

**NOTCH FRACTURE MECHANIC
A REVIEW OF FRACTURE CRITERIA USED IN ELASTIC AND ELASTOPLASTIC CASES**

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ABSTRACT

Notch fracture criteria are described in terms of stress, energy or strain parameters.

KEYWORDS

NOTCH, FRACTURE MECHANIC.

INTRODUCTION

The notch effect on fracture is characterised by the fact that the critical net stress acting on the ligament area below the notch is less than the ultimate strength of the material. This effect is very sensitive to the notch geometry which can be described by 3 parameters : the notch angle, notch radius and notch length. It is maximum for a crack (which is a particular case of notch with a notch radius and notch angle equal to zero) in a structure made with an elastic material.

The stress distribution at notch root in elastoplastic case can be divided into for zones (1) :

* Zone I : very close to the notch tip, the stress distribution increases to the maximum stress.

* Zone II : a transition between zone I et III.

* Zone III : in this zone the stress distribution decreases as a power function of the distance.

By analogy to purely elastic crack tip stress distribution, this part can be considered as a pseudo stress singularity. The stress distribution can be represented by the following relationship :

$$\sigma_{yy} = \frac{K_p}{(2\pi r)^\alpha} \quad (1)$$

where K_p is the so-called notch stress intensity factor and α a power exponent. The limit between zone II and zone III is called the effective distance X_{ef} and it has been shown that it correspond to the limit of the fracture process zone (2).

* Zone IV far from notch tip, the stress value trends to reach the net stress value.

The mechanism of fracture emanating from notch or crack is fundamentally different from the traditional “hot spot” approach (i.e. fracture occurs at the point of maximum stress). It is well known that it needs a fracture process volume. In this volume, the effective stress or fracture

stress can be considered as an average stress which takes into account the stress distribution and the stress gradient. This approach is called "fracture volumetric approach" and can be used with, stress energetic or strain parameters.

LOCAL FRACTURE CRITERION FOR FRACTURE EMANATING FROM NOTCHES

In order to get the fracture effective stress, we have to take into account stress value and the stress gradient in the neighbouring of any point in the fracture process volume. This volume is assumed to be quasi-cylindrical by analogy with notch plastic zone which has a similar shape. The diameter of this cylinder is called the effective distance. The stress value in any point inside the process zone is weighted in order to take into account the distance from notch tip and the relative stress gradient. Fracture stress can be estimated by some average value of the weighted stresses.

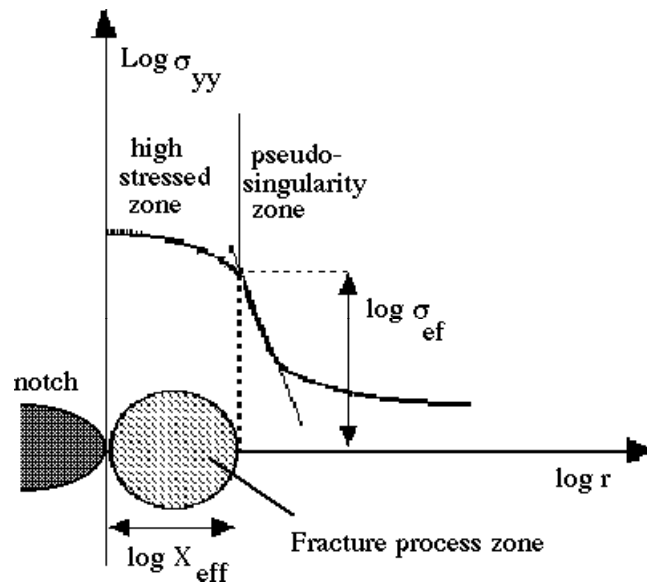


Figure 1 : Schematic presentation of a local stress criterion for fracture emanating from notches.

This leads to a local stress fracture criterion with two parameters : the effective distance X_{ef} and the effective stress σ_{ef} . A graphical representation of this local stress fracture criterion is provided in Figure 2 where the logarithm of the stress normal to the notch plane is plotted versus the logarithm of distance, effective stress and distance are presented. A graphical procedure for determination of X_{ef} has been proposed by [2]. It has been seen that the effective distance is connecting to the minimum of relative stress gradient χ defined by :

$$\chi = \frac{1}{\sigma_{yy}} \cdot \frac{d\sigma_{yy}}{dr} \quad (2)$$

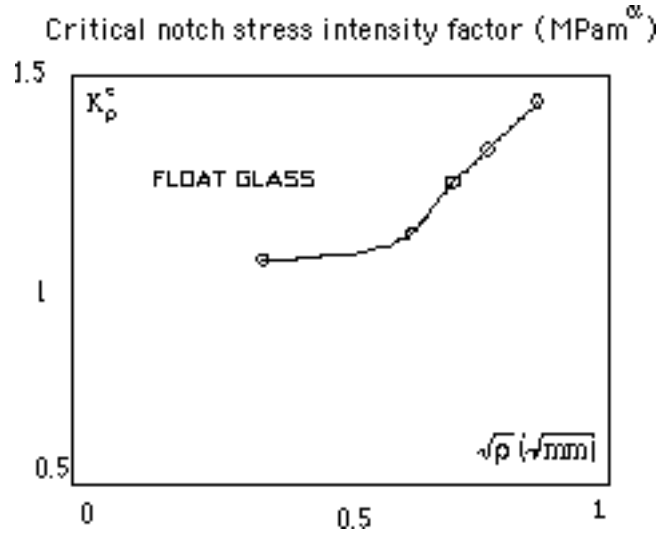


Figure 2 : Evolution of critical notch stress intensity factor versus the square root of notch radius for Float Glass.

The effective stress is defined as the average of the weighted stress inside the fracture process zone

$$\sigma_{ef} = \frac{1}{X_{ef}} \cdot \int_0^{X_{ef}} \widehat{\sigma}_{ij} dx \quad (3)$$

where the weighted stress is given by :

$$\widehat{\sigma}_{ij} = \sigma_{ij} \cdot (1 - \chi x) \quad (4)$$

The fracture criterion is a two parameter criterion. For a CrMoV steel with fine carbides (FC) microstructure (Yield stress 771 MPa), the mean value of the effective stress is 1223 MPa which can be compare to the average maximum local stress at fracture σ_{max} of 1310 MPa. Critical notch stress intensity factor can be used as a maesure of fracture toughness as it can be seen for Float Glass in figure 2.

ENERGETIC FRACTURE CRITERIA FOR NOTCHED COMPONENTS

Influence of notch radius on critical value of energetic parameter J

For a non-linear behaviour, two energy based fracture criteria can be used: the critical non-linear energy release rate of Liebowitz [3] (equ 5), and the energy parameter J of Turner [4] (equ 6).

$$\widetilde{G}_{Ic} = -\frac{1}{B} \cdot \frac{\partial U_{nl}}{\partial a} \quad (5)$$

$$J_{Ic} = \frac{\eta \cdot U_{nl}^c}{B \cdot b} \quad (6)$$

with $b = W - a$ and U_{nl}^c the non-linear work done for fracture. Assuming that $J_{Ic} = \widetilde{G}_{Ic}$ we have :

$$\eta = -\frac{\partial \ln U_{nl}}{\partial a} \quad (7)$$

From this formula we get the evolution of η , as a function of depth and notch root radius :

$$\eta = \eta(a, \rho) \quad (8)$$

The evolution of the η factor as a function of notch root radius shows an absolute minimum with the abscissa ρ_c (figure 3). Similarly, we notice that for radius values below ρ_c , the η factor decreases linearly with ρ . Beyond this critical abscissa, the η factor increases with ρ and becomes approximately constant for a radius ranging between 1.54 and 2 mm. The

difference between the η factor for cracks and notches with the same length can reach 36% of the latter one, which is important and justifies the present approach.

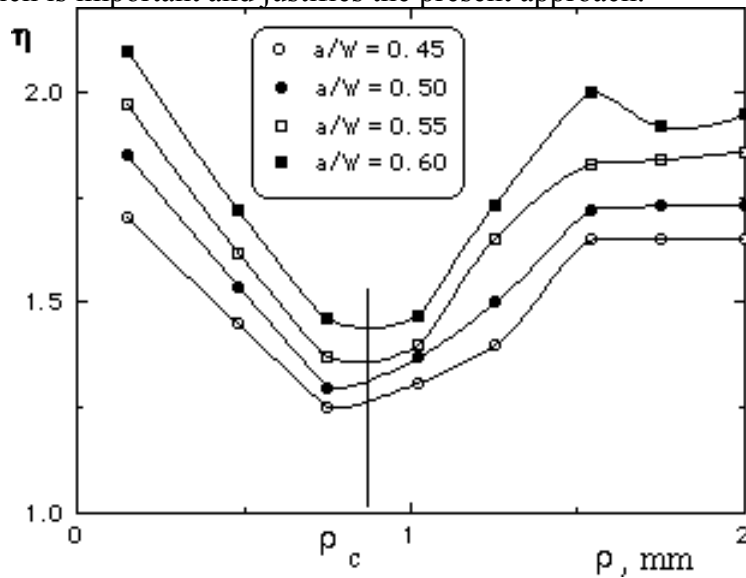


Figure 3 : Evolution of η with notch radius for constant relative depth Charpy U notch specimen.

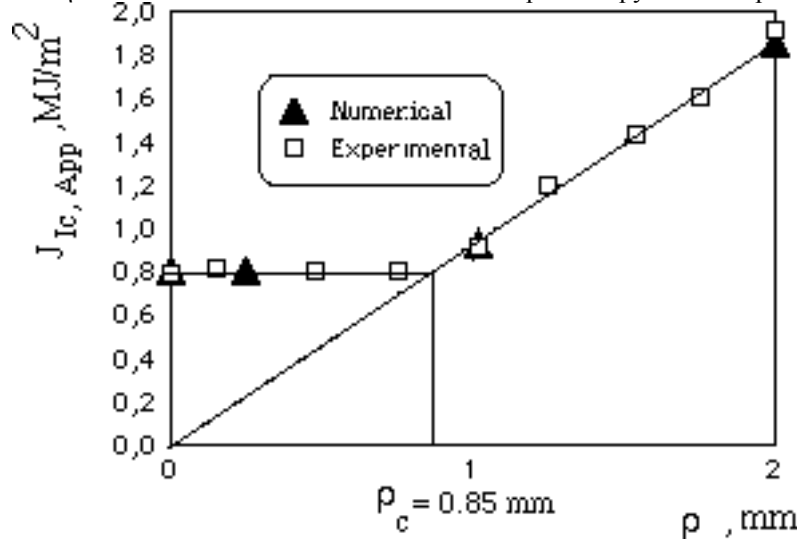


Figure 4 : $J_{Ic,App}$ versus notch radius for Charpy U notch specimen in steel .

In the following, we will call the fracture toughness given from notched specimen “ apparent fracture toughness ” written $J_{Ic,App}$, J_{Ic} is defined only for cracked specimens.

$$J_{Ic,App} = \frac{\eta(a, \rho) \cdot U_{nl}^c(a, \rho)}{B \cdot b} \quad (9)$$

An example of evolution of the apparent fracture toughness versus notch radius is given in figure 4. We can notice that for radii less than a critical value, ρ has no influence on fracture toughness, but for radii beyond this critical value $J_{Ic,App}$ increases linearly.

Strain energy density at notch tip

If we plot the strain energy density versus the notch tip distance in a bilogarithmic graph, we get the following distribution presented in figure 5.

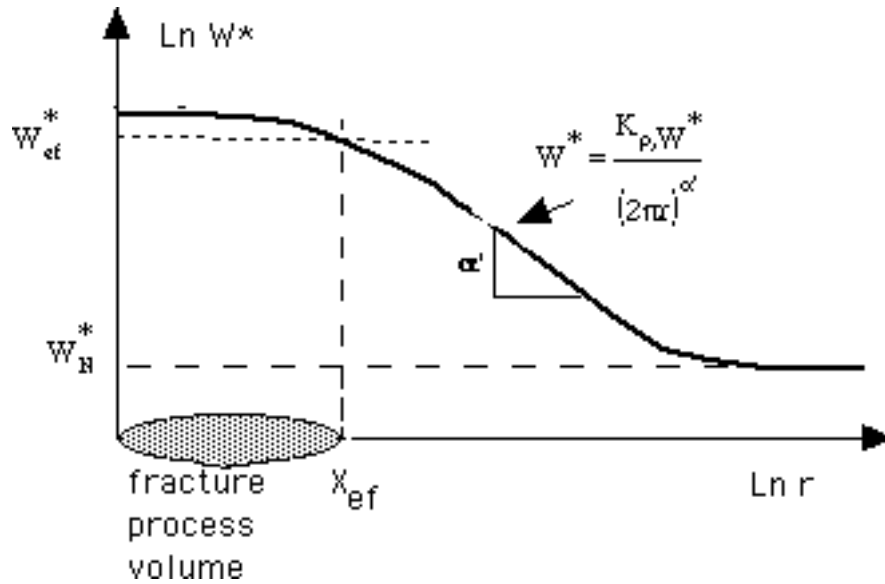


Figure 5 : Distribution of the strain energy density at notch tip

This distribution can be characterised by four parameters:

- W^*_{ef} : the effective strain energy density at notch tip,
- X'_{ef} : the effective distance,
- α' : the slope of the linear part of the strain energy density distribution,
- W^*_N : the net strain energy density.

The strain energy density notch intensity factor has been defined in the area of the "pseudo strain energy density singularities" of the notch in figure 5. This distribution can be considered only for a distance greater than X'_m defined on this figure with the following form :

$$W^* = \frac{K_{p,W^*}}{(2\pi r)^{\alpha'}} \text{ for } r \geq X'_{ef} \quad (10)$$

The strain energy density W^* at notch tip is a mechanical parameter which can be used as fracture criterion. For fracture, the average critical strain energy density W^*_{ef} in the fracture process volume can be used as a local energetic criteria. Fracture occurs when :

$$W^*_{ef} = W^*_c \quad (11)$$

LOCAL STRAIN FRACTURE CRITERION

Strain distribution can be presented similarly that stress distribution in a bilogarithmic graph (figure 6).

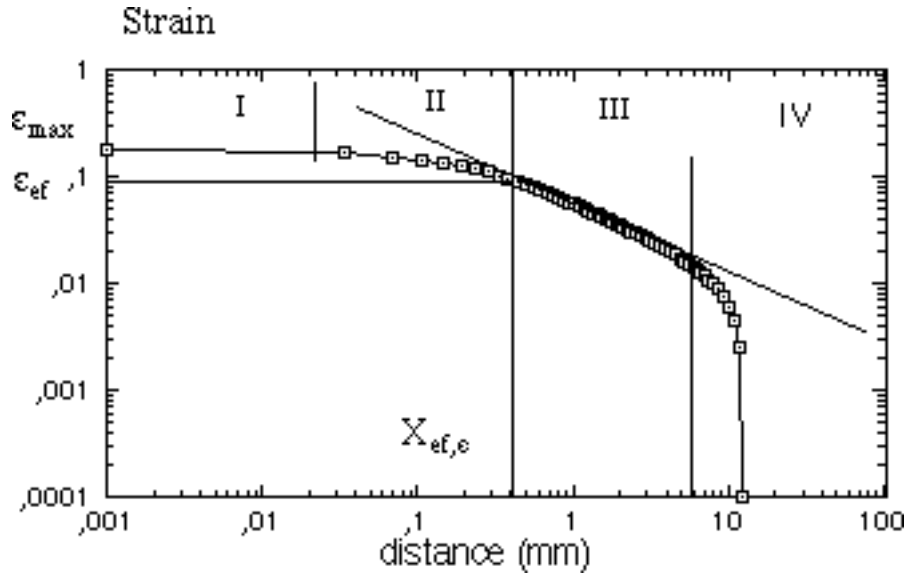


Figure 6 : Exempla of the elastoplastic strain distribution at the notch tip presented in a bilogarithmic graph.

Zone III which can be assimilated to a zone of train pseudo-singularity. In this area the strain-distance relationship has the following form

$$\varepsilon_{yy} = \frac{k_{\rho,\varepsilon}}{(2\pi r)^{\alpha''}} \quad (12)$$

A local strain fracture criterion is based also on the concept of fracture volume process which has been described in the case of local stress fracture criterion . The limit of this fracture process is also the beginning of the strain pseudo singularity which has for abscissa $X_{ef,\varepsilon}$. For the critical event the strain for this abscissa is the critical effective stress $\varepsilon_{ef,c}$.

The product $\varepsilon_{ef,\varepsilon}^c (2\pi \cdot X_{ef,\varepsilon}^c)^{\alpha''}$ is precisely the critical notch strain intensity factor which can be taken as a measure of the fracture toughness.

$$K_{\rho,\varepsilon}^c = \varepsilon_{ef,\varepsilon}^c (2\pi \cdot X_{ef,\varepsilon}^c)^{\alpha''} \quad (13)$$

In other words fracture occurs when the notch strain intensity factor reaches a critical value :

$$K_{\rho,\varepsilon} = K_{\rho,\varepsilon}^c \quad (14)$$

the Notch Ductility Factor (NDF) differs from the critical strain intensity factor by a constant

$$NDF = \varepsilon_{ef}^c \left(X_{ef,\varepsilon}^c \right)^{\alpha''} = \frac{k_{\rho,\varepsilon}^c}{(2\pi)^{\alpha''}} \quad (15)$$

An exempla of evolution of notch ductility factor measured on SENT specimen made in steel is presented in figure 7.

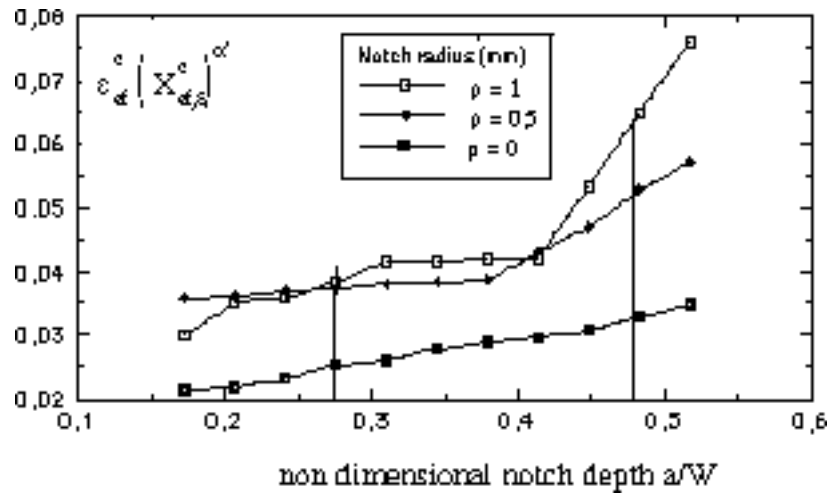


Figure 7. : Evolution of the Notch Ductility Factor with notch radius and non dimensional notch depth.

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