi\varepsilon). \quad (7)$$

The eigenvector **w** is determined by the traction resolution factor η as

$$\mathbf{w}^T = \frac{1}{2}\{\ -i\eta\ \ 1\ \ 0\ \},\ \text{and}\ \eta = (H_{22}/H_{11})^{1/2}. \quad (8.a,b)$$

where H_{ij} (i,j=1,2) are the components of the in-plane matrix **H**. The matrix **H** consists of three independent real components ($H_{12}=H_{21}$)

$$\mathbf{H} = \begin{bmatrix} H_{11} & iH_{12} \\ -iH_{12} & H_{22} \end{bmatrix}, \quad (9)$$

and this matrix can be determined by the Stroh's matrix **B** as follows:

$$\mathbf{H} = \mathbf{B}^{(1)}+\mathbf{B}^{(2)},\ \text{and}\ \mathbf{B} = \frac{1}{\mu D}\begin{bmatrix} \beta_2(1-\beta_2^2) & i(1+\beta_2^2-2\beta_1\beta_2) \\ -i(1+\beta_2^2-2\beta_1\beta_2) & \beta_1(1-\beta_2^2) \end{bmatrix}. \quad (10.a,b)$$

The wave reduction factors β_i (i=1,2) and the Rayleigh wave function D have the form

$$\beta_1 = \sqrt{1-\rho C^2/C_{11}},\ \ \beta_2 = \sqrt{1-\rho C^2/C_{66}},\ \ D = 4\beta_1\beta_2 - (1+\beta_2^2)^2, \quad (11.a,b,c)$$

where C is the crack velocity while C_{11} and C_{66} are the elastic constants. The oscillation index ε is determined by the Dunders parameter β

$$\varepsilon = (1/2\pi)\left[\ln\{(1-\beta)/(1+\beta)\}\right],\ \text{and}\ \ \beta = -H_{12}(H_{11}H_{22})^{1/2}. \quad (12.a,b)$$

The crack opening displacements in the x and y directions behind the crack tip, δ_x and δ_y, are obtained in [3] [4] as

$$\sqrt{\frac{H_{11}}{H_{22}}}\,\delta_y + i\delta_x = \frac{2H_{11}(K_1+iK_2)}{(1+2i\varepsilon)\cosh(\pi\varepsilon)}\sqrt{\frac{r}{2\pi}}\left(\frac{r}{l}\right)^{i\varepsilon}, \quad (13)$$

where l is a characteristic length used to define the stress intensity factors. Usually l = 2a (2a is the entire crack length) is used.

From Eqn. 13, the ratio of the stress intensity factors can be related to the ratio of crack opening displacements, as follows:

$$\alpha = K_2 / K_1 = \lim_{r\to 0} (\eta - S\delta_y/\delta_x)/(\delta_y/\delta_x + \eta S),\ \ S = (\tan Q - 2\varepsilon)/(1 + 2\varepsilon \tan Q), \text{ and } \ Q = \varepsilon \ln(r/l). \quad (14.a,b,c)$$

Taking limit is required to calculate the stress intensity factors. However, it is hard to get accurate results by using numerical results due to the quantity S that has the logarithmic and oscillatory singular terms. For this reason, and to derive explicit formulas for the component separation method, we eliminate the logarithmic and oscillatory singular terms S (S=0), taking tanQ=2ε. This can be achieve, choosing the following special character length:

$$l = \bar{r}/e^{\varepsilon^{-1}\tan^{-1}(2\varepsilon)}, \quad (15)$$

where $\bar{r}$ is the location of the nodal point at which the crack opening displacemets are evaluated. Then the ratio of the stress intensity factors can accurately be evaluated by

$$\alpha=\eta\delta_x/\delta_y. \quad (16)$$

Using Eqn. 16 in Eqn. 6, the explicit formulas for the component separation method can be derived as

$$K_k = \delta_k\sqrt{J'^0_1 / \{\Lambda(\delta_1^2+\delta_2^2)\}} = \delta_k\sqrt{(J'^{0(1)}_1+J'^{0(2)}_1) / \{\Lambda(\delta_1^2+\delta_2^2)\}} = \delta_k\sqrt{G / \{\Lambda(\delta_1^2+\delta_2^2)\}},\ (k=1, 2) \quad (17)$$

where $\delta_1=\delta_y$ and $\delta_2=\eta\delta_x$ are defined as shown in Fig. 2.

The transformation to the stress intensity factors with the characteristic length l'=2a or to those with a desired characteristic length l' can simply be conducted by the following equation:

$$\begin{Bmatrix} K'_1 \\ K'_2 \end{Bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} K_1 \\ K_2 \end{Bmatrix},\ \ \theta = \varepsilon \ln(l'/l). \quad (18.a,b)$$

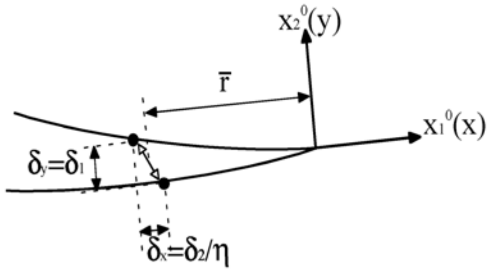
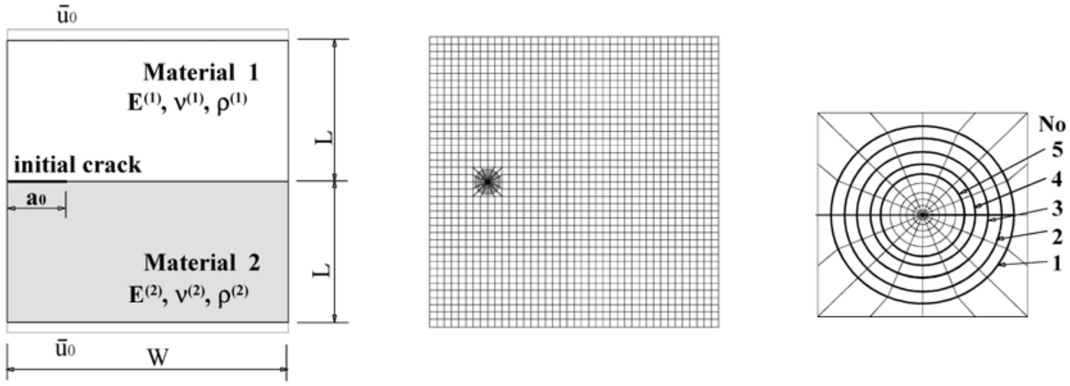


Figure 2: Crack open displacements

NUMERICAL SIMULATION RESULTS

Numerical simulations are carried out for dynamically propagating cracks. Figure 3 shows the computational model. The dimensions of the plate are W=2L=40mm. The initial mesh pattern for the moving finite element method and the dynamic J integral paths are also shown in Fig. 3. We assume that the upper material (material 1) is more compliant than the material 2. The mismatch ratio of Young's moduli is assumed as $\lambda=E^{(2)}/E^{(1)}=3.0$ with $E^{(1)}$=29.4GPa in this paper. The superscripts denote the material number. The mass densities and the Poisson’s ratios of two materials are assumed as the same as $\rho^{(2)}=\rho^{(1)}=2.45\times10^3$kg/m^3, $\nu^{(2)}=\nu^{(1)}=0.286$.

The moving finite element method [2] is used for calculating the separated dynamic J integral. Along with the crack propagating, the mesh pattern for the elements around the crack tip translates in each time step in



(a) Specimen geometry (b) Initial mesh pattern (c) Integral paths

Figure 3: Bimaterial plate with an interfacial edge crack

order to assure enough precision. The moving element method completely satisfies the boundary conditions near the propagating crack tip, while a fixed element method usually violates them. We assume that the crack always propagates along the interface and keeps a constant velocity C. The initial crack length is set as a_0=0.2W.

First, the dynamic finite element analysis of a propagating crack in the homogeneous plate (λ=1.0) subject to constrained displacements is carried out. The velocity is assumed as 40% of the shear wave velocity, C=0.4Cs. Using a fictitious interface along the symmetrical line, the separated dynamic J integrals and the dynamic J integral are evaluated. Both types of integrals show excellent path independence through out the simulation. Then, the dynamic J integral values are converted to the stress intensity factors using the component separation method (see Eqn. 17 with ε=0).

Figure 4 shows the variations of stress intensity factors. The K_1 values are normalized by the static stress intensity factor for a semi-infinite crack in an infinite-width strip [8]: $K_s = \bar{u}_0 E^{(1)} / \sqrt{L\{1-(\nu^{(1)})^2\}}$. The present results agree with the results obtained by moving singular element method [9,10] after certain amount of dynamic crack propagating, and agree with the Nilsson's steady-state solution [8]. Thus, this confirms the validity of the component separation method of the dynamic J integral when the problem reduces to the homogeneous condition (ε=0).

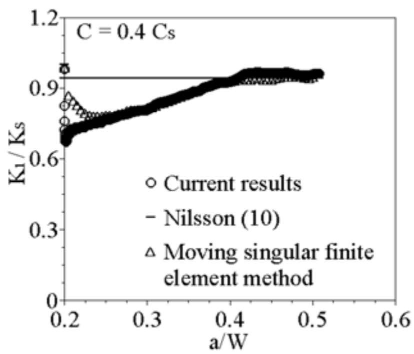
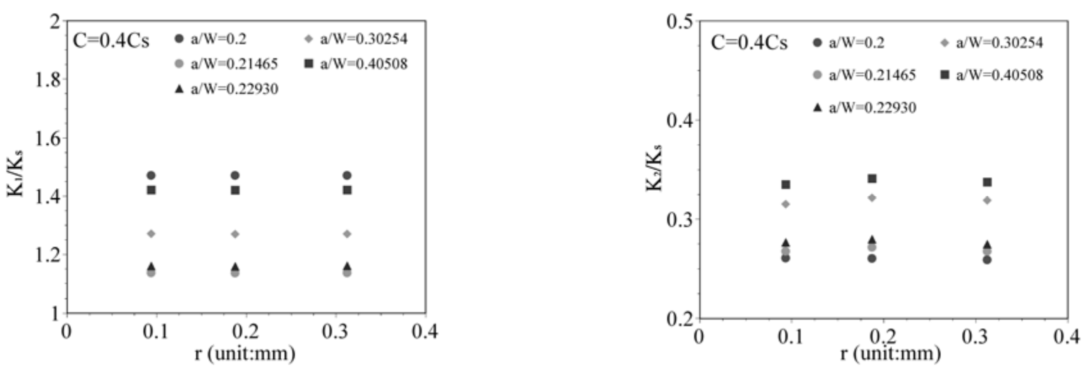
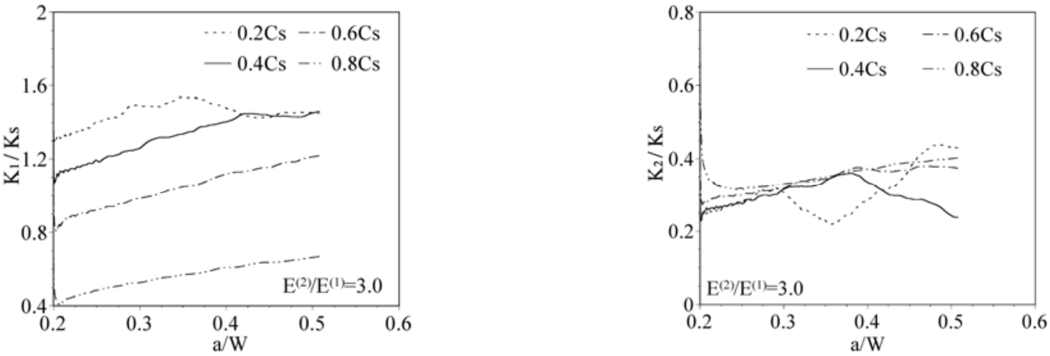


Figure 4: Normalized dynamic stress intensity factors in homogeneous plate



(a) Normalized K_1 (b) Normalized K_2

Figure 5: Stress intensity factors calculated for different crack face node pairs



(a) Variation of normalized K_1 (b) Variation of normalized K_2

Figure 6: Stress intensity factors for dynamically propagating interfacial cracks

Next, in order to check the accuracy of the component separation method, the crack opening displacements at three pairs of the corner-nodes nearest to the propagating crack tip are tested. Figure 5 shows the variations of the stress intensity factors for different node pairs. In all cases, the stress intensity factors for three node pairs are almost constant. This indicates that the component separation method gives very accurate stress intensity factors. Also, the crack opening displacements at the nearest corner node pair give accurate results.

The stress intensity factors for the dynamically propagating interfacial cracks are shown in Fig. 6. We can see that K_2 has a nonzero value in all cases due to the existence of material mismatch. It is interesting to see that, in the early stage of dynamic crack propagation, for increasing crack velocity, the normalized K_1 decreases while the normalized K_2 increases.

CONCLUSIONS

In this paper, first, in order to evaluate accurate mixed-mode stress intensity factors for dynamic interfacial cracks, explicit formulas for the component separation method of the dynamic J integral were derived. This method has more advantages than the M integral method that commonly used for interfacial fracture mechanics problems. The features of the component separation method can be summarized as follows: (1) It can be expressed by the explicit formulas. (2) It does not require any auxiliary solution field. (3) It is applicable using the path independent separated dynamic J integrals. (4) The signs of the stress intensity factors are automatically determined by the signs of the corresponding crack opening displacements. (5) Since its formulas do not include the oscillatory and logarithmic singular terms, the numerical results for the stress intensity factors are stable and accurate. In addition, the moving finite element method and the path-independent separated dynamic J integrals as well as the component separation method demonstrated their great potentials in dynamic interfacial fracture mechanics studies.

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