MODELLING OF TRANSVERSELY ISOTROPIC BLISTER TEST SPECIMEN

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ABSTRACT

In this study, a transversely isotropic elastic layer bonded to a rigid substrate is considered, with the intent of modelling the blister test. The motivation for the study is to incorporate the effect of anisotropy of the layer on the energy release rate associated with the debonding of the layer. A linear elasticity approach is adopted rather than plate theory so the results will be applicable to relatively thick films. In addition to being a model for the blister test, the assumed geometry and loading can also represent the problem of a composite cover plate. The problem is reduced to the solution of a system of singular integral equations of the second kind by using Hankel transforms. These equations are solved for sample cases and numerical results for energy release rates are given.

KEYWORDS

Blister Test, Transversely Isotropic, Energy Release Rate, Singular Integral Equation

INTRODUCTION

Structures consisting of two or more bonded layers are frequently encountered in many engineering applications. Films grown on substrates by different methods such as PVD or CVD can also be regarded to be in this category. It is known that such structures may suffer delamination because of residual or thermal stresses. Bonding strength between the film and the substrate is therefore very important. Williams' Blister test [1] is a method which can be used to determine the bonding strength. Blister test specimen consists of a thin layer bonded to a substrate. In the test, a blister is formed by applying a certain pressure on the lower surface of the layer through an opening in the substrate. The layer is made to separate from the substrate in a controlled manner through the spreading of the circular boundary between the bonded and already separated parts. By calculating the energy release rate associated with this process it is possible to obtain quantitative information on the bonding strength of the layer substrate pair. Many aspects of this test and its variations has been investigated by various researchers. To name a few, Updike [2] who studied the effect of adhesive layer elasticity on bonding strength, Farris and Keer [3] who analyzed Williams' blister test as an interface crack problem and Jensen and Cochelin [4] who studied constrained blister test can be mentioned. In this study (which is based on [6]) a model is constructed to calculate the energy release rate for Williams' blister test applied to a transversely isotropic layer. By virtue of the theory of elasticity approach adopted (which is very similar to that in [7]), the singular nature of stresses at the debonding front are taken into account.

FORMULATION OF THE PROBLEM

Geometry of the problem is given in Figure 1. It was shown in earlier studies (See for example Dahan and Predeleanu [5]) that the solution of axisymmetric elasticity problems for transversely isotropic media can be obtained through the use of a potential function of the Love type.

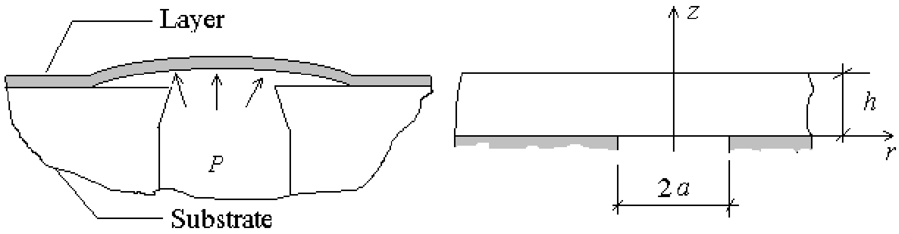

Figure 1: Blister test and test specimen model

This potential function, φ, for the problem under consideration is given as follows:

$$\varphi(r,z)=\int_0^\infty \lambda\left(B_1 e^{m_1\lambda z}+B_2 e^{m_2\lambda z}+B_3 e^{m_3\lambda z}+B_4 e^{m_4\lambda z}\right)J_0(\lambda r)d\lambda, \qquad (1)$$

where $B_k(\lambda)$ are unkowns to be determined by using boundary conditions,

$$m_{1,3}=\left[\frac{(a+c)\pm\sqrt{(a+c)^2-4d}}{2d}\right]^{1/2}, \quad m_{2,4}=-\left[\frac{(a+c)\pm\sqrt{(a+c)^2-4d}}{2d}\right]^{1/2}, \qquad (2)$$

$$a=a_{13}\frac{a_{11}-a_{12}}{a_{11}a_{33}-a_{13}^2}, \; b=\frac{a_{13}(a_{13}+a_{44})-a_{12}a_{33}}{a_{11}a_{33}-a_{13}^2}, c=\frac{a_{13}(a_{11}-a_{12})+a_{11}a_{44}}{a_{11}a_{33}-a_{13}^2}, d=\frac{a_{11}^2-a_{12}^2}{a_{11}a_{33}-a_{13}^2}. \qquad (3)$$

a_{ij} are the compliances defined by

$$\begin{bmatrix}\varepsilon_{rr}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{zz}\\ \gamma_{rz}\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12} & a_{13} & 0\\ a_{12} & a_{11} & a_{13} & 0\\ a_{13} & a_{13} & a_{33} & 0\\ 0 & 0 & 0 & a_{44}\end{bmatrix}\begin{bmatrix}\sigma_{rr}\\ \sigma_{\theta\theta}\\ \sigma_{zz}\\ \tau_{rz}\end{bmatrix}, \qquad (4)$$

Then the non-zero stresses and displacements of interest are given as

$$\sigma_{zz}=\frac{\partial}{\partial z}\left(c\frac{\partial^2\varphi}{\partial r^2}+\frac{c}{r}\frac{\partial\varphi}{\partial r}+d\frac{\partial^2\varphi}{\partial z^2}\right), \tau_{rz}=\frac{\partial}{\partial r}\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}+a\frac{\partial^2\varphi}{\partial z^2}\right), \qquad (5)$$

$$u=-(1-b)(a_{11}-a_{12})\frac{\partial^2\varphi}{\partial r\partial z}, w=a_{44}\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)+(a_{33}d-2a_{13}a)\frac{\partial^2\varphi}{\partial z^2}. \qquad (6)$$

Boundary conditions of the problem are

$$\sigma_{zz}(r,h)=0, \; \sigma_{rz}(r,h)=0, \qquad (7)$$

$$u(r,0)=0, \; w(r,0)=0, \; r\ge a, \qquad (8)$$

$$\sigma_{rz}(r,0)=0, \; \sigma_{zz}(r,0)=-P, \; r\le a. \qquad (9)$$

The problem is subjected to mixed boundary conditions on z=0 surface. In order to reduce the problem to solution of a system of singular integral equations the following auxiliary functions have been defined.

$$G_1(r)=\frac{\partial w}{\partial r}(r,0)\ ,\quad G_2(r)=\frac{1}{r}\frac{\partial(ur)}{\partial r}(r,0). \tag{10}$$

By using equations (1), (5), (6), the homogeneous boundary conditions (7), (8) and the definitions of the auxiliary functions, (10), the unknown $B_k(\lambda)$ functions can be expressed in terms of the auxiliary functions. Then by using the mixed boundary condition (9) the following integral equations can be obtained.

$$\int_0^a \rho G_1(\rho)\int_0^\infty d_{11}(\lambda)\lambda J_0(\lambda r)J_1(\lambda\rho)d\lambda d\rho+\int_0^a \rho G_2(\rho)\int_0^\infty d_{12}(\lambda)\lambda J_0(\lambda r)J_0(\lambda\rho)d\lambda d\rho=P\,, \tag{11}$$

$$\int_0^a \rho G_1(\rho)\int_0^\infty d_{21}(\lambda)\lambda J_1(\lambda r)J_1(\lambda\rho)d\lambda d\rho+\int_0^a \rho G_2(\rho)\int_0^\infty d_{22}(\lambda)\lambda J_1(\lambda r)J_0(\lambda\rho)d\lambda d\rho=0\,, \tag{12}$$

where functions $d_{ij}(\lambda)$ are given in the appendix.

In order to solve these integral equations the asymptotic values of the functions $d_{ij}(\lambda)$ must be determined as λ goes to infinity. After a long analysis these values are found to be as follows:

$$d_{11}{}^\infty=-d_{22}{}^\infty=\frac{M_2m_4(M_3N_1-M_1N_3)+M_4m_2(M_1N_3-M_3N_1)}{(a_{44}-Cm_2{}^2)m_4(M_3N_1-M_1N_3)+(a_{44}-Cm_4{}^2)m_2(M_1N_3-M_3N_1)}, \tag{13}$$

$$d_{12}{}^\infty=-d_{21}{}^\infty=\frac{M_2(a_{44}-Cm_4{}^2)(M_1N_3-M_3N_1)+M_4(a_{44}-Cm_2{}^2)(M_3N_1-M_1N_3)}{A[(a_{44}-Cm_2{}^2)m_4(M_3N_1-M_1N_3)+(a_{44}-Cm_4{}^2)m_2(M_1N_3-M_3N_1)]}, \tag{14}$$

$$A=(1-b)(a_{11}-a_{12}),\ C=(a_{33}d-2a_{13}a),\ t_i=m_i\lambda\,,M_i=m_i(m_i{}^2d-c),N_i=(1-m_i{}^2a),(i=1,2,3,4) \tag{15}$$

By using these asymptotic values the singularities in equations (11) and (12) can be extracted. Further, one can extend the range of outer integrals into the negative range ($-a$,0) by observing that $G_1(r)=-G_1$(r) and $G_2(r)=G_2(-r)$. Doing so the final form of integral equations can be obtained as follows.

$$C_iG_{3-i}(\rho)+\frac{L_i}{\pi}\int_{-a}^{a}G_i(\rho)\left[\frac{1}{\rho-r}+H_i(r,\rho)\right]d\rho+$$
$$\sum_{j=1}^{2}\int_0^a\frac{|\rho|}{2}G_j(\rho)\int_0^\infty\left[d_{ij}(\lambda)-d_{ij}{}^\infty\right]\lambda J_{i-1}(\lambda r)J_{2-j}(\lambda\rho)d\lambda d\rho=Q_i\ ,(i=1,2). \tag{16}$$

where

$$C_1=d_{12}{}^\infty,\ C_2=d_{21}{}^\infty\ ,L_1=d_{11}{}^\infty\ ,\ L_2=-d_{22}{}^\infty\ ,\ H_i=\frac{h_k(r,\rho)-1}{\rho-r},\ (k=1,2) \tag{17}$$

$$h_1(r,\rho)=\begin{cases}\dfrac{\rho^2-r^2}{|r\rho|}K\left(\dfrac{\rho}{r}\right)+\dfrac{r}{\rho}E\left(\dfrac{\rho}{r}\right),\ |\rho|<|r|\\ E\left(\dfrac{r}{\rho}\right),\ |\rho|>|r|\end{cases},\ h_1(r,\rho)=\begin{cases}\left|\dfrac{\rho}{r}\right|E\left(\dfrac{\rho}{r}\right),\ |\rho|<|r|\\ \left(\dfrac{\rho}{r}\right)^2E\left(\dfrac{r}{\rho}\right)-\dfrac{\rho^2-r^2}{r^2}K\left(\dfrac{r}{\rho}\right),\ |\rho|>|r|\end{cases}. \tag{18}$$

This system of singular integral equations can be solved by defining

$$G_2(r)+iG_1(r)=(1-r)^{\alpha}(1+r)^{\beta}F(r),\ \alpha=-\frac{1}{2}-i\omega,\ \beta=-\frac{1}{2}+i\omega,\ \omega=\frac{1}{2\pi}\log\left(\frac{1+\gamma}{1-\gamma}\right)\ \gamma=\frac{d_{12}^{\infty}}{d_{11}^{\infty}}, \tag{19}$$

and following the method given in [6,7]. After solving the unknown function $F(r)$, the stress intensity factors and the energy release rate (ERR) associated with the debonding of the elastic layer can be obtained as;

$$k_1+ik_2=id_{11}^{\infty}\sqrt{1-\gamma^2}F(1),\quad G=\frac{\partial U}{\partial a}=\pi\frac{k_1^2+k_2^2}{4d_{11}^{\infty}}. \tag{20}$$

SAMPLE RESULTS AND DISCUSSION

Figure 2 shows the sample calculation for energy release rate (henceforth ERR) of an epoxy layer, (E=3.1 GPa, ν=0.35), bonded to a rigid substrate. Since a transversely isotropic formulation is done, a slight anisotropy is introduced by reducing the compliances a_{13} and a_{33} by 5%, to obtain numerical results. Also given on this figure are the ERRs for the two limiting cases of the blister test,

$$G_0=\frac{2(1-\nu^2)}{\pi E}P^2a,\quad G_1=\frac{3(1-\nu^2)}{256\pi E}\frac{1}{(h/2a)^3}P^2a. \tag{21}$$

In Eqn. (21) G_0 is the ERR associated with an isotropic infinitely thick layer and G_1 is the ERR associated with an isotropic thin film, bonded to a rigid substrate [1].

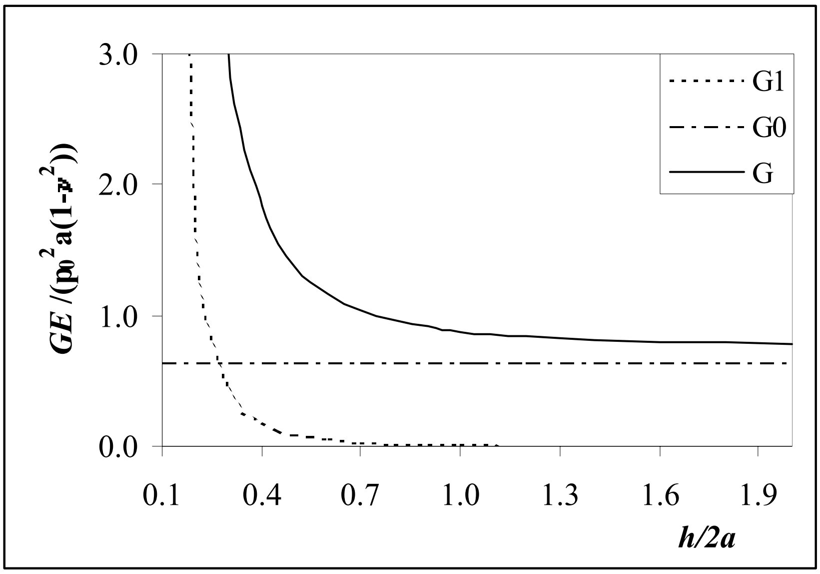

Figure 2: Energy release rates for an isotropic material

In the derivation of G_0 the complex nature of singularity has not been taken into account [1] and the contribution of shear stresses are not considered. On the other hand G_1 is derived by using the classical plate theory and it is a good approximation for small $h/(2a)$ ratios (such as $h/(2a) < 0.1$). The elasticity approach adopted in this study (and in [3]) is most useful in the range between these two extreme cases, where the film thickness is too large to apply classical plate theory but too small to be assumed as infinitely thick.

For $h/(2a) < 0.2$, difficulties are encountered in the numerical solution of the singular integral equations. Because of this, convergence of G value obtained from equation (20) and G_1 value obtained from equation (21) can not be clearly demonstrated. For the other limiting case (large $h/(2a)$), the asymptotic solution is recovered albeit it does not match exactly with G_0. The difference is attributed to omission of the complex singularity and the contribution of shear stresses in the derivation of G_0. The results, however, agree well with those given in [7] for an epoxy layer bonded to an aluminum substrate which is much stiffer than the epoxy.

Figure 3 shows the sample calculations for non-dimensional energy release rate ratio which is defined as

$$G_r = \frac{G}{G_0}. \tag{22}$$

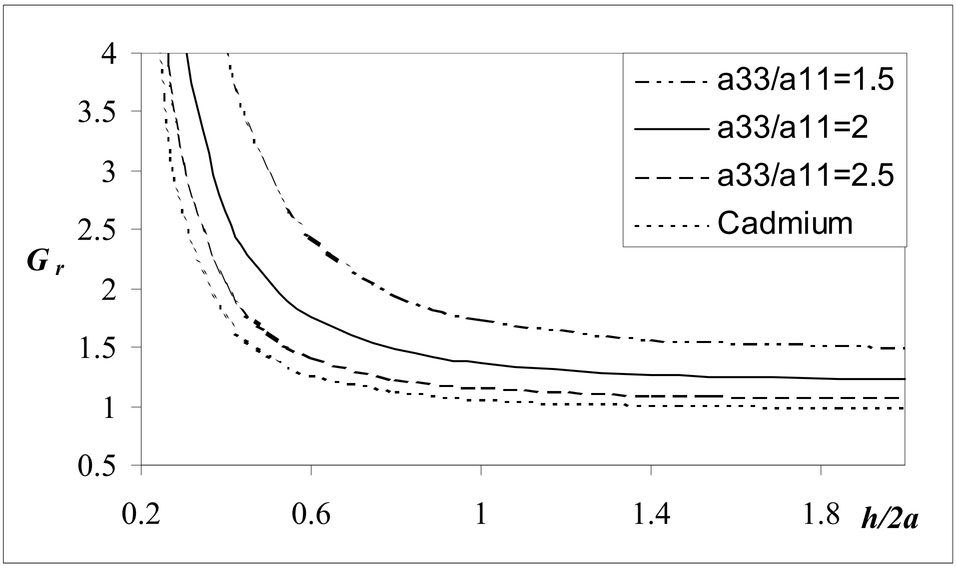

Figure 3: Energy release rate ratio

In these sample calculations cadmium (a_{11}=0.0129, $a_{12}=-1.5\times10^{-3}$, $a_{13}=-9.3\times10^{-3}$, a_{33}=0.0369, a_{44}=0.0640 all in units of GPa^{-1}) and three hypothetical materials are considered. Material properties of hypothetical materials are taken to be equal to those of cadmium except a_{33} which is varied as shown in the legend of Figure 2. The isotropic material is taken to have a Young's modulus of E=77.52 GPa, and a Poisson's ratio of ν=0.116 .These are the in-plane Young's modulus and Poisson ratio of Cadmium.

The results indicate that ERR depends strongly on the elastic constants of a transversely isotropic material. Dependence is stronger when the layer thickness is small compared to the hole diameter. (For layer thicknesses much smaller than the hole diameter, difficulties are encountered in the numerical solution. For such layer thicknesses a thick (or thin plate formulation) could be attempted.) On the other hand, when the layer thickness is greater than 1.25 times the hole diameter, the dependence of ERR on layer thickness practically disappears and infinitely thick layer solutions can be used.

The problem considered in this paper can be extended in several directions. For example the residual stresses in the layer and the elasticity of the substrate can be taken into account. One can also include an adhesive layer between the upper layer and the substrate.

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APPENDIX

$$B_i = \sum_{j=3}^{4} \frac{\Delta_{ij}}{\Delta} R_j \ (i=1,2,3,4),$$

$$H = \begin{bmatrix} M_1 e^{t_1 h} & M_2 e^{t_2 h} & M_3 e^{t_3 h} & M_4 e^{t_4 h} \\ N_1 e^{t_1 h} & N_2 e^{t_2 h} & N_3 e^{t_3 h} & N_4 e^{t_4 h} \\ a_{44} - C{m_1}^2 & a_{44} - C{m_2}^2 & a_{44} - C{m_3}^2 & a_{44} - C{m_4}^2 \\ Am_1 & Am_1 & Am_1 & Am_1 \end{bmatrix},$$ h_{ij} are elements of the matrix H.

$\Delta_{13} = -h_{14}h_{23}h_{42} + h_{13}h_{24}h_{42} + h_{14}h_{22}h_{43} - h_{12}h_{24}h_{43} - h_{13}h_{22}h_{44} + h_{12}h_{23}h_{44}$
$\Delta_{14} = h_{14}h_{23}h_{32} - h_{13}h_{24}h_{32} - h_{14}h_{22}h_{33} + h_{12}h_{24}h_{33} + h_{13}h_{22}h_{34} - h_{12}h_{23}h_{34}$
$\Delta_{23} = h_{14}h_{23}h_{41} - h_{13}h_{24}h_{41} - h_{14}h_{21}h_{43} + h_{11}h_{24}h_{43} + h_{13}h_{21}h_{44} - h_{11}h_{23}h_{44}$
$\Delta_{24} = -h_{14}h_{23}h_{31} + h_{13}h_{24}h_{31} + h_{14}h_{21}h_{33} - h_{11}h_{24}h_{33} - h_{13}h_{21}h_{34} + h_{11}h_{23}h_{34}$
$\Delta_{33} = -h_{14}h_{22}h_{41} + h_{12}h_{24}h_{41} + h_{14}h_{21}h_{42} - h_{11}h_{24}h_{42} - h_{12}h_{21}h_{44} + h_{11}h_{22}h_{44}$
$\Delta_{34} = h_{14}h_{22}h_{31} - h_{12}h_{24}h_{31} - h_{14}h_{21}h_{32} + h_{11}h_{24}h_{32} + h_{12}h_{21}h_{34} - h_{11}h_{22}h_{34}$
$\Delta_{43} = h_{13}h_{22}h_{41} - h_{12}h_{23}h_{41} - h_{13}h_{21}h_{42} + h_{11}h_{23}h_{42} + h_{12}h_{21}h_{43} - h_{11}h_{22}h_{43}$
$\Delta_{44} = -h_{13}h_{22}h_{31} + h_{12}h_{23}h_{31} + h_{13}h_{21}h_{32} - h_{11}h_{23}h_{32} - h_{12}h_{21}h_{33} + h_{11}h_{22}h_{33}$

$$\begin{aligned} \Delta = {} & h_{41}(\ h_{14}h_{23}h_{32} - h_{13}h_{24}h_{32} - h_{14}h_{22}h_{33} + h_{12}h_{24}h_{33} + h_{13}h_{22}h_{34} - h_{12}h_{23}h_{34}) + \\ & h_{31}(-h_{14}h_{23}h_{42} + h_{13}h_{24}h_{42} + h_{14}h_{22}h_{43} - h_{12}h_{24}h_{43} - h_{13}h_{22}h_{44} + h_{12}h_{23}h_{44}) + \\ & h_{21}(\ h_{14}h_{33}h_{42} - h_{13}h_{34}h_{42} - h_{14}h_{32}h_{43} + h_{12}h_{34}h_{43} + h_{13}h_{32}h_{44} - h_{12}h_{33}h_{44}) + \\ & h_{11}(-h_{24}h_{33}h_{42} + h_{23}h_{34}h_{42} + h_{24}h_{32}h_{43} - h_{22}h_{34}h_{43} - h_{23}h_{32}h_{44} + h_{22}h_{33}h_{44}) \end{aligned}$$

$d_{11} = (\Delta_{13}M_1 + \Delta_{23}M_2 + \Delta_{33}M_3 + \Delta_{43}M_4) / \Delta$
$d_{12} = (\Delta_{14}M_1 + \Delta_{24}M_2 + \Delta_{34}M_3 + \Delta_{44}M_4) / \Delta$
$d_{21} = (\Delta_{13}N_1 + \Delta_{23}N_2 + \Delta_{33}N_3 + \Delta_{43}N_4) / \Delta$
$d_{22} = (\Delta_{14}N_1 + \Delta_{24}N_2 + \Delta_{34}N_3 + \Delta_{44}N_4) / \Delta$