MODELLING OF PLASTIC EFFECTS DURING SMALL FATIGUE-CRACK GROWTH

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ABSTRACT

A method is presented for assessment of effects of main plastic factors – reversed plastic yielding of the material and plasticity-induced crack closure – on small fatigue crack behavior. The method uses the closed form analytical solution of the elastic-plastic problem for a rectilinear crack in a plate on the basis of plastic-strip model. This allowed to simplify significantly the calculation of the stress-strain parameters at the crack tip (the effective stress intensity factor range and crack tip opening displacement range). It was shown that the calculated results agree well with those obtained by the more complicated methods, including a method of finite elements. Using the proposed method, the dependence of the small crack growth rate on the loading amplitude and stress ratio, initial crack size and other parameters, has been analyzed. The revealed regularities and tendencies are proved experimentally. In particular, the method is successfully utilized to predict the small fatigue crack growth rate in Fe-3% Si alloy under high amplitude loading.

KEYWORDS

Small fatigue cracks, crack closure, plastic strip model, singular integral equation method, crack tip opening displacement.

INTRODUCTION

Small fatigue crack behavior greatly depends on the plasticity effects. At the relatively high stress level that are usually required to grow small cracks, the criterion of small scale plasticity at the crack tip, relative to the overall size of the crack, is violated. Therefore, conventional methods of linear-elastic fracture mechanics can not be used and the crack growth rate *dl/dN* should be evaluated from deformation or energetic parameters, e.g. crack tip opening displacement range $\Delta\delta$:

$$dl/dN = v(\Delta\delta) \tag{1}$$

The value of $\Delta\delta$ is significantly dependent on the plastic-induced crack closure, that is differently manifested for small and long cracks. To consider these factors, it is necessary to analyze the elastic-plastic situation at the crack tip under cyclic loading. Approximate analysis can be done by known plastic-strip model (Dugdale [1], Panasyuk [2]). This approach was advanced by Budiansky and Hutchinson [3], Newman [4-5], Wang and Blom [6-7] and others authors. A rather simple method of an analytical solution on the basis of strip-model of elastic-plastic problems about fatigue crack growth by its reduction to a singular integral equation, has been proposed earlier by Panasyuk et al [8-9]. This method allows us to decrease the complexity and instability of calculations, typical of numerical methods. The mentioned method is used in this paper to assessment of the stress-strain state near small fatigue crack and prediction of crack growth rate from equation (1).

DEVELOPMENT OF THE MODEL

We consider an internal rectilinear crack of length $2l$ in a plate subjected to cyclic loading ($p=p_{min} \sim p_{max}$), as shown in Figure 1. According to plastic-strip model, plastic zones at the crack tip are replaced by the additional cuts where the boundary normal stresses are equal to flow stress. At $p=p_{max}$ these stresses are taken as equal to $\alpha\sigma_0$, where σ_0 is average between yield stress and ultimate tensile strength of material, α is plastic constraint factor, the accounts for the influence of stress state on tensile yielding at the crack front in accordance with Newman [4]. For plane stress conditions $\alpha=1$, and for simulated plane strain conditions $\alpha=3$. At $p=p_{min}$ the stresses within the cyclic plastic zone are equal to $-\sigma_0$. In the analysis the crack closure, caused by plastic stretches of thickness $h(x)$ on the fatigue crack surfaces, is accounted for. These plastic stretches are the result of the residual plastic deformation formed in the near-surface layers as the crack grows through plastic zone ahead of the crack tip. During unloading, the crack surfaces contact each other along the entire crack length or in specific sections ($l_c < |x| < l$) near the crack tip. Contact stresses are denoted as $\sigma_{con}(x)$.

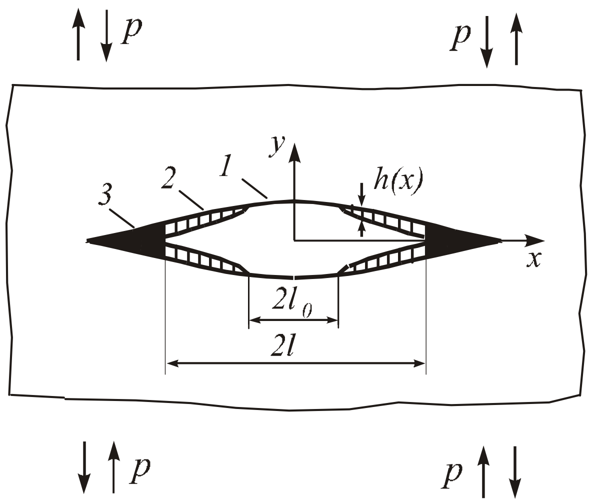

Figure 1: A rectilinear fatigue crack in plate: 1- crack; 2-plastic stretch; 3-plastic zone

Analytical solutions to the corresponding boundary-value problem for a plate with model cuts (crack + plastic zones) have been obtained on the basis of results in [2] (details are given in Panasyuk et al [10]). Specifically, normal displacements of the crack edges at the maximum and minimum loads are found to be:

$$u_{max}(x) = p_{max} \int_0^{l+l_p} H(l+l_p, x, \xi) d\xi - \alpha\sigma_0 \int_l^{l+l_p} H(l+l_p, x, \xi) d\xi \tag{2}$$

$$u_{min}(x) = u_{max}(x) - (p_{max} - p_{min}) \int_0^{l+l_{pf}} H(l+l_{pf}, x, \xi) d\xi + (1+\alpha)\sigma_0 \int_l^{l+l_{pf}} H(l+l_{pf}, x, \xi) d\xi + \int_{l_c}^{l} \sigma_{con}(\xi) H(l+l_{pf}, x, \xi) d\xi \tag{3}$$

where $H(l,x,\xi) = \dfrac{2}{\pi E} ln \dfrac{\sqrt{l^2 - x^2} + \sqrt{l^2 - \xi^2}}{\sqrt{l^2 - x^2} - \sqrt{l^2 - \xi^2}} d\xi$; l_p, l_{pf} is the length of monotone and cyclic plastic zone, respectively; E is the elastic modulus of the material.

Together with the condition of crack edge closing in the contact region

$$u_{min}(x) = h(x) \qquad\qquad l_c \le |x| \le l \tag{4}$$

dependence (3) forms equation to estimate the unknown contact stresses $\sigma_{con}(x)$. By differentiation with respect to x, this equation is reduced to singular integral equation:

$$\int_{l_c}^{l} \frac{\sigma_{con}(\xi)\sqrt{(l+l_{pf})^2-\xi^2}}{x^2-\xi^2}d\xi = \frac{\pi E}{4}\frac{\sqrt{(l+l_{pf})^2-x^2}}{x}f(x) \qquad\qquad l_c \le |x| \le l \tag{5}$$

where

$$f(x) = \frac{\sigma_s}{\pi E}\Big\{H(l+l_p,x,l) - H(l+l_p,x,-l) - 2H(l+l_{pf},x,l) + 2H(l+l_{pf},x,-l) +$$
$$\left(\frac{\pi}{4}\frac{p_{max}-p_{min}}{\sigma_s} - arccos\frac{l}{l+l_{pf}}\right)\frac{8x}{\sqrt{(l+l_{pf})^2-x^2}}\Big\} - \frac{d}{dx}u_{res}(x)$$

Solution of equation (5), according to Muskhelishvili [11], gives the following formula for $\sigma_{con}(x)$:

$$\sigma_{con}(x) = \frac{E}{\pi}\frac{x\sqrt{l^2-x^2}}{\sqrt{x^2-l_c^2}\sqrt{(l+l_{pf})^2-x^2}}\int_{l_c}^{l}\frac{\sqrt{(l+l_{pf})^2-\xi^2}\sqrt{\xi^2-l_c^2}}{\sqrt{l^2-\xi^2}}\frac{f(\xi)}{\xi^2-x^2}d\xi\ . \tag{6}$$

The tensile plastic zone length l_p and cyclic plastic zone length l_{pf} are determined from the conditions that the stress at the point just ahead of the plastic zone is elastic and infinitely approaches the yield stress. Parameter l_c is determined from the condition that the contact stress approaches zero at the end of contact zone.

The developed solution is used for assessment of the parameters of elastic-plastic situation at the fatigue crack tip. Main parameters are as follows: maximum and minimum crack tip opening displacement $\delta_{max}=2u_{max}(l)$; $\delta_{min}=2u_{min}(l)$; and the crack opening stress p_{op}, which is defined at the external load $p=p_{op}$ at which crack surfaces are fully open and process of active deformation of the material immediately ahead of the crack tip begins. The mentioned parameters determine the effective stress intensity factor range $\Delta K_{eff} = (p_{max} - p_{op})\sqrt{\pi l}$ and cyclic crack tip opening displacement range $\Delta\delta=\delta_{max} - \delta_{min}$, which are the basic values for calculating fatigue crack growth rate from linear and nonlinear criteria of fracture mechanics.

NUMERICAL RESULTS AND DISCUSSION

The proposed model is applied to a fatigue crack emanating from a rectilinear defect of length $2l_0$. The defect thickness is assumed to be sufficient to prevent contact of the opposite crack faces even under symmetrical tension-compression loading. Thus, at the initial moment there is no closure effect. Further growth of a fatigue crack is accompanied by the fracture of plastically deformed material ahead of the crack tip and formation of plastic stretches on its surfaces. Their thickness are unknown but are evaluated by step-by-step analysis of crack extension. Specifically, at each step, the size of a stretch formed on the fresh crack surface is assumed to be equal to the value of crack edge displacement $u_{min}(x)$ immediately ahead of the crack tip. This value is calculated by the mentioned above model.

Some of the predicted results for plane stresses ($\alpha=1$) at two stress ratio $R=0$ and $R=-1$ and several operating stress p_{max}/σ_0 are presented in Figures 2-4.

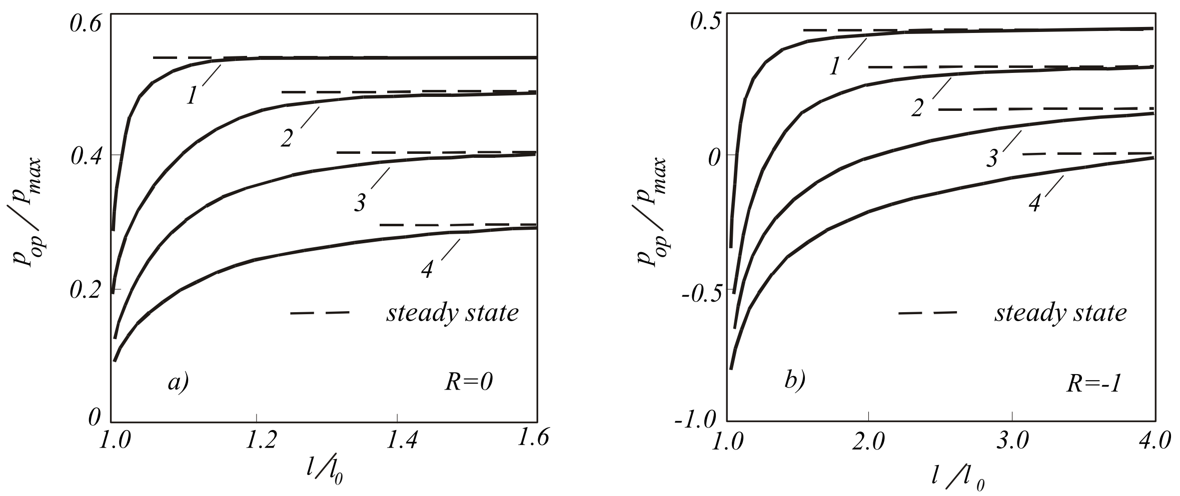

Figure 2: Predicted variation of normalized crack closure loads p_{op}/p_{max} with a crack length: (a) $R=0$; (b) $R=-1$; 1- $p_{max}/\sigma_0=0.2$; 2- $p_{max}/\sigma_0=0.4$; 3- $p_{max}/\sigma_0=0.6$; 4- $p_{max}/\sigma_0=0.8$

With the crack length extension, the thickness of plastic stretch on the crack surfaces increased and this induced a higher crack closure level (see Figure 2). After some increment of a crack length the process stabilizes itself and crack closure ratio p_{op}/p_{max} approaches constant «steady-state» values, which pertain to those for long cracks. These steady-state values are shown in Figure 3 as functions of the maximum stresses p_{max}/σ_0. These results indicate that level of plasticity-induced crack closure becomes lower with increase of the operating loads magnitude, especially at negative load ratios.

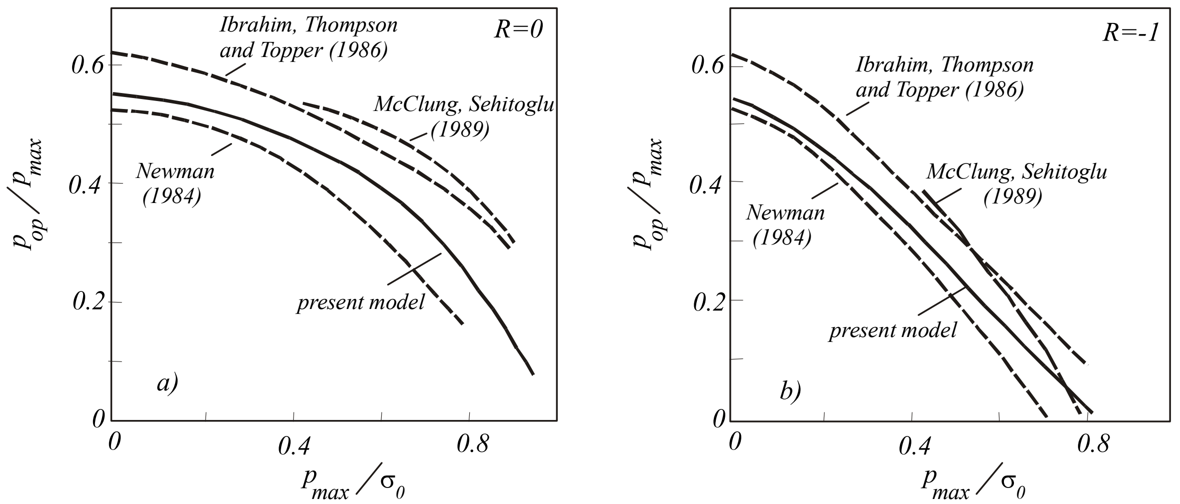

Figure 3: Predicted values of the closure ratio p_{op}/p_{max} as a function of the cyclic loading level p_{max}/σ_0 for a steadily growing crack and comparison of the present results with other models: (a) $R=0$; (b) $R=-1$

In Figure 3 the presented results are compared with several other previous models. The curves $p_{op}/p_{max}\sim p_{max}/\sigma_0$ for a steadily growing crack, which were obtained earlier using the method of finite element (McClung. and Sehitoglu [12]) or generalized plastic strip model (Newman [13], Ibrahim, Thompson and Topper [14]), are also presented. These results agree well for both stress ratio and various loading amplitudes.

The predicted values of normalized cyclic crack tip opening displacement range $\Delta\delta$ are shown as a function of the maximum stress intensity factor K_{max} in Figure 4. Based on presented data, the following conclusions as to behavior of small fatigue crack can be draw.

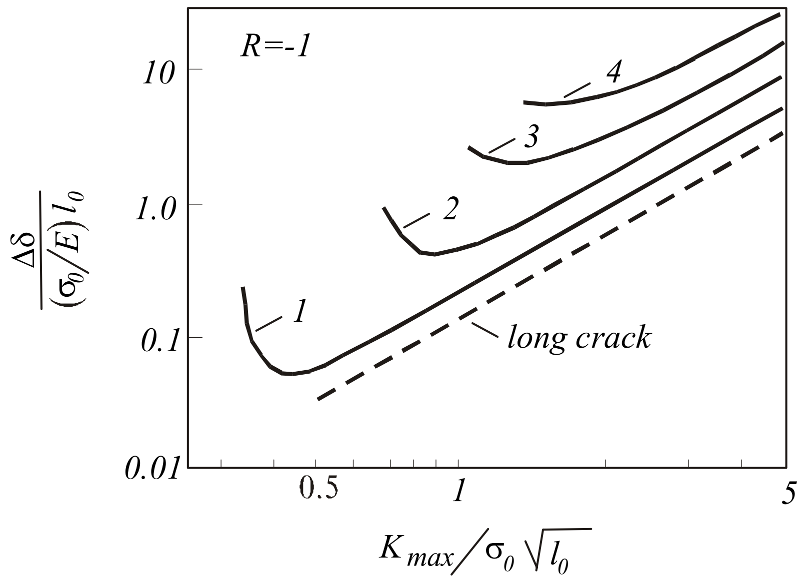

Figure 4: Dependence of normalized crack tip displacement range on maximum stress intensity factor in a cycle

Firstly, the level of plasticity-induced crack closure is lower for small cracks, compared to the steady-state values pertaining to long cracks. Similarly, at the commencement of crack growth, cyclic crack tip opening displacement for the small crack exceed the corresponding steady-state (long crack) values. As a result the growth rate for the small crack would be expected to be higher.

Secondly, as a crack grows, a gradual increase in the magnitude of plastic deformation near crack tip takes place; this promotes the crack growth. At the same time, a gradual increase in the level of plasticity-induced crack closure occurs; this retards the crack growth. The interaction of this two factors caused the non-monotone variation of «crack driving force»: a value of $\Delta\delta$ at first decreases and then increases with the crack growth (see Figure 4). This could causes both retardation and sometimes complete arrest of small cracks, what has been observed frequently by experiment.

Finally, the presented data indicate that for small cracks the actual value of $\Delta\delta$ is determined not only by current magnitude of the external cyclic loading, as is the case for long crack growth, but is significantly dependent on the crack length. We thus conclude that small crack growth rates cannot be predicted without considering the history of their initiation.

EXPERIMENTAL VERIFICATION OF THE MODEL

The proposed model has been used for calculating small fatigue crack growth rate in Fe-3% Si alloy and to compare predicted results with the earlier known experimental data [15]. Nisitani, Kawagoishi and Goto [15] tested cylindrical specimens of diameter 5mm in rotating bending. Maximum bending stresses on the specimen surface were 0.54 ~ 0.86 of the yield strength. The surface cracks initiated from a small blind hole (diameter and depth are 0.3 mm). Figure 5(a) shows relation between the crack growth rate and the stress intensity factor range. Only at low values of loading amplitude (σ_a = 560 ÷ 640 MPa) these relations are described by a unique dependence. At high stress levels, the ΔK -based criterion is unsuitable for evaluating the crack growth rate.

On the basis of the presented model calculation of cyclic crack tip opening displacement range $\Delta\delta$ was carried at the following values of the parameters: $p_{max}=\sigma_a$, $R=-1$; $\sigma_0 = 1300\ MPa$; $\alpha = 1.8$. The initial defect size was taken equal to the hole diameter $2l_0 = 0.3\ mm$. The reason of that was the fact that experimental data correspond to the crack lengths that at least two-fold exceed the hole sizes ($2l \geq 0.6\ mm$). Under such conditions, the elastic-plastic situation at the crack tip slightly depends on the initial defect geometry.

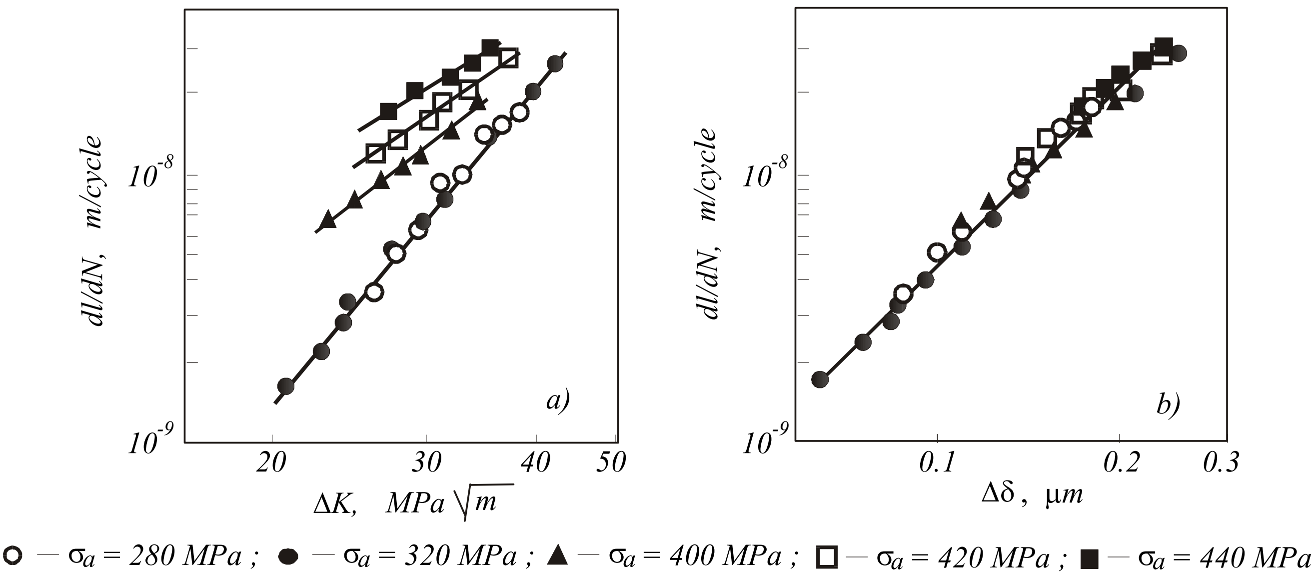

Figure 5: Crack growth rate in Fe-3% Si alloy: (a) depending on ΔK [12], (b) depending on $\Delta\delta$

In Figure 5(b) the dependence of the crack growth rate on $\Delta\delta$ is illustrated. All experimental data are described by a unique dependence. Thus, in this case crack growth can be predicted by the proposed model and deformation fatigue criterion in form equation (1).

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