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MODEL OF MECHANICS FOR FIBER REINFORCED TI ALLOY MATRIX COMPOSITES UNDER THERMOMECHANICAL LOADING

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ABSTRACT

A micromechanical model of fiber-reinforced metal matrix composites is established with considering the coating effects. During the investigation, the coatings and fibers are assumed to have a linearly elastic behavior, and the matrix respond viscoplastically with temperature. Micro-mechanics theories using the concepts of average stress and strain are adopted and developed to integrate matrix, coating and fiber properties for predicting the stress-strain response under thermomechanical loading. Considering the viscoplastic behavior of metal matrix, the Bodner-Partom unified theory is used as the basic constitutive equations of metal matrix. The main problem is reduced to a set of the ordinary differential equations of one order that can be resolved by numerical solution algorithms. In the application, three-layer model is applied to analyze the stress-strain response of a titanium matrix composite. Through the calculation, first, the thermo-residual stresses in the constituents of the material are obtained, then, the mechanical behaviors are estimated under the loading of thermomechanical cycles in the in-phase and out-of-phase cases.

KEYWORDS

thermomechanical loading, metal matrix composite, coating effects, temperature cycle, viscoplasticity

INTRODUCTION

The use of fiber-reinforced metal matrix composites in design has increased significantly in recent years with the development of aerospace industry [1,2]. Therefore, the knowledge of thermomechanical behavior of the materials is very important for designing the structures of the materials. In the past, several of these studies were limited to two-layer models that paid relatively less attention to the effects of coatings on the residual stress and assumed only two linking types: bonding and slipping, existing between metal matrix and fiber. However, the effect of coating could not be ignored, as some relevant investigations indicated [3].

During the investigation, a micromechaical model for fiber-reinforced metal matrix composites with coating materials is presented. The model considers that the coating and fibers have a linearly elastic behavior, but the matrix responds viscoplastically with temperature. Micromechanical theories using the concepts of average stress and strain are adopted and developed to integrate fibers and matrix properties for predicting the response of unidirectional composites. The stress-strain equations are used to the basic three-layer models of metal matrix composites. Then the main problem is reduced to a set of ordinary differential equations. Numerical solution algorithms are developed to achieve the mechanical behavior of the composites by varying thermal and mechanical loading. The thermo-residual stress in fabrication of a fiber-reinforced titanium matrix composite is analyzed and the stress-strain response under the loading of thermomechanical cycles is emphatically estimated in this investigation.

MICRO-MECHANICAL MODEL WITH COATING CONCEPT

The Representative Volume Element (RVE) of unidirectional fiber-reinforced matrix composites is used to describe the stress-strain state. The selection of the RVE and the coordination is shown in Figure 1.

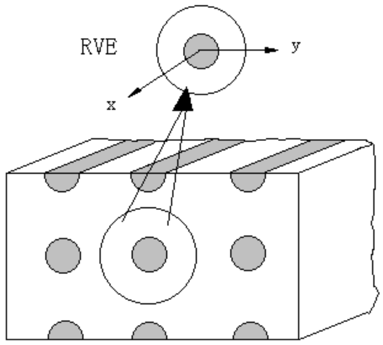

Figure 1: Selection of the RVE and the coordination

The concepts of average stress and strain are adopted in the model [4]. Suppose the stress field in a representative volume, *V*, of a unidirectional composite is denoted as σ_v. The volume average stress σ of the composite are defines by

$$\sigma = \frac{1}{V}\int_V \sigma_v dV = \frac{1}{V}\left(\int_{V^m} \sigma_v dV + \int_{V^c} \sigma_v dV + \int_{V^f} \sigma_v dV\right) \tag{1}$$

where V^f, V^c and V^m are the fiber, coating and matrix volume, respectively. The volume average stresses of the matrix, coating and fibers are described as follows

$$\sigma^m = \frac{1}{V^m}\int_{V^m} \sigma_v dV \ , \quad \sigma^c = \frac{1}{V^c}\int_{V^c} \sigma_v dV \ , \quad \sigma^f = \frac{1}{V^f}\int_{V^f} \sigma_v dV \tag{2}$$

Similarly to the stresses, by introducing ε^m, ε^c and ε^f as the volume average strains of the matrix, coating and fibers, respectively, the stresses σ and the strains ε of the composite can be expressed as

$$\begin{cases} \sigma = v^m \sigma^m + v^c \sigma^c + v^f \sigma^f \\ \varepsilon = v^m \varepsilon^m + v^c \varepsilon^c + v^f \varepsilon^f \end{cases} \tag{3}$$

The superscripts *m, c* and *f* indicate the matrix, coating and fiber, and v^m, v^c and v^f are the volume fractions of the matrix, coating and fiber, respectively.

Assuming that the deformation of the RVE is infinitesimal and the total strain can be decomposed into elastic, thermal and plastic components, and the fibers exhibit transverse-isotropic thermoelastic behaviors and the matrix has thermo-elasto-viscoplastic behaviors. Then, we have the incremental stress-strain relationship for the matrix

$$d\varepsilon^{m} = \underline{S}^{me} d\sigma^{m} + d\varepsilon^{mT} + d\varepsilon^{mp} \tag{4}$$

and the linear elastic stress-strain relationship for the fiber and coating

$$d\varepsilon^{c} = \underline{S}^{ce} d\sigma^{c} + d\varepsilon^{cT} \tag{5}$$

$$d\varepsilon^{f} = \underline{S}^{fe} d\sigma^{f} + d\varepsilon^{fT} \tag{6}$$

The superscript *e*, *p*, and *T* denote the elastic, plastic (inelastic) related components and temperature, respectively. S^{me}, S^{ce} and S^{fe} are the elastic compliance matrices of matrix, coating and fiber, respectively, which read for the plane stress case.

The matrix, coating and fiber thermal strains in Equations (5), (6) and (7) are assumed to be linearly dependent upon the temperature difference ΔT and can be written as

$$\varepsilon^{mT} = \lambda^{m} \Delta T \ , \ \varepsilon^{cT} = \lambda^{c} \Delta T \ , \ \varepsilon^{fT} = \lambda^{f} \Delta T \tag{7}$$

where λ are the thermal expansion coefficients. The strains of the matrix, coating and fiber within a layer along the fiber direction can be considered as:

$$\varepsilon_x^{m} = \varepsilon_x^{c} = \varepsilon_x^{f} \tag{8}$$

In the three-layer model, the transverse matrix stress can be related to the fiber stress and coating stress in a form

$$\sigma_y^{m} = \eta_y^{mc} \sigma_y^{c} + \eta_{yx}^{mc} \sigma_x^{c} \tag{9}$$

$$\sigma_y^{c} = \eta_y^{cf} \sigma_y^{f} + \eta_{yx}^{cf} \sigma_x^{f} \tag{10}$$

where the subscript *x* and *y* denote the direction parallel and normal to the fibers, respectively. η_y^{mc}, η_{yx}^{mc}, η_y^{cf} and η_{yx}^{cf} are the stress partitioning factors.

Combining Equations above, we can obtain the expression for the relationship between matrix stress and coating stress in an incremental form as

$$d\boldsymbol{\sigma}^{m} = \underline{C}_1 d\boldsymbol{\sigma}^{c} + \underline{C}_2 \left(d\varepsilon^{cT} - d\varepsilon^{mT} - d\varepsilon^{mp} \right) \tag{11}$$

and the relationship between coating stress and fiber stress as

$$d\boldsymbol{\sigma}^{c} = \underline{B}_1 d\boldsymbol{\sigma}^{f} + \underline{B}_2 \left(d\varepsilon^{fT} - d\varepsilon^{cT} \right) \tag{12}$$

and the relationship between matrix stress and fiber stress as

$$d\boldsymbol{\sigma}^{m} = \underline{A}_1 d\boldsymbol{\sigma}^{f} + \underline{A}_2 d\varepsilon^{cT} + \underline{A}_3 d\varepsilon^{fT} + \underline{A}_4 \left(d\varepsilon^{mT} + d\varepsilon^{mp} \right) \tag{13}$$

where C_1 , C_2 , B_1 and B_2 are the coefficients related to the Young's modulus, volume percentage and stress partitioning factors of each constituent, and $\underline{A}_1 = \underline{C}_1 \underline{B}_1$, $\underline{A}_2 = -\underline{C}_1 \underline{B}_2 + \underline{C}_2$, $\underline{A}_3 = \underline{C}_1 \underline{B}_2$, and $\underline{A}_4 = -\underline{C}_2$.

The Bodner-Partom unified theory of viscoplasticity is used to establish the basic constitutive models of metal matrix.

(a) Flow law is described as

$$\varepsilon_{ij}{}^{p} = \lambda S_{ij} \tag{14}$$

where S_{ij} is the deviatoric stress related to the stress tensor by $S_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$.

(b) Scalar kinetic equation is described as

$$D_2{}^{p} = D_0{}^{2}\exp\left\{-\left[\frac{(Z)^2}{3J_2}\right]^{n}\right\} \tag{15}$$

with $D_2{}^{p} = \frac{1}{2}\dot{\varepsilon}_{ij}{}^{p}\dot{\varepsilon}_{ij}{}^{p}$, $J_2{}^{p} = \frac{1}{2}S_{ij}S_{ij}$, $Z = Z^{I} + Z^{D}$, $\lambda^2 = D_2{}^{p}/J_2$, and J_2^{p} is the second invariant of the deviatoric stress, and Z^I and Z^D represent the hardening due to isotropic and directional characteristics, respectively.

(c) Isotropic hardening relationship is described as

$$\dot{Z}^{I} = m_1\dot{W}_p\left(Z_1 - Z^{I}\right) - R_1 Z_1\left(\frac{Z^{I} - Z_2}{Z_1}\right)^{r_1} + \dot{T}\left[\left(\frac{Z^{I} - Z_2}{Z_1 - Z_2}\right)\frac{\partial Z_1}{\partial T} + \left(\frac{Z_1 - Z^{I}}{Z_1 - Z_2}\right)\frac{\partial Z_2}{\partial T}\right] \tag{16}$$

with W_p is the inelastic work per volume, and $\dot{W}_p = \sigma_{ij}\dot{\varepsilon}'_{ij}$, $Z^{I}(0) = Z_0$, $\dot{W}_p(0) = 0$.

(d) Kinematic hardening relationship is described as

$$Z^{D} = \beta_{ij}u_{ij} \tag{17}$$

$$\dot{\beta}_{ij} = m_2\dot{W}_p\left(Z_3 u_{ij} - \beta_{ij}\right) - R_2 Z_1\frac{\beta_{ij}}{\sqrt{\beta_{kl}\beta_{kl}}}\left(\frac{\sqrt{\beta_{kl}\beta_{kl}}}{Z_1}\right)^{r_2} + \dot{T}\frac{\beta_{ij}}{Z_3}\frac{\partial Z_3}{\partial T} \tag{18}$$

$$u_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{kl}\sigma_{kl}}} \tag{19}$$

In the equations presented, the quantities D_0, n, m_1, m_2, Z_1, Z_2, Z_3, R_1, R_2, r_1 and r_2 are material parameters to be determined. D_0 is a parameter representing the limiting value of the plastic strain rate under shear, usually assumed to be equal to 10^{-4} $\sec^{-1}$, except for cases of very high strain values. n is a parameter related to the viscosity of dislocation motion and controlling the strain rate sensitively. m_1 and m_2 are material parameters controlling the hardening recovery rate; Z_1 is the saturation value for Z for high inelastic work, i.e. the maximum value of Z (a material constant). Z_2 is a parameter corresponding to the state of full thermal recovery. Z_3 is a variable characterizing the directional (anisotropic) hardening; R_1 and R_2 are parameters characterizing the hardening recovery rate; r_1 and r_2 are indices of the hardening recovery rate.

From equation (14) and (15), we can obtain:

$$\dot{\varepsilon}_{ij}^{p} = D_0\exp\left[-\frac{1}{2}\left(\frac{\left(Z^{I} + Z^{D}\right)^2}{3J_2}\right)^{n}\right]\frac{\sigma'_{ij}}{\sqrt{J_2}} \tag{20}$$

Substituting the relationship between matrix stress and the plastic strain in equation (16)-(19) into equation

(20), we can get the independent differential equations about the metal matrix stress. Then the main problem is reduced to three variables: matrix stress, the isotropic hardening variable and the directional-hardening variable. The equation (16), (18) and (20) that are related to these three variables can constitute a set of ordinary differential equations:

$$\begin{cases} \dot{\sigma}^m = f\left(\sigma^m, Z^I, \beta, T\right) \\ \dot{Z}^I = g^I\left(\sigma^m, Z^I, T\right) \\ \dot{\beta} = g^D\left(\sigma^m, \beta, T\right) \end{cases} \tag{21}$$

Given initial conditions, boundary conditions and a superposition of thermal and mechanical loads, equation (21) can be solved based on the classic fourth order Runge-Kutta method.

STRESS ANALYSIS OF TI MMCS UNDER THERMOMECHANICAL LOADING

A Ti-6Al-4V matrix composite reinforced with SCS-6 carborundum fibers was assumed to be stress free at a fabrication temperature of 900°C and then cooled down to room temperature, 25°C. The composite system analyzed consists of the titanium-based-alloy matrix with a 0.66 volume fraction, the fiber with 0.339 and the coating with 0.001. The coating behaviors are assumed as: the elastic modulus is equal to 200 GPa; the coefficient of thermal expansion is equal to 7.0×10^{-6}/°C. When the temperature drops from 900°C to 25°C, different thermo-residual stresses arise in the constituents of the material. The longitudinal stress in the matrix and coating is positive, and in the fiber is negative. The transverse stress in the matrix is positive, and in the coating and fiber is negative. The shearing stress is the highest in the fiber. For example, Figure 2 gives the variation of the longitudinal residual stress along with the temperature in different constituents of the material.

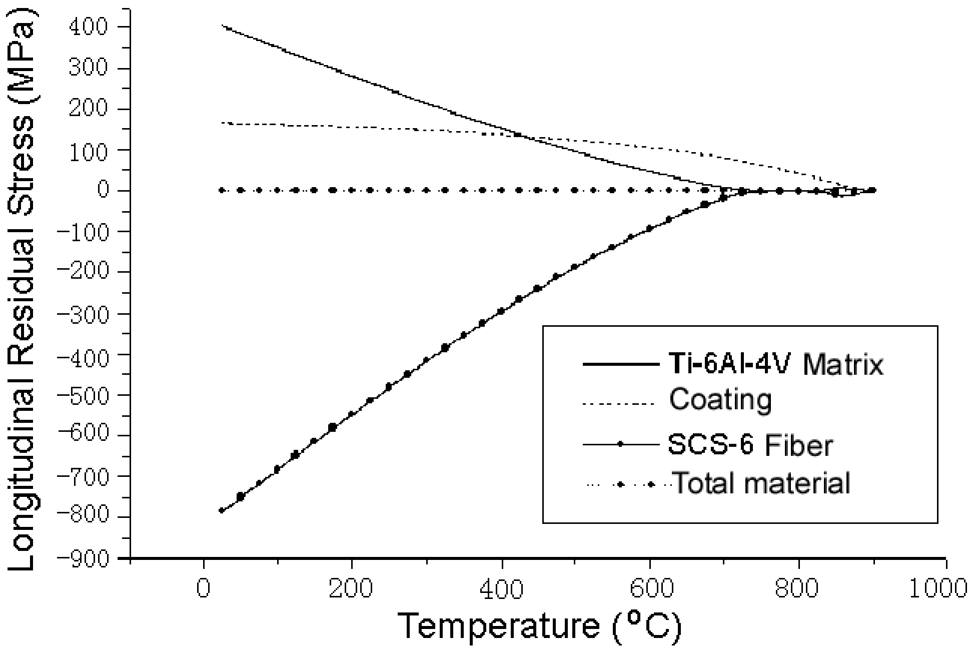

Figure 2: Variation of the residual stress along with the temperature

The mechanical behaviors of the material under thermomechanical loading are obtained by composing tow cycles in in-phase or out-of-phase: temperature cycle of 150~650°C and mechanical strain cycle of -0.004~0.004 in longitudinal direction. In the in-phase case, the maximum tensile strain coincides with maximum temperature. In the out-of-phase case, the maximum compressive strain occurs at maximum temperature. Three cycles are performed in calculation. Before le first cycle, the material is considered at the temperature of 25°C and the thermo-residual stress has exhibited in it. The results of calculation show that in the first cycle of loading there is an inelastic stage because of the thermo-residual stress in the material. The stability of stress-strain response emerged from the second cycle of loading since the plastic deformation is not great in the first cycle. Comparing the in-phase case with the out-of-phase case, one finds that the stresses of the material in the out-of-phase case are greater than that in the in-phase case. Figure 3a shows the longitudinal stress-strain response of the metal matrix from the first cycle to the third cycle in the out-of-phase case. Figure 3b shows the response of the total material in the same case.

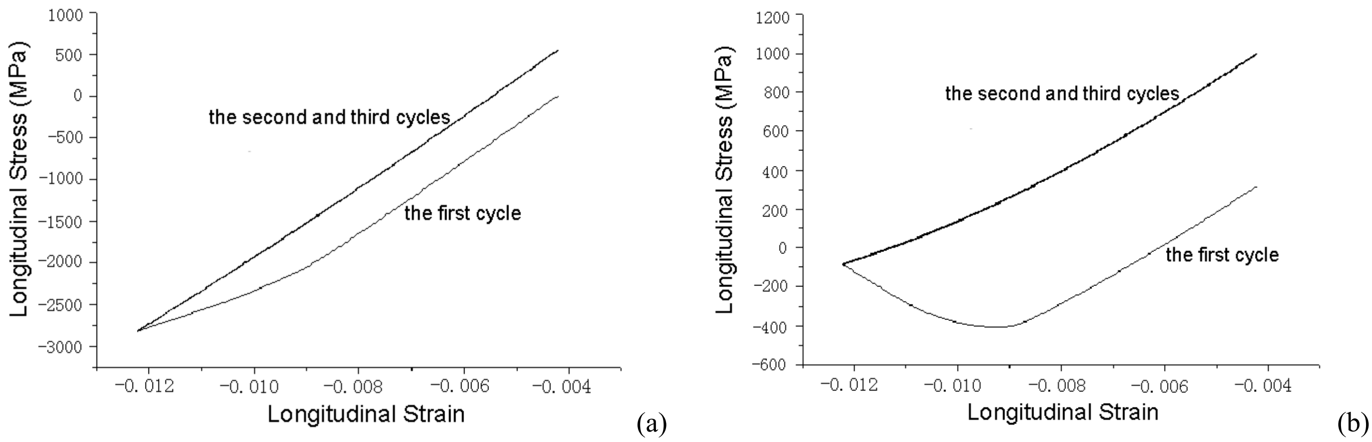

Figure 3: The stress-strain response in the out-of-phase case, (a) matal matrix, (b) total material

CONCLUSIONS

A three-layer micromechanical model of matrix/coat/fiber is used to analyze the stress-strain relationship of fiber-reinforces metal matrix composites under a superposition of thermal and mechanical loads. The concepts of average stress and strain are adopted and developed to integrate matrix, coating and fiber properties for predicting the response of unidirectional composites. The Bodner-Partom unified theory of viscoplasticity is applied to establish the basic constitutive models of metal matrix. The main problem then is reduced to a set of ordinary differential equations. In the application, three-layer model has adopted to analyze the stress-strain response of a titanium matrix composite. Through the calculation, first, the thermo-residual stresses in the constituents of the material are obtained, then, the mechanical behaviors are estimated under the loading of thermomechanical cycles in the in-phase and out-of-phase cases. The results should encourage the utilization of this model in the interface stress analysis and design of composite structures.

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