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MECHANICS-BASED MODELING OF DYNAMIC FRAGMENTATION AND COMPARISON WITH EXPERIMENTS

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ABSTRACT

We describe a new, largely analytical model of the dynamic fragmentation of brittle materials that accounts for the mechanics of the ongoing dynamic deformation and the development of material failure. The model predicts fragment size and time to fragmentation initiation in terms of fundamental material properties and the applied strain rate, in situations where the deformation rate prior to fragmentation initiation can be treated as being uniform. The predictions of the model are compared to some recent careful experiments on dynamic fragmentation; the agreement is shown to be quite good, significantly better than that of two widely-adopted analytical models of dynamic fragmentation that are based on energy balance arguments and which treat dynamic fragmentation as an instantaneous event. Although more extensive theoretical-experimental comparisons are needed, for a wider range of materials and strain rates, the present initial comparisons imply that accounting for the mechanics of the ongoing dynamic deformation and development of material failure in dynamic fragmentation leads to more accurate predictions, and that such modeling need not be heavily numerically-based.

KEYWORDS

dynamic fragmentation; dynamic failure; brittle material; stress waves; characteristics.

INTRODUCTION

If a solid body is subjected to rapid energy input by, for example, impact with or by another body, impingement of a laser or x-ray beam, rapid temperature change, etc., it is often observed that the body will shatter into numerous pieces. Reliable safety assessments of structural components that may be subject to such rapid energy input demand accurate models of this dynamic fragmentation phenomenon, so that key information such as fragment size can be predicted. To date, the most widely applied theoretical models of dynamic fragmentation appear to be based on relatively simple global energy balance arguments to predict fragment size, such as the pioneering study of Grady [1] or its improvement by Glenn and Chudnovsky [2]. Such models assume that the fragmentation event occurs instantaneously, and thus are expected to be limited in validity to situations involving extremely high strain rates.

We are interested in developing models of the dynamic fragmentation process that are analytical to the extent possible, but which also account for the actual time-varying dynamic deformation and the mechanics of the development of material failure that occurs before the final fragmentation event. Not only should such models be applicable over a wider range of applied strain rates, but they should reflect more accurately the actual mechanics of the failure process and hence provide more accurate predictions of fragment size and other pertinent quantities.

The present paper describes one such mechanics-based model of the dynamic fragmentation process for brittle materials that we have recently developed (Drugan [3]), and compares its predictions, together with those of the Grady [1] and Glenn and Chudnovsky [2] models, with the results of the careful impact fragmentation experiments of Piekutowski [4].

MECHANICS-BASED DYNAMIC FRAGMENTATION MODEL

Drugan [3] proposed and analyzed two models of the dynamic fragmentation process in brittle materials: one that treats initially “unflawed” material, and one that treats material containing a distribution of flaws. Here we review the simpler “unflawed” material model. As will be seen, this model assumes that flaws develop at the locations and times predicted by a dynamic instability analysis.

We begin with perhaps the simplest realistic assumption for dynamic fragmentation analysis, as proposed by Grady [1]: a body that is experiencing, due to a rapid energy input that has just occurred, a uniform constant volumetric strain rate that persists until fragmentation initiates. We focus attention on a prospective fragment inside this straining body, perform a dynamic analysis of the ongoing stress and deformation state in the prospective fragment and ask when this deformation becomes unstable, signaling the initiation of fragmentation and thus determining the critical fragment size and time of fragmentation initiation as a function of material properties.

Since we here restrict analysis to brittle fragmentation, we model the material as being homogeneous, isotropic linear elastic but, in addition, we model the surfaces that will become the fracture surfaces by cohesive zones which incorporate a realistic traction-separation relation for fracturing brittle material. For simplicity, we idealize the prospective fragment as being a cube that is connected to the surrounding material in the body by cohesive zones, as illustrated in Figure 1. The advantage of doing this is that, given our assumption of a uniform volumetric strain rate until fragmentation initiation, one observes that the stress and deformation history in each of the three cube normal directions will be identical, and therefore it is sufficient to analyze one of these – i.e., the three-dimensional problem is reduced to being one-dimensional.

We begin by analyzing the dynamic stress and deformation fields within the elastic cube. The three-dimensional version of Hooke’s law for a homogeneous, isotropic linear elastic solid is

$$\sigma_{ij} = \frac{E}{1+\nu}\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right), \tag{1}$$

where E is Young’s modulus, ν is Poisson’s ratio, σ_{ij} and ε_{ij} are components of the stress and infinitesimal strain tensors, respectively, δ_{ij} is the Kronecker delta, and a repeated index implies summation. Using the fact that $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33}$ and the shear strains are all zero, due to our uniform volumetric strain rate assumption, the normal stress-normal strain relation parallel to any cube axis has the form, from (1),

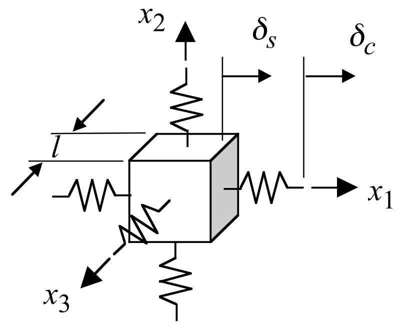

Figure 1. A prospective fragment, consisting of an elastic cube with (initially undetermined) side length l, and half cohesive zones that connect it to the remainder of the body. Displacement of the cube edge relative to its center is denoted by δ_s, and of the half cohesive zone relative to the cube edge by δ_c.

choosing the x_1-direction without loss of generality

$$\sigma_{11} = \hat{E}\varepsilon_{11} = \hat{E}\frac{\partial u_1}{\partial x_1}, \quad \text{where} \quad \hat{E} \equiv \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}, \tag{2}$$

an effective one-dimensional tensile modulus $\hat{E}$ being defined as indicated, and we have employed the relation of the normal infinitesimal strain component to the spatial derivative of the displacement. Substitution of (2) into the x_1-direction equation of conservation of linear momentum gives (when no body forces act) the governing equation for the u_1-component of the displacement field in the elastic cube:

$$c^2\frac{\partial^2 u_1}{\partial x_1^2} = \frac{\partial^2 u_1}{\partial t^2}, \quad \text{where} \quad c \equiv \sqrt{\frac{\hat{E}}{\rho}}, \tag{3}$$

and ρ is mass density. Identical equations apply for the other two coordinate directions, so as observed earlier, it is sufficient to analyze this one direction. Equation (3) has the general solution

$$u_1(x_1,t) = f(x_1 - ct) + g(x_1 + ct), \tag{4}$$

where f and g are arbitrary functions, to be determined by the boundary and initial conditions, and c is the elastic wave speed defined in (3).

We seek the equation governing the half cohesive zone displacement δ_c; this is accomplished by enforcing on (4) the initial conditions in accord with our assumption of a constant and uniform strain rate $\dot{\varepsilon}_0$, and the boundary conditions of normal stress and displacement continuity, at the cube edges, with the cohesive zones. The latter make use of our assumption that the total strain rate (in each direction) of the elastic cube plus its half cohesive zones remains the constant $\dot{\varepsilon}_0$ before fragmentation. One can eliminate all other unknowns to obtain the following governing equation for δ_c (see Drugan [3] for details of the analysis):

$$\boxed{\delta_c'(t) + \frac{1}{\tau}\delta_c(t)\exp\left[1 - \frac{2}{\delta^*}\delta_c(t)\right] = c\,\dot{\varepsilon}_0\,t, \quad \delta_c(0) = 0, \quad \text{valid for } 0 \le t \le \frac{l}{c}}, \tag{5}$$

where $\tau \equiv \delta^*\hat{E}/(2c\sigma_{\max})$ is a constant having time dimensions. In deriving (5), we have employed the normal traction/normal separation relation for the cohesive zones suggested by the *ab initio* atomistic calculations of Rose et al. [5,6]:

$$\sigma_c(2\delta_c) = \sigma_{\max}\frac{2\delta_c}{\delta^*}\exp\left(1 - \frac{2\delta_c}{\delta^*}\right), \tag{6}$$

where σ_{max} is the strength of the cohesive surface, which is attained when $2\delta_c = \delta^*$. Observe that (5) is valid only for times sufficiently short that an elastic wave reflection does not take place; in situations where fragmentation initiation takes longer than this, wave reflections must be accounted for and (5) is modified; see Drugan [3] for details. We here restrict discussion to situations (i.e., to sufficiently high strain rates) in which fragmentation initiation occurs before any elastic wave reflections occur, so that (5) is valid.

There are two unknowns to be determined: the time to fragmentation initiation, and the minimum fragment size. We thus impose two conditions to determine these unknowns: First, we hypothesize that fragmentation initiation will occur at the time t_{cr} when the stress level in the cohesive zone has just attained the cohesive strength σ_{max}. In terms of the cohesive zone displacement, this requires [see (6)]

$$2\delta_c(t_{cr}) = \delta^*. \qquad (7)$$

Second, the instability condition determining the minimum possible fragment size, $l_{\min}$, is the condition that at time t_{cr}, the prospective brittle fragment, which is the elastic cube, has just stopped expanding. This requires, using the assumption of uniform strain rate until fragmentation initiation $\delta_s'(t) + \delta_c'(t) = \dot{\varepsilon}_0 l/2$:

$$\delta_s'(t_{cr}) = 0 \quad \Rightarrow \quad \delta_c'(t_{cr}) = \tfrac{1}{2}\dot{\varepsilon}_0 l_{\min}. \qquad (8)$$

For the very high strain rate regime considered here, defined by the condition that fragmentation initiates for $t_{cr} \leq l_{\min}/c$, (5) applies. [For lower strain rates not satisfying this condition, the modified analysis is given in Drugan [3].] Thus, combining (7) and (8) and using (5) to express $\delta_c'(t)$, we find the minimum fragment size to be:

$$l_{\min} = 2c\left(t_{cr} - \frac{1}{\dot{\varepsilon}_0}\frac{\sigma_{\max}}{\hat{E}}\right), \qquad \text{valid for } 0 \leq t_{cr} \leq \frac{l_{\min}}{c}, \qquad (9)$$

where t_{cr} is obtained by solution of (7) [which in turn requires the numerical solution of (5)]. This can be done once and for all in terms of nondimensionalized quantities; Drugan [3] did this, and found that the resulting predictions for minimum fragment size and time to fragmentation initiation are very accurately fit by the following equations in the very high strain rate regime:

$$\boxed{\tilde{l}_{\min} = 2.1395\tilde{\dot{\varepsilon}}_0^{-0.4264}, \quad \tilde{t}_{cr} = 2.0184\,\tilde{\dot{\varepsilon}}_0^{-0.60016}, \quad \text{valid for } 0 \leq \tilde{t}_{cr} \leq \tilde{l}_{\min}.} \qquad (10)$$

Here, the nondimensionalized quantities are defined as:

$$\tilde{l}_{\min} = \frac{2(1-2\nu)e\sigma_{\max}^2}{(1-\nu)^2 K_{Ic}^2} l_{\min}, \quad \tilde{t}_{cr} = \frac{2(1-2\nu)ce\sigma_{\max}^2}{(1-\nu)^2 K_{Ic}^2} t_{cr}, \quad \tilde{\dot{\varepsilon}}_0 = \frac{(1-\nu)^2 \hat{E} K_{Ic}^2}{2(1-2\nu)ce\sigma_{\max}^3}\dot{\varepsilon}_0, \qquad (11)$$

where all quantities are defined previously except K_{Ic}, the plane strain fracture toughness and e, the natural logarithm base.

REVIEW OF PREVIOUS ENERGY-BALANCE MODELS

A pioneering and widely adopted model of dynamic fragmentation in brittle materials is due to Grady [1]. One postulate of this model, which we have adopted here as noted above, is that a body is initially in a state of rapid uniform expansion – i.e., the strain rate is constant throughout the body. Grady focused attention on a body portion that would ultimately become a fragment, and decomposed its total kinetic energy into two parts: that of the total mass moving with the center of mass of the body portion, and that associated with relative motion of material particles with respect to the body portion's mass center. He postulated that the latter part of kinetic energy is available to drive fragmentation, and predicted fragment size by equating this energy to that required for the total surface energy to form a new fragment, tacitly assuming fragmentation to be instantaneous. For cubic fragments as modeled here, the Grady prediction of fragment size is

$$l = 24^{\frac{1}{3}}\left(\frac{K_{Ic}}{\rho c\dot{\varepsilon}_0}\right)^{\frac{2}{3}}. \tag{12}$$

Glenn and Chudnovsky [2] improved the Grady [1] model by noting that in addition to the local kinetic energy of a prospective fragment, its stored elastic energy is also available for new fracture surface formation. Application of this more complete energy balance argument demands the additional requirement that no fragmentation occurs until the stress state attains a critical value, σ_{max}, at which level instantaneous fragmentation is postulated to occur. The Glenn and Chudnovsky [2] prediction of fragment size is

$$l = 4\sqrt{\frac{\alpha}{3}}\sinh\left(\frac{\phi}{3}\right), \tag{13}$$

where

$$\alpha \equiv \frac{3\sigma_{max}^2}{\hat{E}\rho\dot{\varepsilon}_0^2}, \quad \beta \equiv \frac{3}{2}\left(\frac{K_{Ic}}{\rho c\dot{\varepsilon}_0}\right)^2, \quad \phi \equiv \sinh^{-1}\left[\beta(3/\alpha)^{\frac{3}{2}}\right]. \tag{14}$$

COMPARISON OF THE PRESENT AND ENERGY BALANCE MODELS WITH EXPERIMENT

It should first be observed that it would be difficult to conduct an experiment in which the assumption made by all the analytical models discussed here, namely that a body experiences a uniform and constant strain rate until fragmentation initiation, was precisely met everywhere. However, a set of careful experimental results exist which appear, on the basis of the flash radiographic records, to correspond to a quite uniform fragmentation event, so that our assumption should be met at least in a large portion of the fragmenting body. These are the experimental results of Piekutowski [4]. The experiments involved the high-speed impact of 2017-T4 aluminum spheres with 6061-T6 aluminum plates, causing dynamic fragmentation of the aluminum spheres. Here we compare the measured fragment sizes from the experiments with the predictions of the energy-based models and our new model. The material properties for the 2017-T4 aluminum spheres are:

$$E = 73 \text{ GPa}, \quad \nu = 0.33, \quad \rho = 2791 \text{ kg/m}^3, \quad \sigma_{max} = 430 \text{ MPa}, \quad K_{Ic} = 20 \text{ MPa}\sqrt{\text{m}}. \tag{15}$$

(The material property values are from Kishida and Nakagawa [7], whose dynamic fracture experiments showed a substantial reduction in fracture toughness of this alloy at high strain rates; $K_{Ic} \approx 20$ MPa$\sqrt{\text{m}}$ is a lower bound to their high-strain-rate data.) The strain rates in the experiments of Piekutowski [4] ranged from 0.9 to 2.6 × 10^5/s; he reported only the strain rates for these two extreme tests. Table 1 shows the experimental results from these two tests, together with the predictions of the present model [Equations (10) and (11)], the Grady model [Equation (12)], and the Glenn and Chudnovsky model [Equations (13) and (14)]. Note that, although the present model predicts minimum fragment size, when the strain rate is uniform and material properties variation is neglected, as assumed, all fragments are the same size, so the minimum and average fragment sizes coincide. (The fact that all fragments are not the same size in the experiments shows that our two assumptions are not precisely met.) Also, it is easy to verify that the strain rates in the experiments are in the “high strain rate regime”, that is, they satisfy the needed inequality in (10).

$\dot{\varepsilon}_0$, s^{-1}	l, mm (Piekutowski experiments)	l, mm (present model)	l, mm (Grady model)	l, mm (Glenn & Chudnovsky model)
0.9×10^5	0.814	0.853	1.58	1.39
2.6×10^5	0.506	0.543	0.778	0.731

Table 1. Comparison of experimental results of Piekutowski [4] for the average measured fragment size with the theoretical predictions of the present model [Equations (10), (11)], the Grady model [Equation (12)], and the Glenn and Chudnovsky model [Equations (13), (14)].

DISCUSSION

The comparison in Table 1 of the three dynamic fragmentation models with the experimental data of Piekutowski [4] shows that the present, mechanics-based dynamic fragmentation model agrees much more closely with the experimental measurements than do the two energy-based models which ignore the mechanics of the dynamic fragmentation process. As noted above, the energy-based models assume an instantaneous fragmentation event, so their predictions would be expected to be less accurate the lower the applied strain rate; this is clearly reflected in the comparisons in Table 1, which also shows that the accuracy of the present model changes little with changing strain rate. It should be emphasized that none of these three models involves any "free" parameters: all make specific predictions based only on the set of material constants listed above. Although the comparisons presented here involve a limited set of experimental results, restricted to a single material and a somewhat limited range of strain rates, and much further experimental-theoretical comparison should be conducted, these initial comparisons indeed appear to confirm the importance of accounting for the actual time-varying dynamic deformation and the mechanics of the dynamic failure process in the accurate modeling of dynamic fragmentation.

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