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NEW CONSIDERATIONS AND RESULTS ON CRACK SEPARATION ENERGY RATES IN ELASTIC-PLASTIC FRACTURE MECHANICS

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ABSTRACT

The definition of an energy release rate in elastic-plastic fracture mechanics, denoted by G_P, is proposed, and can be related to the parameter G^{Δ} proposed by Kfouri & Miller in 1976. New results obtained with this parameter and new considerations related to the well known « Paradox of Rice » are presented. In particular we find that this parameter is not zero if we consider the case of an elastoplastic material with a linear isotropic hardening. But it is necessary to consider a very fine mesh and very small crack propagations. Some applications are briefly presented in two cases where the J-approach is not valid.

KEYWORDS

Crack separation rate, Energy release rate, J-approach, Energy approach, Rice's paradox

INTRODUCTION

The J-approach is very limited and cannot be applied as soon as the loading become non proportional. It is necessary to develop other approaches as the two parameters approach or the local approach. Our choice is to reconsider the energetic approach where many parameters called « path independent integrals » have been proposed, without any consensus. We recall the definition of an energy rate in an elastoplastic media and we show that in the case of a stationary crack in an elastoplastic material with linear isotropic hardening, this parameter is not zero, in contradiction with the Paradox of Rice. Then it is possible to use it as a fracture parameter, and we will present two applications where the J- approach is not valid : 1/ the first one concerns the problem of unloading, 2/ the second one concerns the problem of the shallow crack effect.

DEFINITION OF AN ENERGY RELEASE RATE IN AN ELASTOPLASTIC MEDIUM

Brittle fracture in an elastic medium

The Griffith's criterion is widely used to predict whether a crack propagates or not in an elastic medium, considering only progressive and continuous crack propagation. But this approach cannot be used to predict crack initiation in a non-cracked medium, or the discontinuous propagation of a crack. This is the reason why Francfort and Marigo [1] have proposed a new theory where these two phenomena can be predicted. In this theory, we consider a discretisation of the load history where the true evolution of the structure during a load increment is taken into account only through the state of the structure (displacement field U and cracks positions S) at the beginning and at the end of the load increment. Let's call E the energy defined by :

$$\mathrm{E}\,(U, \Delta S) = \mathrm{E_D}(U) + \mathrm{G_c}\ \mathrm{area}(\Delta S)$$

where $\mathrm{E_D}$ is the strain energy, $\mathrm{G_c}$ the toughness and ΔS the newly created surface during the load increment, and Francfort and Marigo postulate that U and ΔS realise the minimum of E.

At this stage two remarks can be made :

- in case of progressive crack propagation, Griffith criterion can be retrieved by restricting ΔS (Δl in 2D) to a sufficiently small propagation dS (dl in 2D) and, if $\mathrm{W_e}$ is the potential energy of the structure, we can define an energy release rate $\mathrm{G_{el}}$ as :

$$\mathrm{G_{el}}(dS) \quad = \quad -\frac{\mathrm{W_e}(dS) - \mathrm{W_e}(\varnothing)}{\mathrm{Aire}(dS)} \quad < \quad G_c$$

- the minimisation principle can be linked to incremental formulations with global internal variables as soon as the first term (elastic energy) along with the kinematic admissibility conditions are identified as the free Helmhotz' energy F and the second term (the energy dissipated when the crack propagates) is identified as the dissipation potential D, see [2].

Brittle fracture in an elastoplastic medium

Exploiting this link, we extend Francfort and Marigo theory in the case of an elastoplastic material by introducing the energetic contribution due to plasticity into the free Helmhotz' energy and the dissipation potential, assuming that fracture mechanisms and plasticity are independent, see [3]. The energy available for propagation is called W, and an energy rate G_P can be defined for a sufficiently small propagation dS, as :

$$\mathrm{G}_p(dS) \quad = \quad -\frac{\mathrm{W}(dS) - \mathrm{W}(\varnothing)}{\mathrm{Aire}(dS)} \quad < \quad G_c$$

The $\mathbf{G}^{\Delta}$ parameter proposed by Kfouri-Miller

The G_P parameter represents the energy available in the structure to obtain a dS propagation, divided by dS. In 1966, Rice considered a continuously growing crack in an elastoplastic material where the flow strength saturates to a finite value at large strain, and demonstrated that this value must be zero, see [4]. This is the « Paradox of Rice ». This result cannot be applied to a stationary crack or to a non bounded flow strength material, but in 1976, Kfouri and Miller, see [5], considered this case and they found a result in agreement with the Paradox of Rice. They proposed a parameter called G^{Δ} defined as ΔW/Δa, ΔW being the work released during a small amount Δa of the crack, and found : G^{Δ} equal 0 when Δa goes to zero. Although the G_P definition is more general, it can be proved that in the case of a 2D elastoplastic media and for a sufficiently small propagation Δa, G_P is equal to G^{Δ}. In 1977, Rice used this result to generalise his paradox, see [6], and now it seems that it is a general and well accepted result. Nevertheless, if we reconsider it 25 years after, the numerical aspects of the modelisation used by Kfouri and Miller seem to be insufficient.

ANALYSIS OF THE G_P DEPENDANCE WITH RESPECT TO Δl

Definition of the problem

Let us consider a Centered Cracked Plate submitted to an increasing loading in mode I. The data related to geometry and material are presented on **Fig. 1**, and the mesh and a zoom of the mesh on the crack tip area are presented on **Fig. 2** and **3**. Due to symmetries only a quarter of the structure is represented, and the plane strain hypothesis is assumed. Different values of the size « Δl » of the element located at the crack tip and corresponding to different meshes are investigated as follows :

- M1 : mesh for Δl = 0.0500 mm, M2 : mesh for Δl = 0.0125 mm,
- M3 : mesh for Δl = 0.0062 mm, M4 : mesh for Δl = 0.0025 mm,
- M5 : mesh for Δl = 0.0012 mm

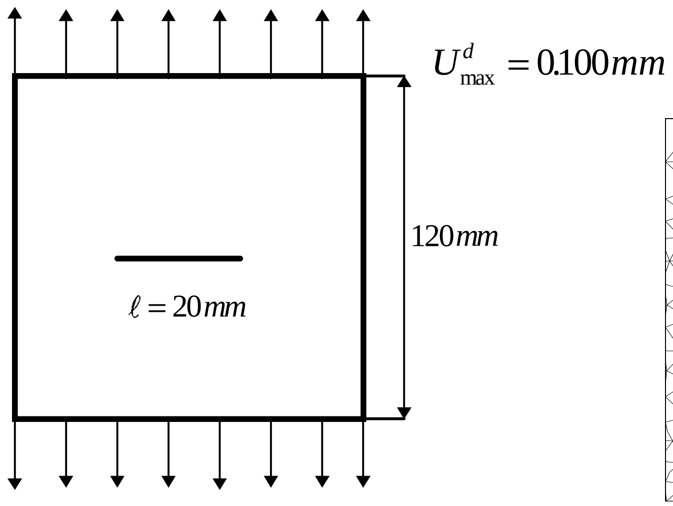
We consider 2 values (one low value, one high value) for the maximum loading :

- U_{max} = 0.016 mm and : U_{max} = 0.100 mm

For each loading and for each mesh we make the computation in 10 steps. After that and for each step, one element is released to obtain the propagation of the crack.

$E = 200\,000$ MPa $\nu = 0{,}3$

$\sigma^y = 480$ MPa $h = 5\,000$ MPa



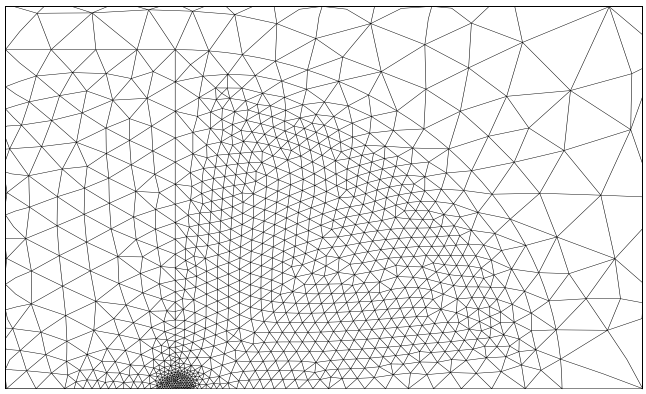


FIG. 1 - Geometry, material **FIG. 2 - Zoom of the mesh in the crack tip area**

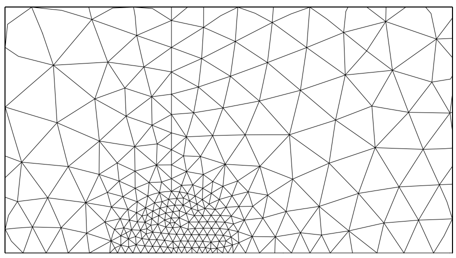


FIG. 3 - Zoom of the mesh near the crack tip

Results obtained

On **Fig. 4** the variation of G_P as a function of the loading is presented for the 4 meshes M2, M3, M4 and M5 and for the applied loading corresponding to the largest maximum value : Ud = 0.100 mm. We can see that the G_P value steadily increases while the loading is increasing, and that the values obtained for the 4 meshes seem to converge to a non zero value. In fact the curve obtained with the mesh M5 is very close to the curve extrapolated with all the results in order to obtain the result corresponding to Δl equal 0.

On **Fig. 5** the variation of G_P as a function of the loading is presented for the 4 meshes M2, M3, M4 and M5 and for the applied loading corresponding to the smallest maximum value : Ud = 0.016 mm. We can see that the G_P value steadily increases while the loading is increasing, but that the values obtained for the 4 meshes are mesh dependent (or Δl dependent) and seem to converge to a zero value.

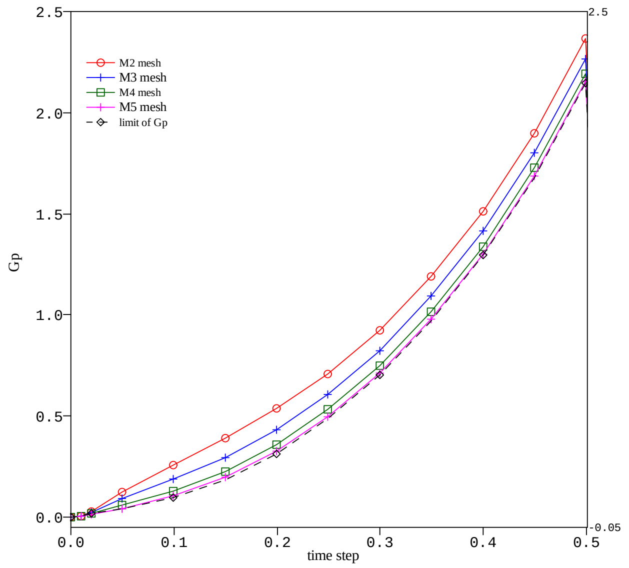


FIG. 4 - Results for different meshes : U = 0.100 mm

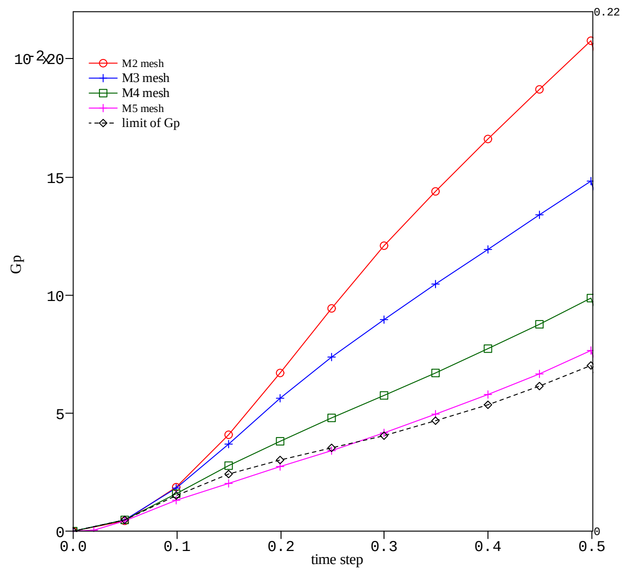


FIG. 5 - Results for different meshes : U = 0.016 mm

Explanation : a very important parameter has to be defined in order to understand these results : Np = Rp/Δl (Rp being the radius of the plastic zone). Obviously, the value of this parameter must be sufficiently high, and because the radius Rp is increasing from zero with the loading, it will be impossible to obtain a precise result on G_p if the loading is very low. That means that the value of Δl (even very low but fixed), will always be too high compared to the loading or to the radius Rp. We can verify this point on **Fig. 6** (zoom of **Fig. 5** for very low loading) where the different curves « Numerical G_p » are distributed, with respect to Δl, between the low curve « Theoretical G_p » (low, but not zero) and the high curve « G-elastic (or J) ». These curves are located in a very large area and we can conclude that the result obtained for a very low loading cannot be precise. We will always have an apparent dependence of the result with respect to Δl. Of course if the loading is higher, this phenomenon disappears and a precise result can be obtained. But for that it is necessary to consider a very fine mesh with very low values of Δl.

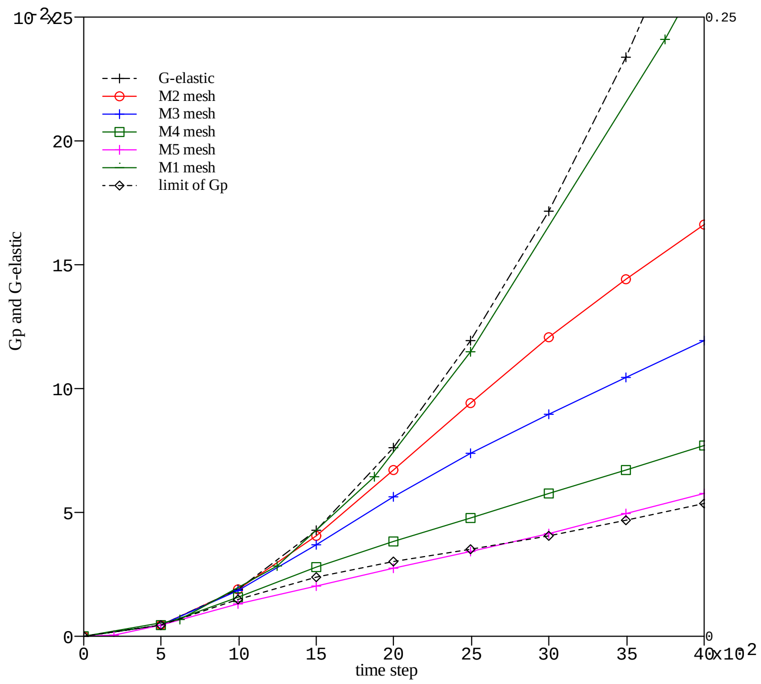


FIG. 6 - Zoom of figure 5

TWO APPLICATIONS WHERE THE J-APPROACH IS NOT VALID

The case of a structure submitted to loading and unloading

Let us consider again the case of the Centred Cracked Plate. Now it is submitted to a loading that is first increasing and then decreasing to zero. As soon as the loading decreases, it becomes non proportional, and the J-approach cannot be applied. On **Fig. 7** the variation of G_P, as a function of the loading, is presented for the applied loading corresponding to different maximum values of the loading. We can see that the G_P value steadily increases while the loading is increasing, and afterwards this value falls down suddenly to zero while the loading is decreasing. Then the G_P value stabilises at this zero value corresponding to the « closure » of the crack tip, when unilateral conditions are taken into account.

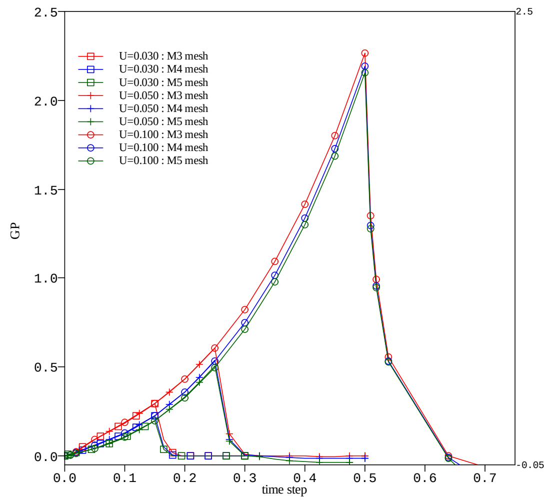


FIG. 7 - Increasing and decreasing of Gp as function of the loading

The variation of the G_P parameter during unloading is very interesting and it is possible to take advantage of that. Indeed, $G_P = 0$ means that there is no energy available in the structure to make the crack propagate. So the conclusion is that in such a case the initiation of the crack is impossible.

The case of the shallow crack effect

The toughness of a material is determined from a test on a CT specimen with a large crack. If we carry out another test on the same material but on a different specimen with a small crack, we find that the toughness is much higher. This is called « the shallow crack effect », and we would like to apply the G_P parameter to the interpretation of this effect. For that, let us consider two SENB specimens with different crack lengths.

Definition of the problem : 1/ Geometries of the SENB : Width : W = 50.mm, Height : H = 420.mm, Length of the crack A : A/W = 0.5 for the large crack, A/W = 0.05 for the small crack. 2/ Material of the SENB : the specimens are made in an A508 forging steel, Young modulus : E = 173528 MPa, Poisson's ratio : $\nu = 0.3$, Yield limit : $\sigma y = 617.8$ MPa, Hardening modulus : H = 1922.6 MPa. The toughness corresponds to a critical value of J, Jc = 42.kN/m in the case of A/W = 0.5 and to Jc = 88.kN/m in the case of A/W = 0.05. Plane strain hypothesis is assumed, and, due to symmetries, only one half of the structure is represented. The different meshes are equivalent to those presented for the CCP specimen.

Results obtained : On **Fig. 8** the variation of J as a function of G_P is presented for the two specimens. We can see that the two curves are very close. So, we can conclude that the G_P parameter seems to be equivalent to J, that is to say it is not able to predict the shallow crack effect. But, as we observe a sudden crack propagation (cleavage fracture), we are now looking at the G^{Δ} value, for finite values of Δl. On **Fig. 9** the variation of J as a function of G^{Δ} is presented for the two specimens, only in the case Δl = 0.200 mm, representative of the effect obtained for large values of Δl. We can see that the curve corresponding to A/W = 0.05 is higher than the curve corresponding to A/W = 0.5. The value : Jc(A/W=0.5) = 42.kN/m, corresponds to the value G^{Δ} = 10.5 kN/m, and to the value Jc(A/W=0.05) = 75. kN/m (to be compared to 88. kN/m). Therefore we can conclude that the G^{Δ} parameter seems to be able to predict the shallow crack effect.

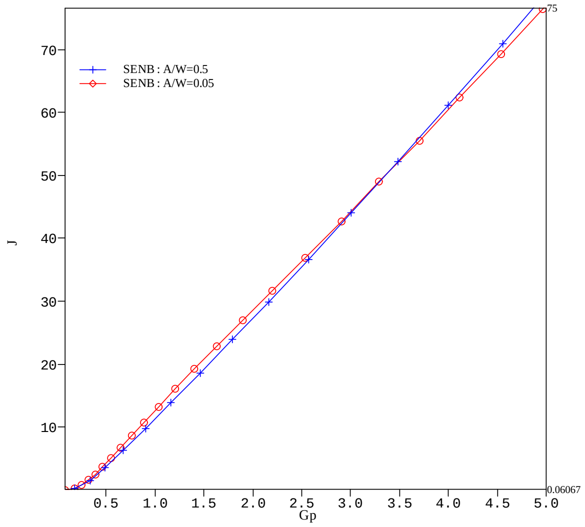


FIG. 8 - J as function of the Gp parameter

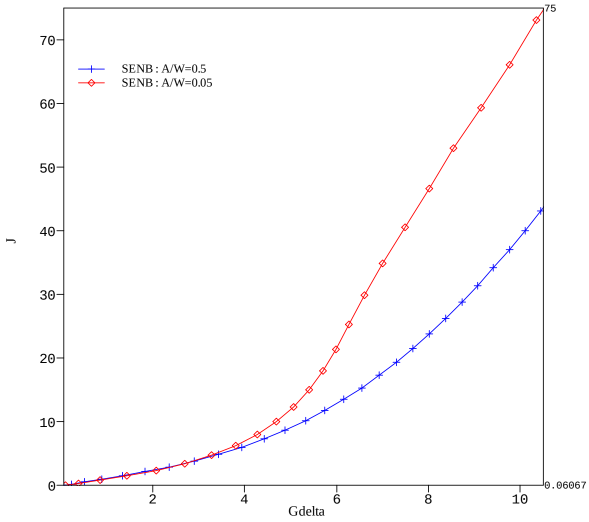


FIG. 9 - J as function of G^{Δ} (Δl = 0.2 mm)

CONCLUSION

An energy release rate G_P has been defined for an elastoplastic material, starting from the elastic fracture theory of Francfort and Marigo. This parameter can be related to the parameter proposed by Kfouri and Miller in 1976. Considering a stationary crack in an elastoplastic material with linear and isotropic hardening, we have obtained the following new results :

- the G_P values are increasing with the loading and if we consider the results obtained for high values of the applied loading, the G_P values tend clearly to a finite value when Δl goes to zero,
- if we consider the results obtained for very low values of the applied loading, these results are necessary mesh dependent, or Δl dependent, and it seems that these values tend to zero when Δl goes to zero,
- this apparent dependence could be explained if we consider the parameter Rp/ Δl corresponding to the mesh refinement in the plastic zone area, which must be sufficiently high,
- the G_P parameter can be used to analyse the case of a structure submitted to a loading that is first increasing and then decreasing ; in particular when $G_P = 0$, the initiation of the crack is impossible,
- an analysis of the shallow crack effect reveals that the G_P parameter is equivalent to J, and cannot explain this effect, but the G^{Δ} parameter corresponding to large crack propagation ($\Delta l > 0.2$ mm) gives a result in agreement with the experimental one.

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