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NOTCH STRENGTH AND NOTCH SENSITIVITY OF POLYMETHYL METHACTYLATE

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Abstract: A formula to predict notch strength (σ_N) of polymethyl methacrylate (PMMA) under different stress concentration factors (K_t) from tensile properties of smooth specimens is given by analyzing the characteristics of stress and strain at notch and crack tip. The predicted values are in good agreement with test results. The curves derived from the formula mentioned above include three parts in different range of K_t. The upper level line controlled by ultimate tensile strength (σ_b) beginning at K_t=1 and ending at the notch sensitivity factor (K_N). The slope part of notch strength decreasing from the K_N to the sharp notch insensitivity factor (K_ρ) and the lower level line controlled by the fracture toughness beginning at the K_ρ. When $K_t > K_N$, the PMMA with the ratio $\sigma_N / \sigma_b < 1$ is sensitive to notch. It is shown that the oriented PMMA has less notch sensitivity than the non-oriented PMMA because of the higher ductility of the former.

Key words: polymethyl methacrylate, notch strength, notch sensitivity, fracture toughness, tensile properties

1. Introduction

PMMA is an important material used in airplane. The scratch induced in its service and the discontinuity of profile due to the need of joining and structure design always exists in elements of PMMA and may be regarded as notch where the stress concentration occurs. The smooth element can be considered as the special notch specimen with notch radius $\rho \to \infty$ and the element with crack can be considered as another special notch specimen with notch radius $\rho \to 0$. Therefore, the notch strength theory not only offers the base for the reliability design of engineering structural elements, but also points out the inner relation between the strength theory of materials and the theory of fracture mechanic. The variation pattern of notch strength of different metals and ceramics under tension, bending and torsion loading have been investigated respectively [1,2]. PMMA is low ductility material, so, it is necessary and important to investigate the fracture of notched PMMA in order to meet the structural design and reliability assessment of PMMA elements. Nevertheless, there is a scant amount of study in the open literature on the notch strength and notch-sensitivity of PMMA [3~5].

In the study, attempt is made to estimate the notch strength and notch sensitivity of notched PMMA from the tensile properties of smooth specimens and the estimated results are checked by the test results of two types of PMMA used in airplane.

2. Expression for notch strength

The following equation between tension notch strength σ_{bN} and stress concentration factor K_t in plane strain state was given in ref [6]. Based on the strain analysis at notch root and the normal strain fracture criterion,

$$\sigma_{bN} = {0.64\sqrt{E\sigma_f\varepsilon_f}}\Big/{K_t} \qquad (1)$$

where E is the Young's elastic modulus, $\varepsilon_f = -\ln(1-\Psi)$, $\sigma_f = \sigma_b(1+\Psi)$ are respectively the fracture strength and fracture ductility, where Ψ is the reduce ratio of the fracture section. PMMA is of low ductility material, when the thickness of the sheet specimen $B>2.5(K_{IC} / \sigma_{0.2})^2 \approx 3$mm, the notch root of the specimen is at the plane strain state, therefore, the notch can be estimated from the tensile properties of smooth specimen according to eqn(1).

Eqn(1) represents a straight line with a minus slope on a logarithmic scale, i.e., $\log K_t$ vs. $\log\sigma_N$ scale. In other words, σ_N decreases with the increasing K_t. However, for some engineering materials including PMMA [6~9], it is found that there is a critical stress concentration factor K_N called notch sensitivity factor, when $Kt<K_N$, σ_N is no less than the σ_b, when $Kt>K_N$, σ_N decreases with increasing Kt according to eqn(1). When Kt further increases with the decreasing radius of notch root ρ and reaches another critical value K_ρ, the notch strength does not further decreases with the increasing Kt and keeps at a value determined by the fracture toughness K_{IC}. The K_ρ is called the sharp notch non-sensitivity factor. Therefore, the complete expression for estimating notch strength can be rewritten in the form of eqn(2)

$$\sigma_N = \begin{cases} \sigma_b & K_t < K_N \\ \dfrac{0.64\sqrt{E\sigma_f\varepsilon_f}}{K_t} & K_N < K_t < K_\rho \\ \dfrac{K_{IC}}{Y\sqrt{\pi a}} & K_t > K_\rho \end{cases} \qquad (2)$$

Where Y is the geometry function for calculating stress intensity factor K_I , 2a is the crack length of Griffith crack, for circumferentially notched round-bar specimen under tension [10]:

$$Y\sqrt{\pi a} = \sqrt{\pi d/2} f(d,D) \qquad (2a)$$

$$f(d,D) = 0.5\sqrt{[1-(d/D)]}[1+0.5(d/D) + 0.375(d/D)^2 - 0.363(d/D)^3 + 0.731(d/D)^4] \qquad (2b)$$

Where D is the diameter of the smooth part of the notch specimen and d is the diameter of the notched section.

3. On the notch sensitivity factor K_N of PMMA

In early studies, the tension notch strength was measured by using notch specimen with a certain notch geometry and stress concentration factor. A parameter called the notch strength ratio ($NSR=\sigma_{bN}/\sigma_b$) was defined as the ratio of notch strength to ultimate strength to assess the notch sensitivity of materials. When $NSR\geq 1.0$, the material would be notch-ductile, when $NSR<1.0$, the material would be notch-brittle. In engineering applications, NSR could be used to qualitatively assess the safety of the structure details without crack-like defect. Nevertheless, NSR is obviously not a material constant. It's hard to predict the variation of notch strength and NSR values when the geometry and stress concentration factor changes. Consequently, let $K_N=NSR\times K_t$, the so-called notch sensitivity factor K_N, a new material constant is deduced for low-ductility materials like PMMA,

$$K_N = {0.64\sqrt{E\sigma_f\varepsilon_f}}\Big/{\sigma_b} \qquad (3)$$

When the stress concentration factor of element is greater than K_N, i.e. $K_b>K_N$, notch strength is lower than ultimate strength σ_b , so NSR<1.0, i.e., PMMA investigated is relatively sensitivity to notch.; when $K_b < K_N$ ， NSR≥1.0 and $\sigma_{bN} \geq \sigma_b$, PMMA is not sensitivity to notch.

4. On the sharp notch non-sensitivity factor K_ρ

According to fracture mechanic theory, the ideal radius at crack tip $\rho = 0$, but the crack tip in engineering materials can be blunted, and there is a critical blunted radius when the material element at crack tip fracture. As for the notch specimen, when the radius of notch root is smaller than a critical value ρ_c , the distribution characteristics of stress and strain near notch root is similar to that near the blunted area of crack tip. Thereby, the material element at notch root is under the approximately similar stress and strain state to that of material element at crack tip before fracture, therefore, the problem of notch becomes the problem of crack and there exist the following equation:

$$K_{IC} = \sigma_N \sqrt{\pi a} Y \tag{4}$$

For V-shape circumferentially notched cylindrical specimen,

$$K_{IC} = \sigma_N \sqrt{\pi d / 2} f(d, D) \tag{4a}$$

Combining eqn.(1), (5) and (5a), we can obtain the expression for K_ρ,

$$K_{IC} = \sigma_N \sqrt{\pi d / 2} f(d, D) \tag{5}$$

$$K_\rho = \frac{0.64\sqrt{\pi d E \sigma_f \varepsilon_f / 2}}{K_{IC}} f(d, D) \tag{5a}$$

As it may be seen, K_ρ is related with the geometry dimension of specimen and is not a material constant. However, the physical meaning for K_ρ is clear: if the notch depth and the dimension of the specimen keep constant, when $K_t \geq K_\rho$, the notch strength of the material keeps constant at the control of K_{IC}. The existence of K_ρ offers the theory base for investigating the problem of crack from the tensile properties of sharp notch specimen at certain condition.

5. Substantiate

For V shape circumferentially notched cylindrical specimen, the deformation at notch root is constricted and the plane strain state can be guaranteed, in this case, the following expression for stress concentration factor exists [11]:

$$K_t = (1.0 - 1.5183\lambda + 0.2530\lambda^2 + 2.2356\lambda^3 - 2.411\lambda^4) K_{tv} \tag{6}$$

$$K_{tv} = (1.035 + 0.0261\eta - 0.1451\eta^2 + 0.0842\eta^3) K_{te} \tag{6a}$$

$$K_{te} = (1.121 - 0.2846\eta + 0.3397\eta^2 - 0.5144\eta^3)(2 + \eta)/\eta \tag{6b}$$

$$\eta = \sqrt{\rho / t} < 1 \text{ , and } \lambda = 2t / D \tag{6c}$$

In order to obtain the high stress concentration factor, let $\lambda = 0.25$， then the notch depth $t = (D - d)/2 = 0.125D$.

The directed (type No: DP3) and non-directed (type No: NP3) PMMA sheet of 10mm depth used for airplane are taken as test material. The PMMA is first cut into the sheet of size of 10×10×100mm, and then accurately machined the sheet into round bar specimen with diameter of 8mm and scale length of 50mm in the middle part of the specimen. Finally, round V-shape notched specimens with constant depth of 1mm and variate notch radius are then made to obtain different stress concentration factor. The smooth round specimens are made with the scale size of 10×6×50mm.

The tensile tests are conducted on universal Electro-hydro-servo test machine at room temperature of 23℃. Take the tensile properties of smooth specimens listed in Table 1 into eqn.(3), then K_N are obtained and listed in Table 1 also. K_{IC} for NP3 and DP3 are respectively 1.3 and 2.3, which are obtained by testing the CT standard specimens under the same testing condition. K_ρ is then obtained by taking K_{IC} into eqn.(5a). So the two predicted horizontal lines for each type of PMMA are obtained and drawn in Fig 1 according to eqn.(2).

Tab. 1 Test results of tensile properties of smooth specimens

	NP3					DP3				
NO.	E /GPa	σ_b /MPa	σ_f /MPa	ε_f /%	K_N	E /GPa	σ_b /MPa	σ_f /MPa	ε_f /%	K_N
1	3.1	76.4	82.8	8.6	1.3	2.9	84.3	108	32.1	2.4
2	3.1	78.1	84.3	8.3	1.2	2.9	84.9	109	32.9	2.4
3	3.1	76.9	83.0	8.3	1.2	2.9	85.3	111	36.1	2. 6
4	3.1	77.3	82.7	7.2	1.1	2.9	87.0	114	37.7	2.6
5	3.1	77.8	84.1	8.4	1.2	2.9	87.3	114	36.5	2.6
average	3.1	77.3	83.3	8.2	1.2	2.9	85.7	111	35.1	2.52

Tab. 2 Test results of notch strength in tensile experiment

	NP3		（DP3） σ_N/MPa			
NO	K_t	σ_N/MPa	(K_t=2.2)	(K_t=2.9)	(K_t=2.7)	(K_t=5)
1	1.6	60.1	90.7	64.8	72.5	68.6
2	1.6	62.9	87.0	66.9	70.4	62.0
3	1.9	50.3	83.5	70.1	80.2	67.6
4	1.9	50.6	87.4	60.5	76.4	66.9
5	2.5	38.8	85.2	58.9	72.7	64.5
6	2.5	39.5	82.6	59.6	67.0	67.6
7	3.1	33.6	83.0	63.4		67.9
8	3.1	35.1	87.4	58.4		63.8
9	6.0	36.1	84.0	58.9		
10	6.0	34.0				

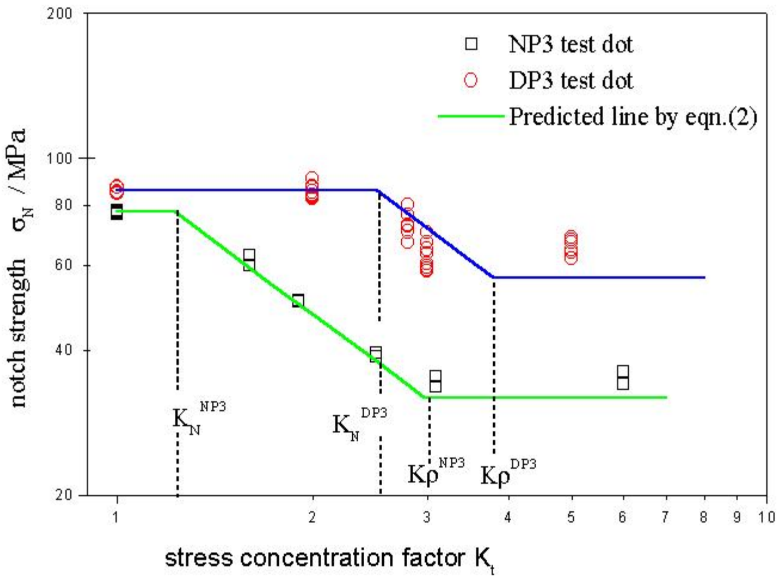

Fig.1 The predicted curve of notch strength and test results of Two kinds of PMMA

Fig 1 shows that the test data are in good agreement with the predicted values. The value of σ_b of the directed PMMA (DP3) is a little higher than that of non-directed PMMA (NP3), but the ductility of DP3 is much higher than that of NP3, therefore, the notch sensitivity factor for DP3 is much greater, K_N=2.5 for DP3 and K_N=1.2 for NP3. When Kt> K_ρ , the notch strength for sharp notch of DP3 is approximately two times of that of NP3 because K_{IC} of DP3 is nearly twice of that of NP3. The above estimated results suggest that DP3 is much safer than NP3 under the same stress concentration.

6. Conclusions

1. The notch strength curves ($\sigma_N \sim K_t$) of PMMA are consisted of three parts, i.e., the upper horizontal lines are controlled by the value of material strength, the lower horizontal lines are controlled by the value of fracture toughness of materials and the middle part of the curves are estimated by the formula of notch strength (eqn.2).
2. The value of K_N estimated by tensile properties was that 1.2 for NP3 and 2.5 for DP3. This values shown that the directed PMMA has less notch sensitivity than that of non-directed PMMA.
3. The sharp notch strength of the directed PMMA is held at higher value because of the action of fracture toughness with higher value.
4. With the same Kt, the notch strength of directed PMMA is much higher than that of non-directed PMMA.

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